the Thomas precession term cancels exactly half of the Dirac moment term in the $\vec{v}_1 \times \vec{E}_1$ expression while leaving alone the Pauli term, resulting in the expression for $\mathcal{H}_2$ in Eq. (7).

[19] Of course, the terms involving the canonical momentum $\vec{p}$ do not contribute here to the field-dependent energy, but are included in $\mathcal{H}_2$ and $\mathcal{H}_3$ to emphasize the interpretation of these terms as spin-orbit and spin-orbit interactions. In the case of molecules, however, terms involving $\vec{p}$ contribute due to the departure from spherical symmetry (see Ref. 12).

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PHYSICAL REVIEW VOLUME 184, NUMBER 1 5 AUGUST 1969

X-Ray Yields in the K and L Series of Low-Z Muonic Atoms

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(Received 12 May 1969)

Experiments are reported which remove the discrepancies between the earlier observed K and L x-ray yields from low-Z muonic atoms and those calculated from theory. We have made direct measurements of the K/L yield ratios for targets of $4 \leq Z \leq 8$ and a determination of the absolute K x-ray yield for $Z = 3$. They agree excellently with the calculated values.

I. INTRODUCTION

A long standing discrepancy in the field of muonic x rays was the anomalously low yield of x rays with energies below approximately 100 keV.[1,2] In muonic atoms, this corresponds to $Z < 8$ for μ-K x rays and $Z < 15$ for μ-L x rays. Because of the low energies of these x rays, the pulses produced in the sodium-iodide detectors used in the experiments would necessarily be small, and could give rise to electronic inefficiencies in the coincidence circuits which might produce such effects. Realizing this, the investigators took many precautions to greatly amplify the low-energy pulses and tested for experimental inefficiencies by using low-energy γ rays from radioactive sources. These tests indicated that no apparent inefficiencies were introduced which might cause coincidence losses due to time jitters resulting from clipping the sodium-iodide pulses or inappropriate discriminator settings. However, the yields given in Ref. 2 were somewhat higher than those given in Ref. 1, although both were considerably lower than the calculated yields.

When a new type of fast discriminator was developed which used the crossover point of an amplifier pulse rather than the front end of the pulse to generate a coincidence pulse, we decided to repeat the yield measurements for low Z materials.[3] Using this new apparatus we found *no* discrepancy between theory[4] and experiment. Furthermore, the experimental ratios of the higher transition yields (i.e., K_β to K_∞) can be compared to theory and they favor an initial capture distribution which is more strongly peaked than the $2l + 1$ distribution.

EXPERIMENTAL PROCEDURE

The experiment was performed with the muon beam from the Chicago cyclotron. The experimental arrangement is essentially the same as in Refs. 1 and 2 except for the coincidence instrumentation and the refinement of the detection apparatus. The pulse from the sodium-iodide crystal was amplified by a Model-101 nonoverload linear amplifier and sent into a Model-501 fast discriminator which generated a fast coincidence

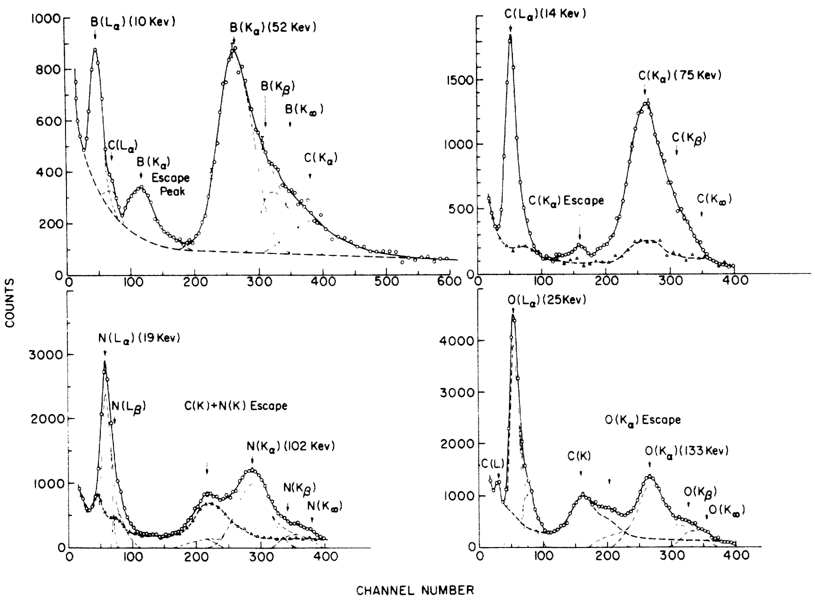

FIG. 1. Some typical μ-K and μ-L spectra. The decomposition of the curves into their various components is shown for B, C, N, and O, also shown by dashed lines are the background runs for C, N, and O. No background run was taken for B, there the dashed background curve is just the assumed value.

pulse from the crossover point of the amplifier pulse. The fast coincidence circuit used was a Model 1401. The manufacturer of all three units was Stirrup, Inc., since taken over by Camberra Industries.

RESULTS

We measured the absolute x-ray yield only for the Li (K) x rays. For the other elements, $4 \leqslant Z \leqslant 8$, we set the gain of the nonoverload amplifier so that both K and L x rays were recorded on the pulse-height analyzer, simultaneously. Thus, we measured the ratio of the total K x rays to that of the total L x rays taken under identical conditions. Figure 1 shows some typical data. In all cases, except for boron, background runs were also taken with the target material removed in order to determine carbon contamination from other sources. The background runs are shown as dotted lines with triangular data points in Fig. 1.

Table I summarizes the experimental results and theoretical calculations and these are plotted in Figs. 2 and 3. Corrections were made, where applicable, for self-absorption in the targets and target holder (where there was one), absorption in the Be window of the NaI counter, air absorption, escape corrections, and crystal efficiency. The self-absorption corrections were determined with radioactive sources. Many of the elements were run with a variety of thicknesses of the target in order to get more accurate results. The errors on the experimental results are not statistical but estimates of the accuracy from all effects including reproducibility of the ratios obtained in various runs for the different thickness targets. The theoretical values listed in Table I are those calculated by Eisenberg and Kessler[4] for two possible initial capture distributions of the muon. The first distribution is derived from the assumption that muons are captured in the $n = 14$ level with initial statistical populations having weights

TABLE I. Summary of experimental results and theoretical calculation of low-Z muonic x-ray yields.

	Energy (keV)		$\frac{K_{\text{total}}}{L_{\text{total}}}$					$\frac{\text{Higher } K}{\text{All } K}$	$\frac{\text{Higher } L}{\text{All } L}$
			Experiment			Theory			
Element	K_α	L_α	Reference 1	Reference 2	Present	$(2l+1)$	$(2l+1)^{0.2l}$		
Be	33.3	6.2			3.1 ± 0.3	3.25	3.2	0.17 ± 0.03	0.22 ± 0.04
B	52.1	9.6			3.1 ± 0.8	2.6	2.4	0.16 ± 0.05	
C	75.0	13.9		6.0	2.0 ± 0.2	2.2	1.85	0.14 ± 0.03	0.20 ± 0.03
N	102	19.0		3.0	1.5 ± 0.2	1.95	1.6	0.16 ± 0.03	0.23 ± 0.03
O	133	25.0	5.7		1.5 ± 0.2	1.9	1.4	0.21 ± 0.04	0.23 ± 0.03
Li	18.7		Absolute K yield = 0.83 ± 0.03			0.81	0.85	0.14 ± 0.03	

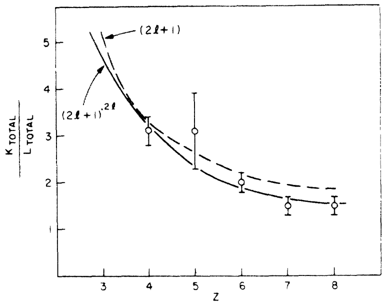

FIG. 2. Comparison of experimental and calculated ratios of K/L x-ray yields. The curves are the calculated ratios of Ref. 4 for the initial capture population distributions of $2l+1$ and $(2l+1)^{0.2l}$.

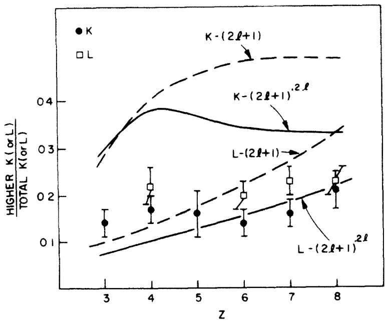

FIG. 3. Comparison of experimental and calculated ratios of the higher transition to the total transition yield.

proportional to $2l+1$. The second distribution is more peaked toward higher l values with statistical weights of $(2l+1)^{0.2l}$. As can be seen, the absolute K x-ray yield of lithium, 0.83 ± 0.03, agrees excellently with either calculated absolute value. Figure 2 shows the comparison between experiment and theory for the ratio of the K/L x-ray yields. Whereas, the ratios are not strongly dependent on the initial capture distribution, the more highly peaked l values (distribution 2) seem to be slightly preferred by experiment. Figure 3 shows the comparison of experimental and calculated ratios of higher transitions yields to the total yield. We see that the experimental data do not agree very well with either distribution, although the flatter nature of the $(2l+1)^{0.2l}$ distribution is closer in character to the observed data.

These measurements have removed the former discrepancies between experiment and theory which seem to have been caused by inefficiencies in the coincidence arrangement. The inefficiencies were apparently caused by a combination of three experimental conditions: duty cycle effects, time jitters arising from using the front end of the NaI pulses, and a bias against small pulses introduced by the pulse-height analyzer which records the largest pulse entering during the coincidence interval. The present data appear to favor a distribution in the initial capture states which is weighted toward the higher l values more strongly than $2l+1$.

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