Comments on the K^+-K^0 Electromagnetic Mass Shift

R. N. CHAUDHURI AND J. S. VAISHYA*
Centre for Advanced Study in Physics, University of Delhi, Delhi-7, India
(Received 30 December 1968; revised manuscript received 25 February 1969)

We analyze the K^+-K^0 electromagnetic mass difference using (i) hard-kaon current algebra of chiral $SU(3)\times SU(3)$ [with a symmetry-breaking term which transforms as $(3, 3^*)\oplus(3^*, 3)$] and (ii) dispersion theory in conjunction with soft-kaon current algebra. The results differ in that the K^* intermediate state appears in the latter approach but not in the former. Its contribution is analyzed in detail and shown to have the correct sign. From our analysis we conclude that the mass shift cannot be of the correct sign and magnitude for a reasonable value of the cutoff unless the contribution of the σ term is large. We also comment on Riazuddin and Sarker's analysis for the VVP vertex which appears in our calculations.

THE calculation of the electromagnetic mass shift of hadrons is complicated by the presence of strong interactions among hadrons. To incorporate the strong-interaction effects, various attempts have been made with approximations by low-energy phenomenology and by other means, but it still remains difficult to understand the $\Delta I=1$ electromagnetic mass differences, e.g., $m_{K^+}-m_{K^0}$, m_p-m_n, etc. Here we make an attempt to analyze the mass shift of kaons within the framework of once-subtracted dispersion theory under the assumption that the dispersion integrals get saturated only by low-lying intermediate states, namely, K, K_A, and K^*. The resulting expression contains, apart from the subtraction term,[1] many form factors. At present, we do not have enough experimental knowledge about these form factors for analysis. One way out appears through the use of the hard-meson current-algebra results for these form factors.[2] These results are obtained under the assumptions of local chiral $SU(3)\times SU(3)$ algebra of vector and axial-vector currents, single-particle dominance, smoothness of three-point primitive functions, and a symmetry-breaking term in the Hamiltonian transforming as $(3, 3^*)\oplus(3^*, 3)$. The subtraction term is evaluated from the soft-kaon current-algebra analysis. We have compared the final expression of the mass shift with that derived through the hard-kaon current-algebra analysis of four-point functions.[3] We find the two to be identical[4] except that the latter contains no contribution due to the K^* intermediate state. Thus, the mass difference is found to be logarithmically divergent in both approaches and to be dependent on the K-K_A mixing parameter δ, the F_K/F_π ratio, and the scalar tadpole (σ_3) contribution. We have analyzed the VVP vertex through the Ward-identity approach and have commented on Riazuddin and Sarker's analysis.[5] We find that the K^* contribution is convergent and small, but has the right sign in contradistinction to the previous analysis of Socolow[6] based on perturbation theory. This happens because part of the subtraction term gets mixed with the K^*-pole term.

The electromagnetic mass difference of kaons to first order in α is given by[7]

$$m_{K^+}{}^2-m_{K^0}{}^2=2m_K\delta m_K$$
$$=\frac{\alpha i}{(2\pi)^3}\int\frac{d^4q}{q^2}\left(g_{\mu\nu}-\lambda\frac{q_\mu q_\nu}{q^2}\right)T_{\mu\nu}(k,q)\,,\quad(1)$$

where $T_{\mu\nu}(k,q)$ is the virtual-photon–kaon scattering amplitude:

$$T_{\mu\nu}(k,q)=i\int d^4x$$
$$\times e^{-iq\cdot x}[\langle K^+,k|T(V_\mu{}^{\rm em}(x),V_\nu{}^{\rm em}(0))|K^+,k\rangle-(K^+\to K^0)]$$
$$=g_{\mu\nu}F_1+q_\mu q_\nu F_2+(q_\mu k_\nu+q_\nu k_\mu)F_3+i(q_\mu k_\nu-k_\mu q_\nu)F_4+k_\mu k_\nu F_5\,.\quad(2)$$

Here the F_i's are functions of q^2 and ν, with $\nu=-q\cdot k$. The gauge invariance of $T_{\mu\nu}(k,q)$ demands

$$F_1+q^2F_2=\nu F_3\,,\quad q^2F_3=\nu F_5\,,\quad\text{and}\quad F_4=0\,.\quad(3)$$

Thus only two of the F_i's are independent. If we write unsubtracted dispersion relations for all F_i, we would, in general, fail to obtain a gauge-invariant result for $T_{\mu\nu}(k,q)$. This difficulty is well known,[8] and it can be overcome by introducing one subtraction in the dis-

---

* Permanent address: Basic Physics Division, National Physical Laboratory, New Delhi-12, India.

[1] H. Harari, Phys. Rev. Letters **17**, 1303 (1966).

[2] I. S. Gerstein, H. J. Schnitzer, and S. Weinberg, Phys. Rev. **175**, 1873 (1968); I. S. Gerstein and H. J. Schnitzer, *ibid.* **175**, 1876 (1968), and references quoted therein; see also S. Weinberg and S. L. Glashow, Phys. Rev. Letters **20**, 224 (1968).

[3] Under the assumption of meson dominance, I. S. Gerstein and H. J. Schnitzer [Phys. Rev. **170**, 1638 (1968)] have obtained the results for the four-point functions of current-generating $SU(2)\times SU(2)$ algebra. These can be generalized to $SU(3)\times SU(3)$ algebra following closely the method of Ref. 2. This current-algebra analysis for the electromagnetic mass shift does not take into account the contribution of the vector-meson intermediate states (see Ref. 4). However, in principle, this should have been included.

[4] Similar observations have been made by K. C. Gupta and J. S. Vaishya [Phys. Rev. **176**, 2125 (1968)] in the context of $\pi^+-\pi^0$ mass shift. See also J. S. Vaishya, *ibid.* **177**, 2512 (1969).

[5] Riazuddin and A. Q. Sarker, Phys. Rev. Letters **20**, 1455 (1968).

[6] R. S. Socolow, Phys. Rev. **137B**, 1221 (1965).

[7] Riazuddin, Phys. Rev. **114**, 1184 (1959); V. Barger and E. Kazes, Nuovo Cimento **28**, 385 (1963).

[8] Unsubtracted dispersion relations for $F_{1,2}$ can lead to a contradictory result at zero photon energy. See, e.g., H. Pagels, Phys. Rev. Letters **18**, 316 (1967); H. Harari, *ibid.* **18**, 319 (1967).

persion relations for F_1 and F_2 at $\nu=0$. For fixed q^2, we assume that[4]

$$F_2(\nu,q^2)-F_2(0,q^2)=\frac{\nu}{\pi}\int\frac{\mathrm{Im}F_2(\nu',q^2)}{\nu'(\nu'-\nu)}d\nu' \tag{4}$$

and

$$F_5(\nu,q^2)=\frac{1}{\pi}\int\frac{\mathrm{Im}F_5(\nu',q^2)}{\nu'-\nu}d\nu'. \tag{5}$$

We now saturate the dispersion integrals by the pole contributions due to K, $K_A(1320)$ or Q, and $K^*(890)$, and evaluate the right-hand sides of Eqs. (4) and (5). In doing so, we define the following vertices:

$$\langle K^a,k|V_\mu{}^c(0)|K^b,k+q\rangle=2if^{abc}F_c(q^2)(2k+q)_\mu\,,$$
$$\langle K^a,k|V_\mu{}^c(0)|K_A{}^b,k+q,\epsilon_Q\rangle$$
$$=2f^{abc}\{[g_{\mu\lambda}((q+k)^2-k^2)+(2k+q)_\mu k_\lambda]C_c(q^2)+[g_{\mu\lambda}q^2+q_\mu k_\lambda]D_c(q^2)\}\epsilon_Q{}^\lambda(q+k)\,, \tag{6}$$

and

$$\langle K^a,k|V_\mu{}^c(0)|K^*,k+q,\epsilon_{K^*}\rangle=2d^{abc}f_c(q^2)\epsilon_{\lambda\mu\alpha\beta}q^\alpha k^\beta\epsilon_{K^*}{}^{*\lambda}(k+q)\,,$$

where F_c, C_c, D_c, and f_c are functions of q^2. Using the definitions from above, we calculate the right-hand sides of (4) and (5), and obtain

$$F_2(\nu,q^2)-F_2(0,q^2)=\nu\Bigg(\frac{F_\rho F_8}{2\nu_K(\nu_K-\nu)}+\frac{m_K{}^2}{6}\frac{f_\rho f_8}{\nu_{K^*}(\nu_{K^*}-\nu)}+\frac{1}{2\nu_Q(\nu_Q-\nu)}\times[\nu_Q{}^2m_Q{}^{-2}(C_\rho-D_\rho)(C_8-D_8)+2\nu_Q(2C_\rho C_8+C_\rho D_8+D_\rho C_8)-q^2(C_\rho+D_\rho)(C_8+D_8)]\Bigg)+(\nu\to-\nu) \tag{7}$$

and

$$F_5(\nu,q^2)=\Bigg(\frac{2F_\rho F_8}{\nu_K-\nu}-\frac{q^2}{6}\frac{f_\rho f_8}{\nu_{K^*}-\nu}+\frac{1}{2(\nu_Q-\nu)}[4C_\rho C_8q^2+q^4m_Q{}^{-2}(C_\rho-D_\rho)(C_8-D_8)]\Bigg)+(\nu\to-\nu)\,, \tag{8}$$

where the subscript 8 stands for the eighth component of the vector-meson octet and $\nu_A=\frac{1}{2}(q^2+m_A{}^2-m_K{}^2)$.

However, note that we do not have enough information about the form factors[9] appearing in (7) and (8). To overcome this difficulty, we make use of the current-algebra analysis for three-point functions of currents and their divergences under the assumption of meson dominance, which has given reasonably good results for many decay widths and form factors.[10] Thus, we use[11]

$$F_c(q^2)=\frac{1}{2(q^2+m_c{}^2)}\Bigg(\frac{g_Q{}^2m_c{}^2}{F_K{}^2m_Q{}^4}q^2(1+\delta)+2(q^2+m_c{}^2)-\frac{q^2g_{Vc}{}^2}{m_c{}^2F_K{}^2}\Bigg), \tag{9}$$

$$C_c(q^2)=\frac{\delta m_c{}^2}{2m_Q{}^2}\frac{g_Q}{F_K(q^2+m_c{}^2)}, \tag{10}$$

$$D_c(q^2)=\Bigg(1-\frac{2+\delta}{2}\frac{m_c{}^2}{m_Q{}^2}\Bigg)\frac{g_Q}{F_K(q^2+m_c{}^2)}. \tag{11}$$

For determining $f_c(q^2)$, we first outline the Ward-identity approach for the VVP vertex, which is similar in spirit to that of Ref. 2 or 5. For this, we define the proper AVV vertex $\Gamma_{\mu\nu\lambda}$, PVV vertex $\Gamma_{\mu\lambda}$, etc., by explicitly displaying the pole structures of $M_{\mu\nu\lambda}$[12]:

$$M_{\mu\nu\lambda}\equiv\int d^4x\,d^4y\,e^{-iq\cdot x+ip\cdot y}\langle T(A_\mu{}^a(x),V_\nu{}^b(y),V_\lambda{}^c(0))\rangle_0$$
$$=i\Delta_{\nu\nu'}{}^{Vb}(p)\Delta_{\lambda\lambda'}{}^{Vc}(k)\Bigg(\Gamma_{\mu'\nu'\lambda'}{}^{abc}(q,p)\Delta_{\mu\mu'}{}^{Aa}(q)g_{Aa}{}^{-1}+\frac{F_aq_\mu}{q^2+m_a{}^2}\Gamma_{\nu'\lambda'}{}^{abc}(q,p)\Bigg)g_{Vb}{}^{-1}g_{Vc}{}^{-1}$$
$$+\text{scalar-meson pole terms}, \tag{12}$$

where $k=p-q$, and $\Delta_{\nu\nu'}{}^{Vb}(p)$ and $\Delta_{\mu\mu'}{}^{Aa}(q)$ are the covariant spin-1 parts of the vector and axial-vector propagators, respectively. In (12), we have not explicitly written down scalar-meson pole terms which arise if some of the vector current is not conserved (as a result, say, of the κ meson in the case of strangeness-changing vector currents). It is easily seen that the technique of algebra of currents gives the Ward-like identity

$$q_\mu\Gamma_{\mu\nu\lambda}{}^{abc}(q,p)=-F_ag_{Aa}{}^{-1}m_{Aa}{}^2\Gamma_{\nu\lambda}{}^{abc}(q,p). \tag{13}$$

---

[9] The low-energy theorem implies that $q^2F_2(0,q^2)|_{q^2=0}=2$ and $F_{\rho,8}(q^2=0)=1$. Now, if one assumes that the off-mass-shell q^2-dependence comes through the electromagnetic form factor $F(q^2)$, i.e., $q^2F_2(0,q^2)=2F(q^2)$, $F_{\rho,8}(q^2)=F(q^2)$, etc., then the expression for the mass shift with the dispersion integral saturated by the K pole only reduces to an expression very similar to the pion-pole contribution to the π^+-π^0 mass difference (Ref. 7). If such is the case, the treatment of $\Delta I=1$ and of $\Delta I=2$ mass shifts do not differ in spirit. In contradistinction, our motivation in the present paper is to analyze the effect of the symmetry breaking which modifies the electromagnetic form factors. For example, in the K form factors $F_c(q^2)$ [Eq. (9)], the effect of the K-K_A mixing has come through the parameter δ (which vanishes for $\delta=-1$), and the expression for $q^2F_2(0,q^2)$ [Eq. (18)] is distinctly different from $2F(q^2)$, etc.

[10] See, e.g., Ref. 2 and H. J. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1967); S. Fenster and F. Hussain, *ibid.* **169**, 1314 (1968); K. C. Gupta and J. S. Vaishya, *ibid.* **170**, 1530 (1968).

[11] It may be pointed out that the K_{l3}-decay form factors depend on the ratios of the wave-function renormalization constants such as Z_κ/Z_K, etc. (see Ref. 2). In contrast, the form factors listed in Eqs. (9)–(11) do not depend on the wave-function renormalization constants. This fact is closely related to the conservation of electromagnetic current.

[12] Here we follow notation similar to that used by H. J. Schnitzer and S. Weinberg (Ref. 10).

TABLE I. Summary of contributions to the K^+-K^0 mass shift for various combinations of F_K/F_π and δ.

F_K/F_π	δ	K pole	Q pole	Soft kaon	K^* pole	Total
1.28	−1.0	$0.89+0.07B$[a]	$-0.05+0.02B$	$-0.36+0.87B$	−0.01	$0.47+0.96B$
	−0.5	$0.89+0.07B$	$-0.02+0.02B$			$0.50+0.96B$
1.17	−1.0	$0.78+0.03B$	$-0.11+0.05B$	$0.74+0.44B$	−0.01	$1.40+0.52B$
	−0.5	$0.83+0.04B$	$-0.05+0.03B$			$1.51+0.51B$
1.00	−1.0	0.22	$-0.24+0.1\ B$	$3.20-0.55B$	−0.02	$3.16-0.45B$
	−0.5	$0.47+0.004B$	$-0.11+0.07B$			$3.54-0.48B$

[a] $B=\ln(\Lambda^2/m_K^2)$, where Λ is a cutoff.

The most general covariant form of $\Gamma_{\mu\nu\lambda}{}^{abc}(q,k)$, satisfying generalized crossing symmetry, is[13,14]

$$\Gamma_{\mu\nu\lambda}{}^{abc}(q,p)=d^{abc}\{[g_1q_\mu\epsilon_{\nu\lambda\alpha\beta}+(g_2k_\nu+g_3p_\nu)\epsilon_{\mu\lambda\alpha\beta}+(g_2p_\lambda+g_3k_\lambda)\epsilon_{\mu\gamma\alpha\beta}]p^\alpha k^\beta+g_4\epsilon_{\mu\nu\lambda\sigma}(p+k)^\sigma\}\,,\quad(14)$$

where the g_i may be functions of p^2, q^2, and k^2. Here, in the spirit of the smoothness hypothesis, we take them to be constant. Substituting (14) in (13), we get

$$\Gamma_{\nu\lambda}{}^{abc}(q,p)=-d^{abc}g_{A_a}F_a{}^{-1}m_{A_a}{}^{-2}\times(g_1q^2+2g_4)\epsilon_{\nu\lambda\alpha\beta}p^\alpha k^\beta.\quad(15)$$

Further, the definition of $\Gamma_{\nu\lambda}$ yields

$$f_c(q^2)=\tfrac{1}{2}(g_1m_K{}^2-2g_4)\frac{g_Qg_{V_c}}{F_Km_Q{}^2(q^2+m_c{}^2)}\,.\quad(16)$$

For determining g_1 and g_4, we obviously require two inputs with different pseudoscalar mesons.[15] We could select them to be $\Gamma(\pi^0\to2\gamma)$ and $\Gamma(\eta_8\to2\gamma)$. However, the latter is not a clean choice in the sense that now we have to consider η^0-X^0 mixing[16] apart from the determination of F_{η_8} and m_{A_8}.[17] From these two imputs, we get[18]

$$g_1\approx0.25m_\pi{}^{-2},\quad g_4\approx-0.71\,,\quad(17)$$

which predicts $\Gamma(K^{*+}\to K^+\gamma)\approx0.01$ MeV. We note that the determination of $f_c(q^2)$ would have been more reliable if $\Gamma(K^{*+}\to K^+\gamma)$ were known experimentally.

We take $F_2(0,q^2)$ as determined by the soft-kaon current algebra[19]:

$$F_2(0,q^2)=F_K{}^{-2}\left(\frac{2F_K{}^2}{q^2}+\frac{2g_Q{}^2}{m_Q{}^2(q^2+m_c{}^2)}-\frac{g_\rho{}^2}{m_\rho{}^2(q^2+m_\rho{}^2)}-\frac{g_8{}^2}{m_8{}^2(q^2+m_8{}^2)}\right),\quad(18)$$

where the so-called σ term is neglected.[20]

Utilizing (7) and (8) with the form factors [Eqs. (9)–(11) and (16)] and the subtraction term [Eq. (18)], as determined from the current-algebra analysis, we can obtain from (1) the electromagnetic mass shift of kaons.[18] Excluding the contribution due to the K^* intermediate state from this expression for the mass shift, we find it to be (a) identical with the expression obtained from the analysis of four-point functions of currents[3] and (b) logarithmically divergent.[21] The K^* contribution is convergent and has the right sign, but the magnitude is small, as shown in Table I. However, the experimental knowledge of $\Gamma(K^{*+}\to K^+\gamma)$ will

[13] Our expression (14) is different from that of Riazuddin and Sarker [Eq. (8) of Ref. 5], where only one coupling g_1 occurs in the AVV vertex. However, we would like to point out that their choice would disallow physical AVV coupling since $\epsilon_\mu(q)\epsilon_\nu(p)\times\epsilon_\lambda(k)\Gamma_{\mu\nu\lambda}(q,p)=0$, where the ϵ's are the polarization vectors for the axial-vector and vector fields.

[14] We have taken the AVV vertex to be $SU(3)$ symmetric. It may be that the octet symmetry-breaking effects, as have been discussed by L. M. Brown, H. Munczek, and P. Singer [Phys. Rev. Letters **21**, 707 (1968)], are important. These introduce four new parameters ϵ_i. However, our result (19) corresponds to the choice $\epsilon_2=0$.

[15] In addition to (13), there are other independent Ward-like identities. They in turn relate g_4 to other parameters, g_2 and g_3 in addition to the scalar-meson couplings (if either of the vector currents is not conserved). If both vector currents are conserved, then these interrelate g_4 and g_3; in other cases nothing much can be learned because of our meager knowledge about scalar mesons.

[16] We consider η^0-X^0 mixing as usual: $X^0=X_1\cos\alpha+\eta_8\sin\alpha$, $\eta^0=-X_1\cos\alpha+\eta_8\sin\alpha$, with $\alpha\simeq-10.3°$, $m_{\eta_8}=567$ MeV, and $m_{X_1}=949$ MeV (see Ref. 5). This relates amplitudes of physical η^0 to that corresponding to the octet member η_8 and the singlet member X_1 through the relation $A(\eta^0\to2\gamma)=A(\eta_8\to2\gamma)\cos\alpha-A(X_1\to2\gamma)\sin\alpha$. Noticing that $\sin\alpha$ is small, we make a rough estimate of $A(X_1\to2\gamma)$ through the vector dominance and $\tilde{U}(12)$ symmetry, which give $A(X_1\to2\gamma)=2\sqrt{2}A(\eta_8\to2\gamma)$. Thus, we obtain $A(\eta_8\to2\gamma)=0.67A(\eta^0\to2\gamma)$.

[17] For obtaining F_{η_8} and m_{A_8}, we utilize the results of T. Akiba and K. Kang, Phys. Rev. **172**, 1551 (1968). They have made use of a modified Weinberg's second sum rule through the octet symmetry-breaking term and generalized Weinberg's first sum rule (Ref. 18). We take $F_{\eta_8}=1.15F_\pi$ and $m_{A_8}=1370$ MeV.

[18] We have used the generalized form for Weinberg's first sum rule: $g_\rho{}^2m_\rho{}^{-2}=g_{A_1}{}^2m_{A_1}{}^{-2}+F_\pi{}^2=g_Q{}^2m_Q{}^{-2}+F_K{}^2=g_8{}^2m_8{}^{-2}=g_{A_8}{}^2\times m_{A_8}{}^{-2}+F_{\eta_8}{}^2$, and the KSRF relation $g_\rho{}^2\simeq2F_\pi{}^2m_\rho{}^2$. We have taken $m_8=930$ MeV as given by the Gell-Mann–Okubo mass formula. All other constants are taken from A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968).

[19] Our soft-kaon result for the mass difference is gauge-invariant [see, e.g., K. Tanaka, Nuovo Cimento **56**, 764 (1968)]. It may be pointed out that our soft-kaon result $F_2(0,q^2)$ corresponds to the point $k^2=0$ instead of $k^2=-m_K{}^2$. We assume that $F_2(0,q^2,k^2=0)=F_2(0,q^2,k^2=-m_K{}^2)$. We may also mention that the one-dimensional dispersion relation in the mass variable k^2 is not sufficient to give any extrapolation to $F_2(0,q^2)$ in this respect.

[20] The contribution of this term in the σ model [M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960)] is related to the tadpole graph describing the transition of a scalar meson of zero momentum to the vacuum, where a photon is emitted and re-absorbed. S. Coleman and S. L. Glashow [Phys. Rev. **134B**, 671 (1969)] have argued that such tadpole contributions are important in understanding the $\Delta I=1$ electromagnetic mass shifts.

[21] See, e.g., R. N. Chaudhuri and D. Bondyopadhyay, Phys. Rev. **177**, 2342 (1969). They take $F_K=F_\pi$ throughout their analysis.

furnish a reliable estimate of δm_K due to the K^* intermediate state through the relation[14,18]

$$\delta m_K|_{K^*} \simeq -1.1\Gamma(K^{*+} \to K^+\gamma). \tag{19}$$

In Table I, we have displayed various contributions to the mass shift δm_K for three values of the F_K/F_π ratio (namely, 1.28, 1.17, and 1.00,) with $\delta = -1.0$ and -0.5. Notice that the soft-kaon current-algebra result, i.e., the subtraction term[1] contribution to δm_K, depends sensitively on the F_K/F_π ratio. Further, in the absence of a σ term, the correct sign and magnitude of the mass shift can be obtained with an unreasonably high value of the cutoff ($\Lambda \sim 300$ GeV) for $F_K/F_\pi \approx 1$. The σ-term contribution depends on the strength of the tadpole vertex. For this, Patil[22] has recently made an approximate estimate in the $SU(3)$-symmetric limit by saturating the unsubtracted dispersion relation through the two-pseudoscalar-meson intermediate state. This gives a contribution which is an order of magnitude smaller than the experimental value, but does have the right sign.[23] Thus a reliable estimate of the σ-tadpole contribution and a knowledge of $\Gamma(K^{*0,+} \to K^{0,+}\gamma)$ are desirable in understanding the K^+-K^0 electromagnetic mass shift.

The authors are grateful to Professor S. N. Biswas for helpful discussions and advice. They are also grateful to Dr. S. H. Patil and Dr. K. Datta for their comments.

[22] S. H. Patil, Phys. Rev. **172**, 1528 (1968); see also V. Barger, Nuovo Cimento **32**, 127 (1964).
[23] S. H. Patil (unpublished).

PHYSICAL REVIEW VOLUME 184, NUMBER 5 25 AUGUST 1969

Coupling Constants and $SU(3)$ Classification of the $Y_0^*(1405)$†

JAE KWAN KIM
Department of Physics, Harvard University, Cambridge, Massachusetts 02138

AND

FRANK VON HIPPEL*
Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California 94305

(Received 10 March 1969)

We present values for the $Y_0^*(1405)$ couplings to the $\bar{K}$-N and π-Σ channels. The *relative sign* is that appropriate to an $SU(3)$ singlet, but the relative magnitude differs by almost a factor of 3 from the corresponding $SU(3)$ prediction.

We present here improved values of the $Y_0^*(1405)$ couplings to the $\bar{K}$-N and π-Σ channels and a determination of their relative sign. There has been considerable theoretical interest in the coupling constants of this s-wave resonance because both dynamical models[1–3] and charge-algebra considerations[3,4] suggest that their ratio should deviate strongly from the value predicted by unbroken $SU(3)$. We discuss first the estimate of these coupling constants and then briefly the status of our theoretical understanding of their ratio.

VALUES OF COUPLING CONSTANTS

There are various definitions of the coupling constants of a resonance, all of which become identical in the limit of a narrow resonance isolated from thresholds. An appropriate definition for an s-wave resonance with finite width is to relate the (coupling constants)[2] to some integral involving the imaginary part of the resonant amplitude. We note that, in the narrow-width approximation, the integral over the imaginary part of a resonant s-wave scattering amplitude between an initial baryon-pseudoscalar state B_i-P_α and a final state B_f-P_β is given by

$$\int \mathrm{Im}T_{\mathrm{res}}(W')dW' = \pi\left(\frac{g_{i\alpha}g_{f\beta}}{4\pi}\right) \times \frac{[(E_i+M_i)(E_f+M_f)]^{1/2}}{2M_{\mathrm{res}}}, \tag{1}$$

where E_i, M_i, and E_f, M_f are the c.m. energies and masses of the initial and final baryons, respectively, $g_{i\alpha}$, $g_{f\beta}$ are the coupling constants of the resonance to the initial and final channels, and W' is the total c.m. energy. This leads us to a definition for the bilinear

† Research sponsored by the Air Force Office of Scientific Research, Office of Aerospace Research, U. S. Air Force, under AFOSR Contract No. F44620-68-C-0075 and the U. S. Atomic Energy Commission, under Contract No. AEC AT930-1-2752.
* A. P. Sloan Foundation Fellow.
[1] R. H. Dalitz, T. C. Wong, and G. Rajasekaran, Phys. Rev. **153**, 1617 (1967).
[2] H. W. Wyld, Jr., Phys. Rev. **155**, 1649 (1967).
[3] C. Weil, Phys. Rev. **161**, 1682 (1967).
[4] M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).