Nuclear Double-Beta Decay and a New Limit on Lepton Nonconservation*

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(Received 21 April 1969)

Nuclear double-β decay is discussed on the assumptions (1) that the process, treated as second-order weak, occurs predominantly without neutrino emission and so violates lepton convervation, and (2) that the matrix element for the process is mainly due to the no-neutrino double-β decay of an isospin-$\frac{3}{2}$ nucleon resonance present, albeit with a small ($\approx 1\%$) probability, in the parent or daughter nucleus. The theoretical lifetimes obtained on this basis are compared with the corresponding experimental lifetimes; the various lifetime ratios are correctly predicted, and a limit $\approx 10^{-4}$ is imposed on the relevant "lepton-non-conservation" parameter. A discussion is also given of the convergence problems associated with the matrix elements for second-order weak processes in general. In addition it is shown that in spite of the presence of virtual-neutrino closed loops, divergences never arise in the matrix elements for no-neutrino double-β decay if the basic nuclear constituents (i.e., quarks or nucleons) have isospin not exceeding $\frac{1}{2}$ and mean separation greater than zero.

1. INTRODUCTION

NOW that nuclear double-β decay has definitely been observed,[1] we can begin to reexamine some of the theoretical questions associated with the phenomenon. Nuclear double-β decay has always been anticipated as a second-order effect of the usual weak ($\Delta Q=\pm 1$, $\Delta S=0$) interaction which gives rise, in first order, to nuclear single-β decay; there is, however, no *a priori* reason to prevent it from being, in part and even predominantly, a first-order effect of some hitherto unrecognized "superweak" ($\Delta=\pm 2$, $\Delta S=0$) interaction.[2] According to the various known conservation laws, it is always possible for two neutrinos to be emitted in the nuclear double-β decay; however, it has yet to be determined whether there occur nuclear double-β events in which no neutrinos are emitted and where, as a consequence, lepton conservation is not valid. These problems are intimately related to the problem of the appropriate description of the free neutrino and of its coupling to other particles, and it is from this point of view that we wish to analyze the present experimental situation. We shall also discuss questions concerning the convergence of second-order weak matrix elements arising in nuclear double-β decay and in certain related processes.

The experiments in which nuclear double-β decay was detected involved mass-spectrometric analyses of tellurium ores of known age.[1] Xenon occluded in the ores was found to contain a relative abundance of Xe^{130} far in excess of that contained in atmospheric xenon. A careful analysis showed that the excess of Xe^{130} could have resulted only from the double-β decay of Te^{130}, and the half-life of Te^{130} was deduced to be $10^{21.34\pm0.12}$ years.[1] In view of the ensuing discussion, it should also be noted that no excess of Xe^{128} was found to accompany the excess of Xe^{130}, and that as a consequence, the half-life for the double-β decay of Te^{128} could be estimated as greater than $10^{23.3}$ years.[1]

Because it is possible to detect only the daughter nucleus by the mass-spectrometric method, no direct conclusion can be drawn from the tellurium ore experiments about the presence or absence of neutrinos in the double-β decay final state. The measured half-life of Te^{130} is, however, in good agreement with a theoretical estimate of $10^{22.5\pm2.0}$ years obtained on the assumption that the nuclear double-β decay is a second-order weak process in which two neutrinos are always emitted so that lepton conservation holds.[3] Thus the measured half-life of Te^{130} seems to provide evidence against the occurrence of any lepton-nonconserving no-neutrino double-β decay, viewed either as a "superweak" or as a second-order weak process.

Pontecorvo has pointed out that this somewhat tentative conclusion may not be valid.[4] In one of the tellurium ore experiments,[5] an excess of Xe^{128} was found to accompany the excess of Xe^{130}, and if this excess of Xe^{128} is attributed to the double-β decay $Te^{128}\rightarrow Xe^{128}$, the corresponding Te^{128} half-life is $10^{22.5\pm0.5}$ years. Now if one makes the reasonable assumption that the nuclear matrix elements for $Te^{128}\rightarrow Xe^{128}$ and $Te^{130}\rightarrow$

* Supported in part by the National Science Foundation and the U. S. Atomic Energy Commission.

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[2] R. G. Winter, Phys. Rev. **83**, 1070 (1951).

[3] S. P. Rosen and H. Primakoff, in *Alpha-, Beta-, and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Co., Amsterdam, 1965), Vol. II, p. 1499; Rept. Progr. Phys. **22**, 121 (1959); Proc. Phys. Soc. (London) **78**, 464 (1961); E. J. Konopinski, in *Theory of Beta Radioactivity* (Oxford University Press, London, 1966), p. 245; E. Greuling and R. C. Whitten, Ann. Phys. (N. Y.) **11**, 510 (1960); T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. **15**, 381 (1965).

[4] B. Pontecorvo, Phys. Letters **26B**, 630 (1968).

[5] N. Takaoka and K. Ogata, Z. Naturforsch. **21a**, 84 (1966).

Xe^{130} are approximately equal, then these matrix elements cancel from the expression for the ratio of the corresponding double-β decay rates, and this ratio reduces to the ratio of the available phase spaces. For two-neutrino double-β decay, the phase space available to the (four) emitted leptons is roughly proportional to the eighth through 11th power of the energy release,[3] and, since the energy release for Te^{130} is three times that for Te^{128}, one would expect a lifetime ratio

$$T_{1/2}(\mathrm{Te}^{128})/T_{1/2}(\mathrm{Te}^{130})\cong 3^{8.4}=10^{4.0},$$

in contrast to the measured lifetime ratio

$$10^{22.5\pm0.5}/10^{21.34\pm0.12}=10^{1.2\pm0.6}.$$

One obvious explanation for this large discrepancy is that most of the excess Xe^{128} found in the experiments of Ref. 5 does not originate from double-β decay; in fact, the argument in favor of the double-β decay origin of this excess Xe^{128} is by no means as strong as the corresponding argument for the excess of Xe^{130},[1,5] particularly since no excess Xe^{128} is found in the experiments of Ref. 1. An alternative and more intriguing explanation noted by Pontecorvo[4] is based on the assumption that both Te^{128} and Te^{130} predominantly undergo no-neutrino double-β decay; in this case the phase space available to the (two) emitted leptons is roughly proportional to the fourth through fifth power of the energy release,[3] and one would estimate that

$$T_{1/2}(\mathrm{Te}^{128})/T_{1/2}(\mathrm{Te}^{130})\cong 3^{4.6}=10^{2.2},$$

in much better agreement with the measured lifetime ratio.

On the basis of this suggestion, Pontecorvo proposed that the decays of Te^{130} and Te^{128} are, predominantly, the first-order effect of a new superweak ($\Delta Q=\pm2$, $\Delta S=0$) interaction which mediates no-neutrino double-β decay, rather than the second-order effect of the usual weak ($\Delta Q=\pm1$, $\Delta S=0$) interaction. In fact, Pontecorvo drew a possible parallel between this superweak ($\Delta Q=\pm2$, $\Delta S=0$) interaction which violates lepton conservation and the Wolfenstein superweak ($\Delta Q=0$, $\Delta S=\pm2$) interaction which violates CP conservation.[6] Rather than considering further the implications of the Pontecorvo superweak interaction, we wish instead, in the present paper, to reexamine an alternative and much older point of view,[3] namely, that while the nuclear double-β decay is predominantly no-neutrino and therefore violates lepton conservation, it is nevertheless a second-order weak process. According to this view, the usual weak ($\Delta Q=\pm1$, $\Delta S=0$) interaction which gives rise in first order to nuclear single-β decay must also violate lepton conservation, though we expect that the violation is in some sense small (see below). The measured half-life of Te^{130} then enables us to set an upper limit on this violation.

[6] L. Wolfenstein, Phys. Rev. Letters **13**, 562 (1964).

In order to introduce a violation of lepton conservation in the usual weak ($\Delta Q=\pm1$, $\Delta S=0$) interaction Hamiltonian, we recall that when the neutrino mass $m_\nu=0$ and the lepton weak current L_λ is of pure "two-component" variety, i.e., when[7]

$$L_\lambda=\psi_e^\dagger\gamma_4\gamma_\lambda(1+\gamma_5)\psi_\nu, \tag{1}$$

the helicity projection operator $(1+\gamma_5)$ ensures that a neutrino emitted together with an electron by one hadron cannot be reabsorbed with simultaneous emission of a second electron by another hadron. Nuclear no-neutrino double-β decay is then forbidden and lepton conservation is valid to all orders in the weak ($\Delta Q=\pm1$, $\Delta S=0$) interaction, irrespective of whether ψ_ν describes a "Majorana" ($\psi_{\bar\nu}\equiv\tilde{\psi}_\nu^\dagger=\psi_\nu$) or a "Dirac" ($\psi_{\bar\nu}\equiv\tilde{\psi}_\nu^\dagger\neq\psi_\nu$) neutrino. If we now modify the lepton current of Eq. (1) by including a term with the helicity projection operator $(1-\gamma_5)$ and suppose that ψ_ν describes a "Majorana" neutrino, viz.,[3]

$$L_\lambda'=\psi_e^\dagger\gamma_4\gamma_\lambda[(1+\gamma_5)+\eta(1-\gamma_5)]\psi_\nu,\quad \tilde{\psi}_\nu^\dagger=\psi_\nu \tag{2}$$

then a neutrino emitted together with an electron by one hadron can be reabsorbed with simultaneous emission of a second electron by another hadron with a probability amplitude proportional to the "lepton-nonconservation" parameter η.[8] It follows that the corresponding probability for nuclear no-neutrino double-β decay is proportional to η^2. The lepton current L_λ' parametrizes lepton nonconservation in a manner complementary to that of parity nonconservation, with $\eta=0$ corresponding to lepton conservation and maximal parity nonconservation, and $\eta=1$ corresponding to parity conservation and maximal lepton nonconservation. Thus for example, L_λ' predicts a longitudinal spin polarization for an electron emitted in a single-β decay, equal to $(v_{el}/c)(1-\eta^2)/(1+\eta^2)$, and this varies from v_{el}/c to 0 as η varies from 0 to 1.

The emission and reabsorption of the virtual neutrino in the lepton-nonconserving double-β decay of a nucleus corresponds to a sum of second-order weak diagrams, each with a virtual-neutrino closed loop (see Fig. 1), and so we might expect that the associated second-order weak matrix element will be divergent. In the standard

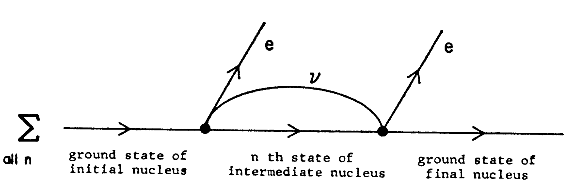

FIG. 1. Second-order weak diagrams for lepton-nonconserving double-β decay.

[7] We use the "Majorana" form of the γ_λ so that $\gamma_\lambda^*=\tilde{\gamma}_\lambda=\gamma_\lambda\times(1-2\delta_{\lambda4})$.

[8] We suppose in this paper that η is real: $\eta=\eta^*$. If η should differ from η^* the replacement of L_λ by L_λ' in the $\mathcal{H}_{\mathrm{weak}}$ of Eq. (5) below will yield a weak interaction which not only violates lepton conservation but also violates CP conservation.

calculation,[3] however, the virtual neutrino is emitted together with an electron by one neutron in the nucleus, and is reabsorbed with simultaneous emission of a second electron by another neutron in the same nucleus. The reciprocal of the mean separation between the two neutrons then serves as an effective cutoff for the energy of the virtual neutrino, and the matrix element in question is actually finite. Underlying this result is the fact that the only charge states of the nucleon are zero and one; thus it is not possible for a single nucleon to emit and reabsorb the virtual neutrino with simultaneous emission of two electrons. In other words, there is a direct connection between (i) the convergence of the matrix element for lepton-nonconserving nuclear double-β decay, and (ii) the isospin and mean separation of the basic nuclear constituents, i.e., the nucleons in the standard picture: As long as this isospin does not exceed $\frac{1}{2}$ and the mean separation is greater than zero, the matrix element is finite.

This situation is to be contrasted with other second-order weak processes which can however all be considered in the limit of lepton conservation. Thus, in two-neutrino double-β decay, there are no virtual-neutrino closed loops, so that the associated matrix element is finite, and would in fact remain so even if the mean separation between the basic nuclear constiuents were taken as zero.[3] Further, in processes where an electron-positron pair is emitted, e.g., $O^{16*} \rightarrow O^{16}+e^-+e^+$, the virtual neutrino can be emitted and reabsorbed by the same basic constituent of the nuclues, i.e., the same nucleon in the standard picture. There is then, no mean separation available to provide a natural cutoff for the energy of the virtual neutrino in the second-order weak matrix element, and this matrix element diverges quadratically.

An immediate limit on the "lepton-nonconservation" parameter η of Eq. (2) can be obtained by comparing the measured half-life of Te^{130} with a theoretical estimate obtained on the basis of the standard two-nucleon mechanism; this yields the limit $\eta \approx 10^{-3}$ [see Eq. (27) below]. The theoretical lifetimes are inversely proportional to η^2, directly proportional to the square of the mean separation of a typical pair of nucleons in the parent or daughter nucleus $(\cong A^{1/3}/m_\pi)^2$, and directly proportional to the square of the reciprocal overlap between the wave functions of these nuclei; for the double-β decay $Te^{130} \rightarrow Xe^{130}$, each of these last two factors is quite large. It follows that if the expression for the theoretical lifetime can be reasonably modified in such a way as to reduce these last two factors, then a sharper limit on η can be set.

One way of achieving this modification is to take advantage of a paper by Kerman and Kisslinger, wherein it is argued rather convincingly that the deuteron contains about a 1% probability admixture of the nucleon resonance with $J^{(P)}=\frac{5}{2}^+$, $I=\frac{1}{2}$, and $m=1688$ MeV, namely $N^*(1688)$.[9] Generalizing this to other nuclei and to other nucleon resonances, we contemplate the possibility that the various parent and daughter nuclei involved in double-β decay contain about 1% probability admixture of the nucleon resonance with $J^{(P)}=\frac{3}{2}^+$, $I=\frac{3}{2}$, and $m=1236$ MeV [$N^*(1236)$]. Since the $N^*(1236)$ has isospin $\frac{3}{2}$ it can emit and reabsorb the virtual neutrino with simultaneous emission of two electrons, viz.,

$$N^{*-}(1236)\rightarrow p+e^-+e^-,\ n\rightarrow N^{*++}(1236)+e^-+e^-. \quad (3)$$

Thus, if the processes of Eq. (3) contribute to nuclear double-β decay, the change of two units of charge within the nucleus can involve one particle instead of two, and the overlap between the wave functions of the parent and daughter nuclei is larger than in the standard two-nucleon mechanism. This situation will always occur if the nucleus contains particles with isospin greater than $\frac{1}{2}$. To avoid the divergence problem that will arise if the nucleon resonances and the nucleons are treated as "elementary," i.e., as point particles, we estimate the lifetimes of the processes of Eq. (3) on the assumption that each of these nucleon resonances and nucleons is a composite of three isospin-$\frac{1}{2}$ quarks. Then instead of the mean separation between nucleons in a nucleus, the lifetime estimate will contain the mean separation between quarks in a three-quark composite, and the latter mean separation is much smaller than the former. Taking all factors into account, we expect the corresponding limit on η to be an order of magnitude smaller than the limit obtained on the basis of the two-nucleon mechanism.

Certain aspects of the general convergence problem of second-order weak matrix elements are studied in the next section, while the limit on η obtained on the basis of the $N^*(1236)$ mechanism is described in Sec. 3. Section 4 summarizes our results.

2. CONVERGENCE OF SECOND-ORDER WEAK MATRIX ELEMENTS

As we have already indicated, the matrix elements for a second-order weak process between two hadron states in which a virtual neutrino is emitted and then reabsorbed may be divergent. To study this problem, we consider those processes in which a pair of charged leptons emerges, and we divide them into two clasess. In the first class, the lepton pair carries off zero charge ($\Delta Q=0$; e.g., $O^{16*}\rightarrow O^{16}+e^-+e^+$, or $\pi^0\rightarrow e^-+e^+$, or $K_L{}^0\rightarrow \mu^-+\mu^+$), and in the second, the lepton pair carries off two units of charge ($\Delta Q=\pm 2$; e.g., no-neutrino double-β decay). As we have already noted, the matrix elements for the $\Delta Q=0$ processes diverge, while the matrix elements for the $\Delta Q=\pm 2$ processes converge in any composite hadron model where the basic hadronic

[9] A. K. Kerman and L. S. Kisslinger, Phys. Rev. **180**, 1483 (1969); see also H. Arenhövel and M. Danos, Phys. Letters **28B**, 299 (1968); L. Kisslinger, Phys. Rev. **29B**, 211 (1969).

constituents have isospin not exceeding $\frac{1}{2}$ and mean separation greater than zero.

We begin by considering the second-order weak processes for which $\Delta Q=0$ and $\Delta S=0$, namely,

$$\alpha \begin{matrix} \nearrow \text{any } \gamma_n^{(+)}+\nu+e^- \searrow \\ \searrow \text{any } \gamma_n^{(-)}+\nu+e^+ \nearrow \end{matrix} \beta+e^-+e^+, \qquad (4)$$

where α, $\gamma_n^{(+)}$, $\gamma_n^{(-)}$, and β are hadrons; e.g., $\alpha=O^{16*}$, $\gamma_n^{(+)}=F^{16}$, F^{16*} $\cdots$, $\gamma_n^{(-)}=N^{16}$, N^{16*} $\cdots$, $\beta=O^{16}$ or $\alpha=\pi^0$, $\gamma_n^{(+)}=\pi^+$, ρ^+ $\cdots$, $\gamma_n^{(-)}=\pi^-$, ρ^- $\cdots$, and $\beta=$vacuum. From a practical point of view, it is of course true that when strangeness is conserved the decay $\alpha\rightarrow\beta+e^-+e^+$ can be engendered by electromagnetic interactions alone, and that contributions to it arising from Eq. (4) (associated with, e.g., parity violation effects) will be extremely hard to detect. The process of Eq. (4) has no bearing on the matter of lepton conservation, and so we shall neglect the (in any case, small) parameter η of Eq. (2).

The current-current weak-interaction Hamiltonian density for strangeness-conserving semileptonic processes can be written

$$\mathcal{H}_{\text{weak}}(x)=(G/\sqrt{2})[L_\lambda(x)J_\lambda^{(+)}(x)+J_\lambda^{(-)}(x)L_\lambda^\dagger(x)], \qquad (5)$$
$$J_\lambda^{(-)}(x)=[J_\lambda^{(+)}(x)]^\dagger, \quad J_\lambda^{(\pm)}(x)=V_\lambda^{(\pm)}(x)+A_\lambda^{(\pm)}(x)$$

where G is the weak-coupling constant, $L_\lambda(x)$ is the lepton weak current given in Eq. (1), and $V_\lambda^{(\pm)}(x)$ and $A_\lambda^{(\pm)}(x)$ are polar and axial-vector hadron weak currents specified by CVC (conserved vector current), PCAC (partially conserved axial-vector current), and appropriate equal-time commutation relations. $\mathcal{H}_{\text{weak}}(x)$ is normally used to treat first-order weak processes but we shall use it here to treat second-order weak processes as well. The matrix element for the process of Eq. (4) is then given by

$$\mathfrak{M}(\alpha\rightarrow\beta+e^-+e^+)=\left(\frac{G}{\sqrt{2}}\right)^2\int\int d\mathbf{x}d\mathbf{y}\sum_{\mathbf{p}_\nu,S_\nu}\sum_{n;\mathbf{p}_n,S_n}\left[\frac{\langle e^+|L_\mu^\dagger(\mathbf{x})|\nu\rangle\langle\beta|J_\mu^{(-)}(\mathbf{x})|\gamma_n^{(+)}\rangle\langle e^-\nu|L_\lambda(\mathbf{y})|0\rangle\langle\gamma_n^{(+)}|J_\lambda^{(+)}(\mathbf{y})|\alpha\rangle}{E_\nu+E_{e^-}+E_n^{(+)}-E_\alpha}\right.$$
$$\left.+\frac{\langle e^-|L_\lambda(\mathbf{y})|\nu\rangle\langle\beta|J_\lambda^{(+)}(\mathbf{y})|\gamma_n^{(-)}\rangle\langle e^+\nu|L_\mu^\dagger(\mathbf{x})|0\rangle\langle\gamma_n^{(-)}|J_\mu^{(-)}(\mathbf{x})|\alpha\rangle}{E_\nu+E_{e^+}+E_n^{(-)}-E_\alpha}\right], \qquad (6)$$

where

$$S_\nu\equiv\mathbf{S}_\nu\cdot\hat{p}_\nu, \quad S_n\equiv\mathbf{S}_n\cdot\hat{p}_n,$$
$$E_\alpha=m_\alpha, \quad E_n^{(\pm)}=[(\mathbf{p}_n)^2+(m_n^{(\pm)})^2]^{1/2},$$
$$E_{e^\pm}=[(\mathbf{p}_\pm)^2+m_e^2]^{1/2}, \quad E_\nu=|\mathbf{p}_\nu|,$$

and where the intermediate hadron states $|\gamma_n^{(\pm)}\rangle$ form a complete set. Thus, summing over this complete set,

$$\mathfrak{M}(\alpha\rightarrow\beta+e^-+e^+)$$
$$=\left(\frac{G}{\sqrt{2}}\right)^2\int\int d\mathbf{x}d\mathbf{y}\sum_{\mathbf{p}_\nu,S_\nu}\left[\frac{\langle e^+|L_\mu^\dagger(\mathbf{x})|\nu\rangle\langle e^-\nu|L_\lambda(\mathbf{y})|0\rangle}{E_\nu+E_{e^-}+\langle E_n^{(+)}\rangle-E_\alpha}\right.$$
$$\times\langle\beta|J_\mu^{(-)}(\mathbf{x})J_\lambda^{(+)}(\mathbf{y})|\alpha\rangle$$
$$+\frac{\langle e^-|L_\lambda(\mathbf{y})|\nu\rangle\langle e^+\nu|L_\mu^\dagger(\mathbf{x})|0\rangle}{E_\nu+E_{e^+}+\langle E_n^{(-)}\rangle-E_\alpha}$$
$$\left.\times\langle\beta|J_\lambda^{(+)}(\mathbf{y})J_\mu^{(-)}(\mathbf{x})|\alpha\rangle\right], \qquad (7)$$

with the $\langle E_n^{(\pm)}\rangle$ being appropriate average values. Using Eq. (1) for L_λ, remembering that the field operators for e^+ and e^- anticommute, and summing over S_ν, we obtain from Eq. (7)

$$\mathfrak{M}(\alpha\rightarrow\beta+e^-+e^+)=\left(\frac{G}{\sqrt{2}}\right)^2\int\int d\mathbf{x}d\mathbf{y}$$
$$\times\int\frac{d\mathbf{p}_\nu}{(2\pi)^3}\left(l_{\lambda\mu}+i\mathbf{l}_{\lambda\mu}'\cdot\frac{\mathbf{p}_\nu}{E_\nu}\right)e^{-i(\mathbf{p}_+\cdot\mathbf{x}+\mathbf{p}_-\cdot\mathbf{y})}$$
$$\times\left[\frac{e^{i\mathbf{p}_\nu\cdot(\mathbf{x}-\mathbf{y})}}{E_\nu+E_{e^-}+\langle E_n^{(+)}\rangle-E_\alpha}\langle\beta|J_\mu^{(-)}(\mathbf{x})J_\lambda^{(+)}(\mathbf{y})|\alpha\rangle\right.$$
$$\left.-\frac{e^{i\mathbf{p}_\nu\cdot(\mathbf{y}-\mathbf{x})}}{E_\nu+E_{e^+}+\langle E_n^{(-)}\rangle-E_\alpha}\langle\beta|J_\lambda^{(+)}(\mathbf{y})J_\mu^{(-)}(\mathbf{x})|\alpha\rangle\right], \qquad (8)$$

where

$$l_{\lambda\mu}\equiv(u_e^\dagger(\mathbf{p}_-)\gamma_4\gamma_\lambda\gamma_4\gamma_\mu(1+\gamma_5)u_e^*(\mathbf{p}_+)),$$
$$\mathbf{l}_{\lambda\mu}'\equiv(u_e^\dagger(\mathbf{p}_-)\gamma_4\gamma_\lambda\gamma_4\boldsymbol{\gamma}\gamma_4\gamma_\mu(1+\gamma_5)u_e^*(\mathbf{p}_+)).$$

We now recall that the $\sum_{n;\mathbf{p}_n,S_n}\cdots$ in Eq. (6) is dominated by contributions from those $|\gamma_n^{(\pm)}\rangle$ which have a small four-momentum difference and a small mass difference *vis-à-vis* $|\alpha\rangle$; such $|\gamma_n^{(\pm)}\rangle$ are characterized by $|\mathbf{p}_n|\approx[(m_n^{(\pm)})^2-m_\alpha^2]/2m_\alpha$ and $m_n^{(\pm)}\approx m_\alpha$, i.e., by $E_n^{(\pm)}\approx[(m_n^{(\pm)})^2+m_\alpha^2]/2m_\alpha\approx m_\alpha$. Thus

$$E_\nu+E_{e^\mp}+\langle E_n^{(\pm)}\rangle-E_\alpha$$
$$\approx E_\nu+(m_\beta-m_\alpha)+m_\alpha-m_\alpha\cong E_\nu, \qquad (9)$$

so that, inserting Eq. (9) into Eq. (8), and carrying out the integration over $\mathbf{p}_\nu$,

$$\mathfrak{M}(\alpha\rightarrow\beta+e^-+e^+)$$
$$=-\left(\frac{G}{\sqrt{2}}\right)^2\left\{\left(\frac{1}{2\pi^2}\right)l_{\lambda\mu}\langle\beta|\int\int d\mathbf{x}d\mathbf{y}[J_\lambda^{(+)}(\mathbf{y}),J_\mu^{(-)}(\mathbf{x})]_-\times\frac{e^{-i(\mathbf{p}_-\cdot\mathbf{y}+\mathbf{p}_+\cdot\mathbf{x})}}{|\mathbf{y}-\mathbf{x}|^2}|\alpha\rangle-\left(\frac{1}{4\pi}\right)\mathbf{l}_{\lambda\mu}'\cdot\langle\beta|\int\int d\mathbf{x}d\mathbf{y}\times[J_\lambda^{(+)}(\mathbf{y}),J_\mu^{(-)}(\mathbf{x})]_+\frac{e^{-i(\mathbf{p}_-\cdot\mathbf{y}+\mathbf{p}_+\cdot\mathbf{x})}}{|\mathbf{y}-\mathbf{x}|^3}(\mathbf{y}-\mathbf{x})|\alpha\rangle\right\}, \quad (10)$$

where $[\ \ ]_-$ and $[\ \ ]_+$ denote a commutator and an anticommutator, respectively. Eq. (10) shows that $\mathfrak{M}(\alpha\rightarrow\beta+e^-+e^+)$ receives a quadratically divergent contribution from the term containing $[J_\lambda^{(+)}(\mathbf{y}), J_\mu^{(-)}(\mathbf{x})]_-$ if, as predicted on the basis of any of the proposed equal-time current-current commutation relations, $[J_\lambda^{(+)}(\mathbf{y}),J_\mu^{(-)}(\mathbf{x})]_-=f_{\lambda\mu}^{(-)}(\mathbf{x})\delta^{(3)}(\mathbf{y}-\mathbf{x})+\cdots$. On the other hand, a term of the form $f_{\lambda\mu}^{(+)}(\mathbf{x})\delta^{(3)}(\mathbf{y}-\mathbf{x})$ in the expression for $[J_\lambda^{(+)}(\mathbf{y}),J_\mu^{-}(\mathbf{x})]_+$ contributes nothing to $\mathfrak{M}(\alpha\rightarrow\beta+e^-+e^+)$ because of the factor $(\mathbf{y}-\mathbf{x})$ in the second integral of Eq. (10). To investigate these matters in more detail and to specify explicitly the operators $f_{\lambda\mu}^{(\mp)}(\mathbf{y})$, we consider the nonrelativistic impulse approximation for $J_\lambda^{(\pm)}(\mathbf{y})$. In this approximation, $J_\lambda^{(\pm)}(\mathbf{y})$ is written as a sum of terms each of which acts only on one of the basic constituents (i.e., quarks or nucleons) of the hadrons α and β, viz.,

$$J_\lambda^{(\pm)}(\mathbf{y})=\sum_n\tau_n^{(\pm)}(\Gamma_\lambda)_n\delta^{(3)}(\mathbf{y}-\mathbf{r}_n),$$
$$[\tau_n^{(+)},\tau_m^{(-)}]_-=\delta_{nm}\tau_n^{(3)},$$
$$\tau_n^{(+)}\tau_n^{(+)}=\tau_n^{(-)}\tau_n^{(-)}=0 \quad (11)$$
$$(\Gamma_\lambda)_n=g_V\delta_{\lambda4}+g_Ai(\sigma_\lambda)_n(1-\delta_{\lambda4}),$$
$$i\boldsymbol{\sigma}=\gamma_4\boldsymbol{\gamma}\gamma_5,\quad g_V=1.0,\quad g_A\cong1.2$$

where the $\tau_n^{(\pm)}$ appear because these basic constituents are assumed to have isospin $\frac{1}{2}$. Equation (11) yields

$$[J_\lambda^{(+)}(\mathbf{y}),J_\mu^{(-)}(\mathbf{x})]_-=\{\tfrac{1}{2}\sum_n(\tau_n^{(3)}[(\Gamma_\lambda)_n,(\Gamma_\mu)_n]_++[(\Gamma_\lambda)_n,(\Gamma_\mu)_n]_-)\delta^{(3)}(\mathbf{x}-\mathbf{r}_n)\}\delta^{(3)}(\mathbf{y}-\mathbf{x}),$$
$$[J_\lambda^{(+)}(\mathbf{y}),J_\mu^{(-)}(\mathbf{x})]_+=\{\tfrac{1}{2}\sum_n(\tau_n^{(3)}[(\Gamma_\lambda)_n,(\Gamma_\mu)_n]_-+[(\Gamma_\lambda)_n,(\Gamma_\mu)_n]_+)\delta^{(3)}(\mathbf{x}-\mathbf{r}_n)\}\delta^{(3)}(\mathbf{y}-\mathbf{x})$$
$$+2\sum_{n,m}\tau_n^{(+)}\tau_m^{(-)}(\Gamma_\lambda)_n(\Gamma_\mu)_m(1-\delta_{nm})\delta^{(3)}(\mathbf{y}-\mathbf{r}_n)\delta^{(3)}(\mathbf{x}-\mathbf{r}_m), \quad (12)$$

$$[(\Gamma_\lambda)_n,(\Gamma_\lambda)_n]_\pm=[g_V^2\delta_{\lambda4}-g_A^2(1-\delta_{\lambda4})](1\pm1),\quad [(\Gamma_\lambda)_n,(\Gamma_4)_n]_\pm=g_Vg_Ai(\sigma_\lambda)_n(1\pm1),\quad \lambda\neq4$$
$$[(\Gamma_\lambda)_n,(\Gamma_\mu)_n]_\pm=-g_A^2\epsilon_{\lambda\mu\nu}i(\sigma_\nu)_n(1\mp1),\quad \lambda\neq4,\ \mu\neq4,\ \lambda\neq\mu$$
$$(\Gamma_\lambda)_n(\Gamma_\mu)_m=g_V^2\delta_{\lambda4}\delta_{\mu4}-g_A^2(\sigma_\lambda)_n(\sigma_\mu)_m(1-\delta_{\lambda4})(1-\delta_{\mu4})$$
$$+ig_Vg_A[(\sigma_\mu)_m\delta_{\lambda4}(1-\delta_{\mu4})+(\sigma_\lambda)_n\delta_{\mu4}(1-\delta_{\lambda4})],\quad n\neq m$$

so that

$$f_{\lambda\mu}^{(\mp)}(\mathbf{x})=\tfrac{1}{2}\sum_n\{\tau_n^{(3)}[(\Gamma_\lambda)_n,(\Gamma_\mu)_n]_\pm+[(\Gamma_\lambda)_n,(\Gamma_\mu)_n]_\mp\}\delta^{(3)}(\mathbf{x}-\mathbf{r}_n); \quad (13)$$

$f_{\lambda\mu}^{(\mp)}(\mathbf{x})$ is different from zero since either $[(\Gamma_\lambda)_n, (\Gamma_\mu)_n]_+$ or $[(\Gamma_\lambda)_n,(\Gamma_\mu)_n]_-$ is different from zero. The combination of Eqs. (12), (13), and (10) gives

$$\mathfrak{M}(\alpha\rightarrow\beta+e^-+e^+)=-\left(\frac{G}{\sqrt{2}}\right)^2\left\{\left(\frac{1}{2\pi^2}\right)l_{\lambda\mu}\langle\beta|\int d\mathbf{x}f_{\lambda\mu}^{(-)}(\mathbf{x})e^{-i(\mathbf{p}_-+\mathbf{p}_+)\cdot\mathbf{x}}|\alpha\rangle\int d\mathbf{y}\frac{\delta^{(3)}(\mathbf{y}-\mathbf{x})}{|\mathbf{y}-\mathbf{x}|^2}\right.$$
$$-\left(\frac{1}{4\pi}\right)\mathbf{l}_{\lambda\mu}'\cdot\langle\beta|\int d\mathbf{x}f_{\lambda\mu}^{(+)}(\mathbf{x})e^{-i(\mathbf{p}_-+\mathbf{p}_+)\cdot\mathbf{x}}|\alpha\rangle\int d\mathbf{y}\frac{\delta^{(3)}(\mathbf{y}-\mathbf{x})}{|\mathbf{y}-\mathbf{x}|^3}(\mathbf{y}-\mathbf{x})$$
$$\left.-\left(\frac{1}{4\pi}\right)\mathbf{l}_{\lambda\mu}'\cdot\langle\beta|2\sum_{n,m}\tau_n^{(+)}\tau_m^{(-)}(\Gamma_\lambda)_n(\Gamma_\mu)_m(1-\delta_{nm})\frac{e^{-i(\mathbf{p}_-\cdot\mathbf{r}_n+\mathbf{p}_+\cdot\mathbf{r}_m)}}{|\mathbf{r}_n-\mathbf{r}_m|^3}(\mathbf{r}_n-\mathbf{r}_m)|\alpha\rangle\right\}, \quad (14)$$

and since

$$\int d\mathbf{y}\frac{\delta^{(3)}(\mathbf{y}-\mathbf{x})}{|\mathbf{y}-\mathbf{x}|^2}=\lim_{a\to 0}\int\frac{d\mathbf{r}}{r^2}\left(\frac{e^{-r^2/a^2}}{\pi^{3/2}a^3}\right)$$

$$=\lim_{a\to 0}\left(\frac{2}{a^2}\right)=\infty\,, \quad (15)$$

$$\int d\mathbf{y}\frac{\delta^{(3)}(\mathbf{y}-\mathbf{x})}{|\mathbf{y}-\mathbf{x}|^3}(\mathbf{y}-\mathbf{x})=\lim_{a\to 0}\int\frac{d\mathbf{r}}{r^3}\left(\frac{e^{-r^2/a^2}}{\pi^{3/2}a^3}\right)\mathbf{r}=0\,,$$

the quadratic divergence in $\mathfrak{M}(\alpha\to\beta+e^-+e^+)$ indeed arises from the equal-time commutation relation $[J_\lambda^{(+)}(\mathbf{y}),J_\mu^{(-)}(\mathbf{x})]_-=f_{\lambda\mu}^{(-)}(\mathbf{x})\delta^{(3)}(\mathbf{y}-\mathbf{x})$ with nonzero $f_{\lambda\mu}^{(-)}(\mathbf{x})$. Physically, the fact that $f_{\lambda\mu}^{(-)}(\mathbf{x})\neq 0$ corresponds to the fact that the same basic constituent of the hadron can emit and reabsorb the virtual neutrino when an electron-positron pair is emitted so that no natural cutoff for the energy of this virtual neutrino is available. We remark that had we approached the calculation of $\mathfrak{M}(\alpha\to\beta+e^-+e^+)$ using covariant rather than noncovariant perturbation theory, and had we retained only its leading part by means of the Bjorken procedure, we would have found just the quadratically divergent term in Eq. (10) containing $[J_\lambda^{(+)}(\mathbf{y}), J_\mu^{(-)}(\mathbf{x})]_-$, and thus drawn conclusions identical to those reached above. Finally, we note the last term on the right-hand side of Eq. (14) which gives a finite contribution to $\mathfrak{M}(\alpha\to\beta+e^-+e^+)$ and which arises from $[J_\lambda^{(+)}(\mathbf{y}),J_\mu^{(-)}(\mathbf{x})]_+$ [see Eqs. (12) and (10)]; it may well turn out that this term will constitute the main part of a physically sensible, second-order weak calculation of $\mathfrak{M}(\alpha\to\beta+e^-+e^+)$.

We proceed to consider the second-order weak processes with $\Delta Q=\pm 2$ and $\Delta S=0$, namely, the no-neutrino double-β decay

$$N_i \begin{matrix}\nearrow \text{any } N_n+e_1^-+\nu \searrow \\ \searrow \text{any } N_n+e_2^-+\nu \nearrow\end{matrix} N_f+e_1^-+e_2^-\,, \quad (16)$$

where N_i, N_n, and N_f are hadrons, e.g., $N_i=\mathrm{Te}^{130}$, $N_n=\mathrm{I}^{130}$, I^{130*}, $\cdots$, $N_f=\mathrm{Xe}^{130}$. By means of an analysis analogous to that given in Eqs. (5)–(9), but with the $\mathcal{H}_{\mathrm{weak}}$ of Eq. (5) containing the L_λ' of Eq. (2) instead of the L_λ of Eq. (1), we obtain an expression for $\mathfrak{M}(N_i\to N_f+e_1^-+e_2^-)$ similar to that in Eqs. (10) and (8), but with the essential difference that $J_\mu^{(-)}(\mathbf{x})$ is replaced by $J_\mu^{(+)}(\mathbf{x})$ and $(1+\gamma_5)$ in $l_{\lambda\mu}$ and $\mathbf{l}'_{\lambda\mu}$ is replaced by 2η. Since $[J_\lambda^{(+)}(\mathbf{y}),J_\mu^{(+)}(\mathbf{x})]_-=0$, it follows that $\mathfrak{M}(N_i\to N_f+e_1^-+e_2^-)$ is convergent and dependent only on $[J_\lambda^{(+)}(\mathbf{y}), J_\mu^{(+)}(\mathbf{x})]_+$. In addition we now have, using Eq. (11),

$$[J_\lambda^{(+)}(\mathbf{y}),J_\mu^{(+)}(\mathbf{x})]_+ =2J_\lambda^{(+)}(\mathbf{y})J_\mu^{(+)}(\mathbf{x})=2\sum_{n,m}\tau_n^{(+)}\tau_m^{(+)}(\Gamma_\lambda)_n(\Gamma_\mu)_m \times\delta^{(3)}(\mathbf{y}-\mathbf{r}_n)\delta^{(3)}(\mathbf{x}-\mathbf{r}_m) \quad (17)$$

$$=2\sum_{n,m}\tau_n^{(+)}\tau_m^{(+)}(\Gamma_\lambda)_n(\Gamma_\mu)_m(1-\delta_{nm}) \times\delta^{(3)}(\mathbf{y}-\mathbf{r}_n)\delta^{(3)}(\mathbf{x}-\mathbf{r}_m)\,,$$

where the last equality is a consequence of $\tau_n^{(+)}\tau_n^{(+)}=0$. Thus, $\mathfrak{M}(N_i\to N_f+e_1^-+e_2^-)$ is proportional to

$$\langle N_f|2\sum_{n,m}\tau_n^{(+)}\tau_m^{(+)}(\Gamma_\lambda)_n(\Gamma_\mu)_m \times(1-\delta_{nm})\frac{e^{-i(\mathbf{p}_1\cdot\mathbf{r}_n+\mathbf{p}_2\cdot\mathbf{r}_m)}}{|\mathbf{r}_n-\mathbf{r}_m|^3}(\mathbf{r}_n-\mathbf{r}_m)|N_i\rangle$$

$$\cong-\tfrac{1}{3}i(\mathbf{p}_1-\mathbf{p}_2)\langle N_f|2\sum_{n,m}\tau_n^{(+)}\tau_m^{(+)}(\Gamma_\lambda)_n(\Gamma_\mu)_m \times(1-\delta_{nm})\left(\frac{1}{|\mathbf{r}_n-\mathbf{r}_m|}\right)|N_i\rangle\,, \quad (18)$$

where the approximate equality follows since in all double-β decays of interest $|N_i\rangle$ and $|N_f\rangle$ are characterized by zero spin and the same parity. We note for the sake of completeness that a term proportional to

$$(E_1-E_2)\langle N_f|2\sum_{n,m}\tau_n^{(+)}\tau_m^{(+)}(\Gamma_\lambda)_n(\Gamma_\mu)_m \times(1-\delta_{nm})\left(\frac{1}{|\mathbf{r}_n-\mathbf{r}_m|}\right)|N_i\rangle \quad (19)$$

also contributes to $\mathfrak{M}(N_i\to N_f+e_1^-+e_2^-)$; this term arises if one does not totally neglect E_1 and E_2 compared to E_ν in the energy denominators of the second-order perturbation expression for $\mathfrak{M}(N_i\to N_f+e_1^-+e_2^-)$.[3]

3. ESTIMATES OF THE DEGREE OF LEPTON NONCONSERVATION

For an initial estimate of the degree of lepton nonconservation in weak interactions, we use the standard two-nucleon mechanism for no-neutrino double-β decay.[3] Two neutrons within the nucleus undergo a second-order weak transition into two protons and simultaneously emit an electron pair. Since the decay rate is proportional to η^2 [see Eq. (2)] the measured half-life of Te^{130} enables us to place a limit on the size of η.

In order to sharpen the limit, we consider an alternative mechanism in which an $N^*(1236)$ within the nucleus undergoes no-neutrino double-β decay directly [Eq. (3)]. Even though we assume that the probability of finding the $N^*(1236)$ inside the nucleus is only about 1%, we gain more than enough from other factors to render the associated nuclear matrix element appreciably greater than in the two-nucleon mechanism. As a result, we improve the limit on η by an order of magnitude.

A. Two-Nucleon Mechanism

The half-life of the no-neutrino double-β decay of a nucleus on the basis of the two-nucleon mechanism can be expressed as

$$\frac{\ln 2}{T_{1/2}}=2\pi\int\int\frac{d\epsilon_1}{(2\pi)^3}\frac{d\epsilon_2}{(2\pi)^3}\delta(\epsilon_0-\epsilon_1-\epsilon_2) \times(\epsilon_1+1)^2(\epsilon_2+1)^2P(\epsilon_1,\epsilon_2)\,, \quad (20)$$

where ϵ_1 and ϵ_2 are the kinetic energies of the electrons and ϵ_0 is the energy release, all in units of m_e. The most general form of $P(\epsilon_1,\epsilon_2)$ can be calculated along the lines described in Sec. 2 using the nuclear matrix elements specified in Eqs. (18) and (19) with $(\Gamma_\lambda)_n(\Gamma_\mu)_m$ given in Eq. (12). Making the further assumption that pairs of like nucleons in the nucleus are predominantly in singlet spin states, so that

$$\langle N_f| \sum_{n,m=1}^{A} \frac{\tau_n^{(+)}\tau_m^{(+)}}{r_{nm}}(1-\delta_{nm})|N_i\rangle$$

$$\cong -\tfrac{1}{3}\langle N_f| \sum_{n,m=1}^{A} \frac{\tau_n^{(+)}\tau_m^{(+)}}{r_{nm}}\boldsymbol{\sigma}_n\cdot\boldsymbol{\sigma}_m(1-\delta_{nm})|N_i\rangle, \quad (21)$$

we find, specializing some general results obtained in an earlier paper by the present authors,[3] that

$$P(\epsilon_1,\epsilon_2)\cong\eta^2G^4(2\pi Z/137)^2(1-e^{-2\pi Z/137})^{-2}$$
$$\times\{(16/E_1E_2)(E_1-E_2)^2(E_1E_2-1)\}$$
$$\times[(g_A{}^4/9R^2)|\langle N_f|(\mathbf{Y}^{(+)})^2|N_i\rangle|^2],$$
$$E_i\equiv\epsilon_i+1,\quad i=1,2$$

$$\left\langle\frac{1}{r_{nm}}\right\rangle\equiv\langle N_f| \sum_{n,m=1}^{A} \frac{\tau_n^{(+)}\tau_m^{(+)}}{r_{nm}}\boldsymbol{\sigma}_n\cdot\boldsymbol{\sigma}_m(1-\delta_{nm})|N_i\rangle \Big/$$
$$\langle N_f| \sum_{n,m=1}^{A} \tau_n^{(+)}\tau_m^{(+)}\boldsymbol{\sigma}_n\cdot\boldsymbol{\sigma}_m(1-\delta_{mn})|N_i\rangle, \quad (22)$$
$$\langle r_{nm}\rangle\cong R,$$
$$\sum_{n,m=1}^{A} \tau_n^{(+)}\tau_m^{(+)}\boldsymbol{\sigma}_n\cdot\boldsymbol{\sigma}_m(1-\delta_{nm})=(\sum_{n=1}^{A}\tau_n^{(+)}\boldsymbol{\sigma}_n)^2\equiv(\mathbf{Y}^{(+)})^2,$$

where $\langle r_{nm}\rangle$ is the mean separation between any two nucleons in the initial or the final nucleus, $R=(1.2\times10^{-13}$ cm) $A^{1/3}$ is the nuclear radius, and $\tau_n^{(+)}\tau_n^{(+)}=0$ (since the nucleons have isospin $\frac{1}{2}$). Thus, combining Eqs. (20) and (22), we obtain the half-life of the no-neutrino double-β decay of a nucleus calculated on the basis of the two-nucleon mechanism[3]

$$T_{1/2}\cong[10^{19.9}/\eta^2f(\epsilon_0)](137/2\pi Z)^2(1-e^{-2\pi Z/137})^2$$
$$\times(A/130)^{2/3}|\langle N_f|(\mathbf{Y}^{(+)})^2|N_i\rangle|^{-2}\text{ years},\quad (23)$$

where

$$f(\epsilon_0)\equiv\epsilon_0{}^2(\epsilon_0{}^5+13\epsilon_0{}^4+77\epsilon_0{}^3+70\epsilon_0{}^2).\quad (24)$$

We also record the corresponding electron-electron angular correlation function[3]

$$C(\hat{p}_1\cdot\hat{p}_2)\cong1+[p_1p_2/(E_1E_2-1)]\hat{p}_1\cdot\hat{p}_2\cong1+\hat{p}_1\cdot\hat{p}_2.\quad (25)$$

The nucleus Te^{130} has $\epsilon_0=5.0$ and so its half-life is predicted from Eqs. (23) and (24) to be $\cong10^{13.4}\eta^{-2}|\langle N_f|(\mathbf{Y}^{(+)})^2|N_i\rangle|^{-2}$ years. If all of the measured half-life $T_{1/2}(Te^{130})=10^{21.34}$ years is attributed to no-neutrino double-β decay, then

$$\eta\cong10^{-4.0}/|\langle N_f|(\mathbf{Y}^{(+)})^2|N_i\rangle|,\quad (26)$$

whence, if the nuclear matrix element $\langle N_f|(\mathbf{Y}^{(+)})^2|N_i\rangle$ is estimated to have a magnitude ≈0.1,

$$\eta\approx10^{-3}.\quad (27)$$

This limit on η is considerably lower than the limit obtained in other ways; for example, the nonobservation of the process $\bar{\nu}+Cl^{37}\to Ar^{37}+e^-$ yields $\eta\cong0.2$, while the measured longitudinal spin polarization of electrons emitted in single-β decay yields $\eta\cong0.1$.

With the value of $\eta|\langle N_f|(\mathbf{Y}^{(+)})^2|N_i\rangle|$ given in Eq. (26), we can now estimate the half-lives of Te^{128} and Ca^{48}, for which $\epsilon_0=1.7$ and 8.6, respectively. From Eqs. (23) and (24) we obtain

$$T_{1/2}(Te^{128})\cong T_{1/2}(Te^{130})f(5.0)/f(1.7)$$
$$=(10^{21.34}\text{ years})10^{2.40}=10^{23.7}\text{ years}\quad (28a)$$

and

$$T_{1/2}(Ca^{48})\cong T_{1/2}(Te^{130})\left(\frac{54}{22}\right)^2\left(\frac{1-e^{-2\pi\times22/137}}{1-e^{-2\pi\times54/137}}\right)^2$$
$$\times\left(\frac{48}{130}\right)^{2/3}\frac{f(5.0)}{f(8.6)}\quad (28b)$$
$$=(10^{21.34}\text{ years})10^{-0.75}=10^{20.6}\text{ years}.$$

This estimate of the Te^{128} half-life is much closer to the experimental value of $10^{22.5\pm0.5}$ years than is the estimated Te^{128} half-life for two-neutrino double-β decay: $\cong10^{25.3}$ years $[\cong T_{1/2}(Te^{130})\times10^{4.0}=(10^{21.34}$ years) $\times10^{4.0}$, see Sec. 1]. The estimate of the Ca^{48} half-life is not inconsistent with the experimental lower limit of $10^{21.3}$ years[10] since it may well be that the nuclear matrix element $\langle N_f|(\mathbf{Y}^{(+)})^2|N_i\rangle$ is actually some two to three times smaller in $Ca^{48}\to Ti^{48}$ than in $Te^{130}\to Xe^{130}$.

B. $N^*(1236)$ Mechanism

As we have already mentioned, it has been suggested that the deuteron and presumably most other nuclei contain about a 1% probability admixture of the $N^*(1688)$.[9] If this is indeed the case, then it is likely that the various nuclei involved in double-β decay also contain a similar probability admixture of $N^*(1236)$. This particle, having isospin $\frac{3}{2}$, can undergo no-neutrino double-β decay directly, as set down in Eq. (3), and its decay amplitude will therefore contribute to the amplitude for the nuclear no-neutrino double-β decay. In fact, the contribution of the $N^*(1236)$ may well dominate the nuclear amplitude, as we shall now show.

If the $N^*(1236)$ were treated as "elementary," i.e., as a point particle, then, as noted above, the emission and reabsorption of a virtual neutrino in the processes of Eq. (3) would give rise to a divergence in the associated second-order weak-matrix element. To overcome this difficulty, we adopt a quark model for the $N^*(1236)$ and the nucleon. Because the isospin of the (nonstrange)

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[10] The limit quoted is from R. K. Bardin, P. J. Gollon, J. D. Ullman, and C. S. Wu, Phys. Letters **26B**, 112 (1967), where references to earlier experimental work are also given.

quarks is $\frac{1}{2}$, no single quark can emit and reabsorb the virtual neutrino and simultaneously emit two electrons, and so Eq. (3) represents a two-quark process. The mean separation between the two quarks involved then serves to cut off the divergence in the same way as in the two-nucleon case.

On the average, the mean separation between any two quarks within the $N^*(1236)$ or within the nucleon is about equal to the charge radius of the proton, i.e., about 0.1 the mean separation between any two nucleons in medium-to-heavy nuclei. This property compensates for the small probability of finding an $N^*(1236)$ inside the nucleus. In addition, because only one particle of the nucleus is now involved in the double-β decay, the overlap between initial and final nuclear wave functions is larger than in the two-nucleon mechanism so that, in general, the nuclear matrix element for no-neutrino double-β decay is estimated to be larger on the $N^*(1236)$ mechanism than on the two-nucleon mechanism.

In the $N^*(1236)$ mechanism, using the quark model, the matrix element for the first process of Eq. (3), $\mathfrak{M}(S_{N^*},S_p)$, can be calculated by the same method as is used in the two-nucleon mechanism.[3] The general form of this matrix element is given by

$$\mathfrak{M}(S_{N^*},S_p)=(\eta G^2/6\pi)(u_e^\dagger(\mathbf{p}_2)i\gamma_4\boldsymbol{\gamma}u_e^*(\mathbf{p}_1))\cdot(\mathbf{p}_1-\mathbf{p}_2)\times\mathbf{M}(S_{N^*},S_p)\,,$$

$$\begin{aligned}\mathbf{M}(S_{N^*},S_p)&\equiv g_Vg_A\langle p;S_p|\sum_{n,m=1}^{3}\frac{\tau_n^{(+)}\tau_m^{(+)}}{r_{nm}}\times\tfrac{1}{2}(\boldsymbol{\sigma}_n+\boldsymbol{\sigma}_m)(1-\delta_{nm})|N^{*-};S_{N^*}\rangle\\&=g_Vg_A\langle 1/r_{nm}\rangle\times\langle p;S_p|I^{(+)}\mathbf{Y}^{(+)}|N^{*-};S_{N^*}\rangle,\end{aligned}\quad(29)$$

$$\sum_{n,m=1}^{3}\tau_n^{(+)}\tau_m^{(+)}\boldsymbol{\sigma}_m(1-\delta_{nm})=(\sum_{n=1}^{3}\tau_n^{(+)})(\sum_{n=1}^{3}\tau_n^{(+)}\boldsymbol{\sigma}_n)\equiv I^{(+)}\mathbf{Y}^{(+)}\,,$$

where $\langle r_{nm}\rangle$ is the mean separation between any two quarks within the $N^*(1236)$ or the nucleon, $\tau_n^{(+)}\tau_n^{(+)}=0$ (since the quarks have isospin $\frac{1}{2}$), S_{N^*} and S_P are the z components of the spin of N^{*-} and p, and $\mathbf{M}\ (S_{N^*},S_p)$ does not contain terms proportional to g_V^2 or to g_A^2 because the $N^*(1236)\leftrightarrow$ nucleon transition involves a spin change of one unit. Then, evaluating $\mathbf{M}(S_{N^*},S_p)$ by means of $SU(6)$-type spin-isospin wave functions with perfect radial overlap for the quarks in the $N^*(1236)$ and in the nucleon,[11] we get

$$\begin{aligned}[M(S_{N^*},S_p)]_{x\pm iy}&=\mu C_{S_{N^*}\pm1S_p}{}^{\frac{3}{2}1\frac{1}{2}}\,,\\ [M(S_{N^*},S_p)]_z&=\mu C_{S_{N^*}0S_p}{}^{\frac{3}{2}1\frac{1}{2}}\,,\\ \mu&\equiv g_Vg_A\langle 1/r_{nm}\rangle\end{aligned}\quad(30)$$

[11] R. H. Dalitz, in *High Energy Physics*, edited by C. DeWitt and M. Jacob (Gordon and Breach Science Publishers, Inc., New York, 1965), p. 253.

where the $C_{m_1m_2m_3}{}^{j_1j_2j_3}$ are Clebsch-Gordan coefficients. Using Eqs. (29) and (30), we obtain the half-life of the no-neutrino double-β decay of a nucleus calculated on the basis of the $N^*(1236)$ mechanism

$$T_{1/2}\cong\frac{10^{17.5}}{\eta^2g(\epsilon_0)}\left(\frac{137}{2\pi Z}\right)^2(1-e^{-2\pi Z/137})^2\left(\frac{\langle r_{nm}\rangle}{0.7\times10^{-13}\text{cm}}\right)^2\times(P(N^*))^{-1}|\langle\Phi_f|\Phi_i\rangle|^{-2}\text{ years},\quad(31)$$

where

$$g(\epsilon_0)\equiv\epsilon_0^2(\epsilon_0^5+14\epsilon_0^4+81\epsilon_0^3+221\epsilon_0^2+228\epsilon_0+140)\,,\quad(32)$$

and where $P(N^*)$ is the probability of finding an $N^*(1236)$ in the nucleus, and $|\langle\Phi_f|\Phi_i\rangle|^2$ is an overlap factor between the initial and final nuclear wave functions. This overlap factor is normalized to unity for identical momentum distributions (1) of the N^* and the nucleon into (out of) which the N^* transforms, and (2) of the other $A-1$ nucleons. We also obtain the corresponding electron-electron angular-correlation function

$$\begin{aligned}(C\hat{p}_1\cdot\hat{p}_2)&=1-\left(\frac{p_1p_2}{E_1E_2}\right)\left(\frac{(E_1+E_2)^2E_1E_2}{(E_1E_2+1)(p_1^2+p_2^2)+p_1^2p_2^2}\right)\hat{p}_1\cdot\hat{p}_2\\&\quad+\left(\frac{p_1p_2}{E_1E_2}\right)^2\frac{E_1^2E_2^2}{(E_1E_2+1)(p_1^2+p_2^2)+p_1^2p_2^2}(\hat{p}_1\cdot\hat{p}_2)^2\\&\cong(1-\hat{p}_1\cdot\hat{p}_2)(1-\tfrac{1}{3}\hat{p}_1\cdot\hat{p}_2)\,.\end{aligned}\quad(33)$$

Equations (31) and (32) with $\langle r_{nm}\rangle\cong0.7\times10^{-13}$ cm yield a half-life $\cong10^{10.8}\eta^{-2}[P(N^*)]^{-1}|\langle\Phi_f|\Phi_i\rangle|^{-2}$ years for $Te^{130}(\epsilon_0=5)$; comparison of this with the measured half-life of $10^{21.34}$ years yields

$$\eta\cong10^{-5.3}/[P(N^*)]^{1/2}|\langle\Phi_f|\Phi_i\rangle|\,,\quad(34)$$

whence, taking $P(N^*)\approx0.01$ and estimating (rather conservatively) $|\langle\Phi_f|\Phi_i\rangle|\approx(0.1)^{1/2}$,

$$\eta\approx10^{-4}.\quad(35)$$

This limit on η is lower than the limit on η in Eq. (27) by an order of magnitude. The half-lives of Te^{128} ($\epsilon_0=1.7$) and Ca^{48} ($\epsilon_0=8.6$) predicted on the basis of Eqs. (31) and (32) are

$$\begin{aligned}T_{1/2}(Te^{128})&\cong T_{1/2}(Te^{130})g(5.0)/g(1.7)\\&=(10^{21.34}\text{ years})10^{2.13}=10^{23.5}\text{ years},\end{aligned}\quad(36a)$$

$$\begin{aligned}T_{1/2}(Ca^{48})&\cong T_{1/2}(Te^{130})\left(\frac{54}{22}\right)^2\left(\frac{1-e^{-2\pi\times22/137}}{1-e^{-2\pi\times54/137}}\right)^2\frac{g(5.0)}{g(8.6)}\\&=(10^{21.34}\text{ years})10^{-0.82}=10^{20.5}\text{ years},\end{aligned}\quad(36b)$$

in essential agreement with Eqs. (28a) and (28b) and with the experimental data. Finally, it is instructive to record the half-life for the double-β decay of a free N^* as calculated from Eqs. (31), (32), and (35)—we find $T_{1/2}(N^{*-}\to p+e^-+e^-)\approx10^{17.5}/(10^{-4})^2(600)^7$ years

$=4\times10^{13}$ sec; this is greater by a factor of 10^{23} than the half-life for the single-β decay of a free N^* and by a factor of 10^{37} than the half-life for the pionic decay of a free N^*.

4. SUMMARY AND DISCUSSION

We have shown that if the transition $Te^{130}\rightarrow Xe^{130}$ is assumed to be largely due to lepton-nonconserving no-neutrino double-β decay, and if the dominant mechanism in this decay involves an $N^*(1236)\leftrightarrow$ nucleon transition within the nucleus, then, granting the validity of the above estimate of $P(N^*)|\langle\Phi_f|\Phi_i\rangle|^2$, the limit on the "lepton-nonconservation" parameter is $\eta\approx10^{-4}$. This limit is an order of magnitude smaller than the one obtained in an analogous way from the standard two-nucleon mechanism and is very much smaller than the limits on η which are obtained from the analysis of first-order weak processes. We also note that the electron-electron angular-correlation functions of Eqs. (33) and (25) are quite different so that a measurement of the electron-electron angular correlation could conceivably decide whether the $N^*(1236)$ mechanism is indeed dominant over the two-nucleon mechanism.

Although the measure half-life[1] for $Te^{130}\rightarrow Xe^{130}$ is in good agreement with theoretical estimates for the lepton-conserving two-neutrino double-β decay,[3] we have chosen, pursuing a suggestion of Pontecorvo,[4] to interpret this process as a lepton-nonconserving no-neutrino double-β decay because of an apparent anomaly in the case of Te^{128}. If the excess of Xe^{128} found in a recent mass-spectrometric experiment[5] can be ascribed to the double-β decay $Te^{128}\rightarrow Xe^{128}$, it corresponds to a half-life for Te^{128} whose ratio to the measured half-life of Te^{130} is consistent with the no-neutrino process, but is three orders of magnitude smaller than that expected for the two-neutrino process. At present, the arguments in favor of ascribing the excess Xe^{128} to $Te^{128}\rightarrow Xe^{128}$ are not conclusive, and they may not survive further experimentation. In the event that they do survive, however, we will be forced to accept the existence of lepton-nonconserving no-neutrino double-β decay, since it is extremely unlikely that the nuclear matrix elements for the lepton-conserving two-neutrino double-β decays of Te^{128} and Te^{130} differ in magnitude by a factor as large as $10^{3/2}$.[12] We emphasize, however, that by far the best way to settle the question of the presence or absence of neutrinos in double-β decay is to observe the electron-electron energy-sum spectrum.[3] This spectrum will exhibit two peaks if the half-lives of no-neutrino and two-neutrino double-beta decay are not too different: a broad relatively low peak in the vicinity of $\frac{1}{2}\epsilon_0$ from the two-neutrino process and a narrow relatively high peak at ϵ_0 from the no-neutrino process. A search for these peaks and a determination of the corresponding branching ratio is obviously of crucial interest.

In the above discussion we parametrized a possible lepton nonconservation by inclusion in the lepton weak current of an "opposite-helicity" term: $\eta\psi_e^\dagger\gamma_4\gamma_\lambda(1-\gamma_5)\psi_\nu$; however, we kept our "Majorana" neutrino massless: even though a nonvanishing η in general implies a nonvanishing m_ν proportional to η. Alternatively, we could have parametrized lepton nonservation by supposing that $\eta=0$ and $m_\nu\neq0$; lifetime calculations of no-neutrino double-β decay would then lead essentially to the formulas derived above but with η replaced by $(m_\nu\langle E_\nu^{\text{virtual}}\rangle^{-1})\approx(m_\nu\langle_{nm}\rangle)$. The experimental limit $m_\nu<2\times10^{-4}m_e$[13] and the estimate $\langle r_{mn}\rangle \cong0.7\times10^{-13}$ cm $=\frac{1}{2}m_\pi^{-1}$ then yield $(m_\nu\langle r_{nm}\rangle)<4\times10^{-7}$ which is some 250 times smaller than the limit required to account for the measured lifetime of Te^{130} [see the limit on η in Eq. (35)]. Thus the effect of any possible nonzero m_ν can be safely neglected in the treatment of lepton-nonconserving no-neutrino double-β decay.

Still another possible parametrization of lepton nonconservation involves inclusion in the lepton weak current of an "opposite-helicity" scalar or tensor term $\eta_S\psi_e^\dagger\gamma_4(1-\gamma_5)\psi_\nu$ or $\eta_T\psi_e^\dagger\gamma_4\sigma_{\lambda\mu}(1-\gamma_5)\psi_\nu$, instead of the above-used "opposite-helicity" vector term $\eta\psi_e^\dagger\gamma_4\gamma_\lambda \times(1-\gamma_5)\psi_\nu$. The limits that one can then set on η_S or η_T are similar numerically to those set on η [Eq. (35) or (27)] and are very much smaller than the limits presently available on any $1+\gamma_5$ and/or $1-\gamma_5$ scalar or tensor weak currents as determined by an analysis of single-β decay.

As a final remark, we would like to emphasize that, in spite of the presence of virtual-neutrino closed loops, divergences never arise in second-order weak matrix elements with $\Delta Q=\pm2$ if the basic hadronic constituents (i.e., quarks or nucleons) have isospin not exceeding $\frac{1}{2}$ and a mean separation greater than zero; this circumstance permits a quantitative estimation of lifetimes in no-neutrino double-β decay in terms of a single "lepton-nonconservation" parameter. Unfortunately, divergences arising from virtual-neutrino closed loops are present in the second-order weak matrix elements with $\Delta Q=0$, and this necessitates the introduction of auxiliary "cutoff" or "cancellation" procedures in order to obtain quantitative estimates of the second-order weak contributions to the rates of processes such as $O^{16*}\rightarrow O^{16}+e^-+e^+$, $\pi^0\rightarrow e^-+e^+$, $K_L{}^0\rightarrow\mu^-+\mu^+$, etc.

ACKNOWLEDGMENT

We wish to thank Dr. O. A. Schaeffer for a very helpful communication on the lower limit of the half-life of Te^{128} as determined from the experiments of Ref. 1.

[12] These nuclear matrix elements are of the form

$$M(Te^{128}\rightarrow Xe^{128})\cong\sum_n \langle Xe^{128}|Y^{(+)}|I_n{}^{128}\rangle\cdot\langle I_n{}^{128}|Y^{(+)}|Te^{128}\rangle/[E(I_n{}^{128})-E(Te^{128})]$$

and

$$M(Te^{130}\rightarrow Xe^{130})\cong\sum_n \langle Xe^{130}|Y^{(+)}|I_n{}^{130}\rangle\cdot\langle I_n{}^{130}|Y^{(+)}|Te^{130}\rangle/[E(I_n{}^{130})-E(Te^{130})],$$

and, on the basis of any reasonable nuclear model, are expected to have about the same magnitude.

[13] K. E. Bergkvist, in *Proceedings of the CERN Topical Weak Interaction Conference* (CERN, Geneva, 1969), p. 91.