

\*Research supported in part by the National Research Council of Canada and the Ontario Department of University Affairs.

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<sup>1</sup>E. A. Burke, Phys. Rev. 130, 1871 (1963) (referred to as B in this paper).

<sup>2</sup>S. Larsson, Phys. Rev. 169, 49 (1968) (referred to as L).

<sup>3</sup>R. F. Gentner and E. A. Burke, Phys. Rev. 176, 63

(1968).

<sup>4</sup>E. A. Burke, J. Math. Phys. 6, 1691 (1965). As pointed out in Ref. 31 of Ref. 3 the numerical values of  $l$  and  $j$  in Eq. (5) and  $L$  and  $J$  in Eq. (21) should be interchanged.

<sup>5</sup>H. M. James and A. S. Coolidge, Phys. Rev. 49, 688 (1936) (referred to as JC).

<sup>6</sup>H. M. James and A. S. Coolidge, Phys. Rev. 55, 873 (1939).

## Spin Orientation of Photoelectrons: Erratum and Addendum\*

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(Received 9 May 1969)

An effect overlooked in a recent calculation is pointed out and included in the theory. The resulting corrections amount to less than 1%.

A recent paper,<sup>1</sup> called I in the following and dealing with the alternative spin orientations of photoelectrons ejected by circularly polarized light, regrettably overlooked a well-known phase normalization requirement on the photoelectron wave functions. It seems worthwhile to rectify this error and evaluate its consequences, also because the calculation of I applies to a variety of experiments, one of which has been successfully performed.<sup>2</sup>

In calculations of the probability of alternative final channels for an outgoing particle, the outgoing large- $r$  wave-function term  $\exp(ikr)$  of a scattering eigenstate must have the same phase as the corresponding term of a free-particle wave function.<sup>3</sup> In our problem, this phase normalization allows for the influence of electron scattering by the atomic field and specifically for the influence of spin-orbit coupling after the photoabsorption proper. By disregarding this phase normalization, paper I took into account the spin-orbit coupling influence only upon the photoabsorption but not in later stages of the process. The latter influence is weak owing to the weakness of the coupling itself, but it would suffice to produce some spin polarization if the major influence upon photoabsorption were absent.

Appropriate phase normalization multiplies the real wave function  $P(\epsilon p j'; r)$  in (I1) by a factor  $\exp(i\delta_j')$ , where  $\delta_j'$  indicates the phase shift of  $P$ . We need not know the actual values of  $\delta$  for the alternative  $j'$  values but only their difference

$$\delta = \delta_{1/2} - \delta_{3/2} = \pi \Delta\tau \sim 0.1 \text{ rad}, \quad (1)$$

where  $\Delta\tau$  is defined by (I8). Renormalization of the wave function  $P$  causes factors  $\exp(i\delta_{3/2})$  and  $\exp(i\delta_{1/2})$  to be inserted in front of  $R_3$  and  $R_1$ , respectively, in (I3). The same insertion in (I4) prevents this equation from being satisfied exactly. The squares of matrix elements in (I5) must now be interpreted as absolute squares, owing to the complex character of the corrected matrix (I3); moreover, the real parameter  $x$  defined by (I6) must be replaced by the complex parameter

$$z(\epsilon) = \frac{x - ie^{-i\frac{1}{2}\delta} (\sin\frac{1}{2}\delta)^{\frac{2}{3}}(x-2)}{1 + ie^{-i\frac{1}{2}\delta} (\sin\frac{1}{2}\delta)^{\frac{2}{3}}(x-2)} \\ = \frac{x + (\sin\frac{1}{2}\delta - 3i \cos\frac{1}{2}\delta)(\sin\frac{1}{2}\delta)^{\frac{2}{3}}(x-2)(x+1)}{1 + (\sin\frac{1}{2}\delta)^{\frac{4}{3}}(x-2)(x+1)} \quad (2)$$

and the polarization formula (I5) itself becomes

$$P(z) = P(x, \delta) = (1 + z + z^*) / (2 + |z|^2) \\ = \frac{1 + 2x}{2 + x^2} + \frac{8}{9} \frac{(x-2)(x+1)}{2 + x^2} \sin^2\frac{1}{2}\delta. \quad (3)$$

These corrected results differ from those of I by terms proportional to  $\sin\frac{1}{2}\delta \sim 0.05$  or  $\sin^2\frac{1}{2}\delta \sim 0.002$ . The corrections also vanish at  $x=2$  and  $x=1$  where  $R_1$  and  $R_3$  vanish, respectively. (This result reflects the circumstance that the scattering eigenchannels with  $j' = \frac{1}{2}$  and  $j' = \frac{3}{2}$  no longer interfere when either of their probability amplitudes vanishes.) The maximum of  $P$  lies near

$x = 1$ , as in I, but falls a little short of unity. Conversely, we have  $P = (\frac{8}{9})\sin^2\frac{1}{2}\delta \neq 0$  for  $x = \pm\infty$ .

In principle, both  $x$  and  $\delta$  could be determined, as functions of the photoelectron energy  $\epsilon$ , by fitting (3) to sufficiently accurate measurements of  $P(\epsilon)$ . This procedure would be more difficult, of course, than the determination of  $x(\epsilon)$  suggested in I. In practice,  $\delta(\epsilon)$  can presumably be disregarded.

The calculation of I pertains to the average spin polarization of all photoelectrons, which are assumed to be collected irrespectively of their direction of emission. Different degrees of polar-

ization would, of course, be observed by collecting electrons ejected in limited solid angles, except in the full polarization limit  $P = 1$ . [This limit may be unattainable; e. g.,  $(x = 1, \delta = 0)$  would yield  $P = 1$  but  $\delta = 0$  implies  $\Delta\tau = 0$  in (I18) and hence  $x = \infty$  except for  $\bar{R}_O = 0$ .] To calculate the polarization of photoelectrons ejected at an angle  $\theta$  from the light beam, the diagonal and off-diagonal elements of (I3) should be multiplied by  $\sin\theta \exp(i\varphi)$  and  $2^{1/2}\cos\theta$ , respectively.

I thank Howard Stewart for calling my attention to the oversight in paper I.

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\*Work supported in part by the U. S. Atomic Energy Commission Contract No. COO-1674-19.

<sup>1</sup>U. Fano. Phys. Rev. 178, 131 (1969).

<sup>2</sup>M. S. Lubell and G. W. Raith, Proceedings of the Sixth International Conference on the Physics of Electronic and Atomic Collisions, Cambridge, 1969 (to be published). I thank these authors for advance communication of their results.

<sup>3</sup>Textbooks and journal articles have discussed the continuum functions appropriate to alternative boundary conditions from different points of view. See, e. g., G. Breit and H. A. Bethe, Phys. Rev. 93, 888 (1954); L. D. Landau and E. M. Lifshitz, Quantum Mechanics (Pergamon Press, Inc., New York, 1965), 2nd ed., Sec. 134. Briefly, each spherical wave function has,

at large  $r$ , outgoing wave terms proportional to  $\exp(ikr)$  and ingoing terms proportional to  $\exp(-ikr)$ . States of particles emerging from a reaction in a specified channel have wave functions called  $\psi^-$  with the phase of  $\exp(ikr)$  matching that of a free-particle wave, the phase of  $\exp(-ikr)$  being thereby fixed indirectly. On the other hand, states of particles incident on a target from a specified channel have wave functions  $\psi^+$  with the phase of  $\exp(-ikr)$  matching that of a free-particle wave. The distinction between  $\psi^+$ ,  $\psi^-$ , and real standing wave  $\psi$  is irrelevant to calculations of total reaction probabilities with summation over final channels. To verify this, calculate the total cross section from the matrix (I3) with the added factors  $\exp(i\delta_{j'})$ ; these factors will cancel out.