gives

$$I(W)=\frac{\pi^4(W-4m_\pi)^{9/2}}{W}\frac{\sqrt{2}}{5}.$$

It can be shown that this formula is valid up to $W=6m_\pi$ with a precision of 20%.

Thus we finally have

$$\frac{\sigma_{e^+e^-\to 2\pi^+2\pi^-}(W)}{\sigma_{e^+e^-\to\pi^+\pi^-}(W)}=\left(\frac{g^2}{4\pi}\right)^2 C^2 \frac{(W-4m_\pi)^{9/2}}{[1-(W^2/4m_\rho^2)(1-\delta)]^2(W^2-4m_\pi^2)^{3/2}m_\pi^{3/2}} 2\times10^{-4}.$$

Of course, this ratio for $W^2=m_\rho^2$ also gives the branching ratio of the corresponding decay modes of the ρ^0 meson. For $g^2/4\pi\approx 2$, $\delta=1$, and $\xi=1$ (σ model), we have

$$\Gamma_{\rho^0\to 2\pi^+2\pi^-}/\Gamma_{\rho^0\to\pi^+\pi^-}\approx 10^{-4},$$

to be compared with the experimental[7] upper limit 1.5×10^{-3}. For completeness we give here also the results for other decay modes:

$$\Gamma_{\rho^0\to 2\pi^0\pi^+\pi^-}/\Gamma_{\rho^0\to 2\pi^+2\pi^-}=\tfrac{1}{4}, \quad \Gamma_{\rho^+\to 2\pi^+\pi^-\pi^0}/\Gamma_{\rho^0\to 2\pi^+2\pi^-}=\tfrac{3}{4}.$$

Thus, if chiral predictions are correct, the branching ratios for the 4π decay modes of the ρ meson are extremely small.

Note added in manuscript. Since completing this paper we have been informed that B. Renner (private communication) has done a similar calculation using hard-pion techniques.

[7] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968).

PHYSICAL REVIEW VOLUME 184, NUMBER 5 25 AUGUST 1969

$SU(3)$ Model of Double Octets*

R. H. Graham and J. W. Moffat
Department of Physics, University of Toronto, Toronto, Canada

(Received 20 January 1969; revised manuscript received 7 April 1969)

On the assumption that both peaks of the A_2 have $J^P=2^+$, an $SU(3)$ double octet model is proposed. A sum rule predicts a K_N' at $\simeq$1300 MeV and implies that the two octets can be manifestations of an octet of fundamental double poles. Predictions of partial widths for the decays of the two octets are in reasonable agreement with experiment. The sum rule is also well satisfied for two octets of $J^P=1^+$ mesons and for two octets of $J^P=1^-$ mesons, provided the ρ' and $K^{*\prime}$ mesons are degenerate in mass with the $\rho(765)$ and $K^*(890)$.

THERE is increasing evidence for the splitting of the A_2 meson.[1,2] In the following, we shall explore the consequences for $SU(3)$ if both the A_2 peaks have $J^P=2^+$, as seems to be favored by the missing-mass experiments at CERN.[1,2] If indeed both A_{2L} and A_{2H} have $J^P=2^+$, then this would suggest that there are two $J^P=2^+$ octets, and we may anticipate that there is octet-octet mixing.[3] We shall adopt the position that the $f^0(1260)$ and $f^{0\prime}(1515)$ correspond to the isoscalar members of the two octets; this is in contrast to the usual picture,[4] which takes the f^0 and $f^{0\prime}$ to be mixtures of an octet and a singlet. In order to complete the two octets, we predict a partner for the $K_N(1420)$ which we call the K_N'.

We shall choose the following form for the mass Lagrangian:

$$L=L_0^M+L_B^M, \tag{1}$$

where

$$L_0^M=m_{10}^2\,\mathrm{Tr}(M_1^\dagger M_1)+m_{20}^2\,\mathrm{Tr}(M_2^\dagger M_2) \tag{2}$$

and

$$\begin{aligned}L_B^M=\tfrac{1}{2}m_{12}^2[(M_1^\dagger)_3{}^\alpha(M_2)_\alpha{}^3+(M_1^\dagger)_\alpha{}^3(M_2)_3{}^\alpha\\ -\tfrac{2}{3}\,\mathrm{Tr}(M_1^\dagger M_2)]+\tfrac{1}{2}m_1^2[(M_1^\dagger)_3{}^\alpha(M_1)_\alpha{}^3\\ +(M_1^\dagger)_\alpha{}^3(M_1)_3{}^\alpha-\tfrac{2}{3}\,\mathrm{Tr}(M_1^\dagger M_1)]\\ +(1\leftrightarrow 2).\end{aligned} \tag{3}$$

Here M_1 and M_2 denote the matrices corresponding to the two fundamental $J^P=2^+$ octets; m_{10} and m_{20} are the "bare" masses of the members of the octets, m_1^2 and m_2^2 are the usual Gell-Mann–Okubo mass-splitting

* Supported in part by the National Research Council of Canada.

[1] B. French, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 91.

[2] G. E. Chikovani *et al.*, Phys. Letters **28B**, 526 (1969).

[3] Octet-octet mixing has been considered by P. G. O. Freund, Phys. Rev. Letters **12**, 348 (1964).

[4] S. L. Glashow and R. H. Socolow, Phys. Rev. Letters **15**, 329 (1965).

TABLE I. Decays of the $J^P=2^+$ octets.

Process	Predicted width (MeV)[a]	Experimental width (MeV)
$f'\to 2\pi$	0*	$<10\pm3$
$f'\to K\bar{K}$	29	53_{-23}^{+28}
$f'\to 2\eta$	17	$<29\pm9$
$f\to 2\pi$	167	145 ± 25
$f\to K+\bar{K}$	0.7	$<36\pm6$
$f\to 2\eta$	0	...
$A_{2L}\to \pi\eta$	5	...
$A_{2L}\to K\bar{K}$	2	...
$A_{2H}\to \pi\eta$	6	...
$A_{2H}\to K\bar{K}$	3	...
$K_N\to K\pi$	24	45_{-6}^{+7}
$K_N\to K\eta$	0.8	$1.8_{-1.0}^{+1.1}$
$K_N'\to K\pi$	14	...
$K_N'\to K\eta$	0.2	...
$A_{2L}\to \rho\pi$[b]	55	...
$A_{2H}\to \rho\pi$[b]	82	...
$K_N\to K^*\pi$	30*	30_{-5}^{+6}
$K_N\to \rho K$	8	9.8_{-3}^{+3}
$K_N'\to K^*\pi$	5	...

[a] Input values are starred.
[b] Although the total widths for A_{2L} and A_{2H} listed in Ref. 4 are of the order of 30 MeV, the possibility of these being much larger is raised in Ref. 1.

parameters for the two separate octets, and m_{12} is the transition mass.

In analogy with the usual treatment of ω-ϕ mixing,[5] mass matrices for the A_2, K_N, and f^0 doublets can be constructed from the Lagrangian given in Eqs. (2) and (3). By diagonalizing these mass matrices, we obtain six equations for six unknowns. These equations for the respective members of the octets are

$$\begin{aligned}
A_2:\ & m_{10}^2-\tfrac{1}{3}m_1^2+m_{20}^2-\tfrac{1}{3}m_2^2=m_{A_L}^2+m_{A_H}^2,\\
& (m_{10}^2-\tfrac{1}{3}m_1^2)(m_{20}^2-\tfrac{1}{3}m_2^2)-\tfrac{1}{9}m_{12}^4=m_{A_L}^2m_{A_H}^2;\\
f^0:\ & m_{10}^2+\tfrac{1}{3}m_1^2+m_{20}^2+\tfrac{1}{3}m_2^2=m_{f^0}^2+m_{f^{0\prime}}^2,\\
& (m_{10}^2+\tfrac{1}{3}m_1^2)(m_{20}^2+\tfrac{1}{3}m_2^2)-\tfrac{1}{9}m_{12}^4=m_{f^0}^2m_{f^{0\prime}}^2;\\
K_N:\ & m_{10}^2+\tfrac{1}{6}m_1^2+m_{20}^2+\tfrac{1}{6}m_2^2=m_{K_N}^2+m_{K_N'}^2,\\
& (m_{10}^2+\tfrac{1}{6}m_1^2)(m_{20}^2+\tfrac{1}{6}m_2^2)-(1/36)m_{12}^4\\
& \qquad =m_{K_N}^2m_{K_N'}.
\end{aligned}$$

From these equations, we can find the mass of the missing K_N resonance

$$m_{K_N'}^2=\tfrac{1}{4}(m_{A_L}^2+m_{A_H}^2)+\tfrac{3}{4}(m_{f^0}^2+m_{f^{0\prime}}^2)-m_{K_N}^2. \quad (4)$$

If we substitute the mean experimental masses[6] into the right-hand side of (4), we get $m_{K_N'}=1311$ MeV. A possible candidate[7] for this resonance has been observed in the $K\pi$ mass spectrum in the reaction $K^+p\to K^0\pi^+p$ at 3 and 3.5 GeV/c with a mass $m_{K_N'}=1260\pm30$ MeV, and since it is observed in the $K\pi$ mode, this limits its J^P to the series 0^+, 1^-, 2^+, $\cdots$.

Within the experimental uncertainties of the masses, we find that $m_{10}^2\simeq m_{20}^2$, and this result does not depend sensitively upon the experimental errors of the masses. Because of the error in the Gell-Mann–Okubo formula, i.e., the neglect of the 27-plet symmetry-breaking terms, we shall work with the mean experimental masses. We find that $m_0^2\simeq 1.8$ GeV^2 and $m_1^2+m_2^2\simeq 0.8$ GeV^2.

In order to describe the decays, we shall assume that the Lagrangian for the decay into two pseudoscalar mesons has the form

$$\begin{aligned}
& g_1\,\mathrm{Tr}(M_1\{P,P\})+g_2\,\mathrm{Tr}(M_2\{P,P\})\\
& \qquad +g_1'(M_1)_3{}^3\,\mathrm{Tr}(PP)+g_2'(M_2)_3{}^3\,\mathrm{Tr}(PP). \quad (5)
\end{aligned}$$

For the decay into a vector meson and a pseudoscalar meson, we take

$$f_1\,\mathrm{Tr}(M_1[V,P])+f_2\,\mathrm{Tr}(M_2[V,P]). \quad (6)$$

In Eqs. (5) and (6), the P and V are matrix representations of the pseudoscalar and vector meson octets. Also, g_1, g_2 and f_1, f_2 are the $SU(3)$ symmetric coupling constants and g_1', g_2' are the coupling constants describing a model of $SU(3)$ symmetry breaking. Because the f^0 and $f^{0\prime}$ suffer the largest splitting, we have taken the symmetry-breaking coupling constants to influence only these particles. We have not assumed any symmetry breaking in Eq. (6), because there are insufficient data to determine it at present.

In view of the simple interpretation that we have adopted of the $SU(3)$ breaking in Eq. (5) and the neglect of breaking in Eq. (6), the predictions can be considered to be in reasonable agreement with experiment.

We shall now describe the decays of the two octets and calculate the partial widths. In general, the physical A_2, K_N, and f^0 mesons are mixtures of the corresponding members of M_1 and M_2 depending on the three angles θ_{A_2}, θ_{K_N}, and θ_f. These mixing angles may be determined by m_{10}^2, m_{20}^2, m_1^2, m_2^2, and m_{12}^2. Since we have assumed that $m_{10}^2\simeq m_{20}^2\simeq m_0^2$, we are unable to solve explicitly for m_{12}^2, but only for the combination $m_1^2m_2^2-m_{12}^4$ which appears to be small. In view of this and the fact that $m_1^2+m_2^2$ turns out to be smaller than $2m_0^2$, we expect m_{12}^2 to be small, and, in fact, it is perfectly consistent to take $m_{12}^2=0$ in our model. This leads to $\theta_{A_2}=\theta_f=\theta_{K_N}=\pm\frac{1}{2}\pi$, which corresponds to having zero mixing between the octets. In reality, we expect there to be some mixing, but we shall adopt this simple model to describe the decays.

By virtue of our solution $m_1^{02}\simeq m_2^{02}$, it is reasonable to assume that $g_1=g_2=g$ and $f_1=f_2=f$. We shall also assume that $g'=-g_1'=g_2'$.[8]

[5] See, for example, P. A. Carruthers, *Introduction to Unitary Symmetry* (Wiley-Interscience, Inc., New York, 1966).
[6] N. Barash-Schmidt *et al.*, University of California Lawrence Radiation Laboratory Report No. UCRL-8030, 1968 (unpublished).
[7] B. French, Ref. 1, p. 116.
[8] With this choice of coupling constants and the mixing angle $\theta=\frac{1}{2}\pi$, our model corresponds to the singlet-octet-mixing model of $J^P=2^+$ decays when, in that model, the mixing angle is $\frac{1}{4}\pi$. It should also be pointed out that we have neglected the mixing between the $\eta(549)$ and the $\eta'(958)$.

The following values for the coupling constants $g=-15.25$ GeV^{-1} and $g'=6.82$ GeV^{-1} yield the predicted partial decay widths for decay into two pseudoscalar mesons shown in Table I.[9] The experimental value

$$\Gamma(K_N \to K^*\pi)=30 \text{ MeV}$$

is used to determine f with the result $f=35.45$ GeV^{-2}. The predicted partial decay widths for decay into a vector meson and a pseudoscalar meson are also shown in Table I. In calculating the partial widths, the physical masses were used in the phase-space factors.

We shall now study the consequences of a double octet model of $SU(3)$ for the $J^P=1^-$ and 1^+ meson resonances. There is mounting evidence for the existence of a $J^P=1^+$ meson $A_{1.5}(1180)$ as well as the controversial $A_1(1060)$.[1] We shall assume that the $D(1285)$ and the $E(1420)$ are the isoscalar members of the two octets of $J^P=1^+$ mesons, and that the $K_A(1230)$, $K_A'(1320)$, $A_1(1060)$, and $A_1'(1180)$ complete the two octets. Another possible K_A resonance has been reported at 1780 MeV with $I=\frac{1}{2}$, but the J^P values are still in doubt and we shall assume that it is not a 1^+ object.

In the case of the $J^P=1^+$ mesons, the sum rule analogous to (4) can be written

$$m_{K_A}{}^2+m_{K_A'}{}^2=\tfrac{1}{4}(m_{A_1}{}^2+m_{A_1'}{}^2)+\tfrac{3}{4}(m_D{}^2+m_E{}^2). \quad (7)$$

If we use the mean experimental masses,[6] the left-hand side gives 3.31 GeV^2 and the right-hand side gives 3.39 GeV^2, yielding a difference of 2%. Thus, the sum rule is remarkably well satisfied by the $J^P=1^+$ mesons.

We discovered in the case of the $J^P=2^+$ mesons that the "bare" masses of the two octets (denoted by $m_{10}{}^2$ and $m_{20}{}^2$) were approximately equal. In the present case of two $J^P=1^+$ octets, we again find that $m_{10}{}^2 \simeq m_{20}{}^2 \simeq m_0{}^2$ within the experimental errors; the values $m_0{}^2 \simeq 1.5$ GeV^2 and $m_1{}^2+m_2{}^2 \simeq 0.6$ GeV^2 are obtained. Since there are insufficient data on the $J^P=1^+$ meson partial decay widths, we shall not attempt to calculate these at present.

In the case of the $J^P=1^-$ mesons, we shall choose the $\omega(783)$ and $\phi(1019)$ mesons as the isoscalar members of the two octets and complete the octets with the $\rho(765)$, $K^*(890)$ and predicted missing members ρ' and $K^{*\prime}$. The mass formula in this case is

$$m_{K^*}{}^2+m_{K^{*\prime}}{}^2=\tfrac{1}{4}(m_\rho{}^2+m_{\rho'}{}^2)+\tfrac{3}{4}(m_\omega{}^2+m_\phi{}^2). \quad (8)$$

If we choose $m_{K^{*\prime}} \simeq m_{K^*}$ and $m_{\rho'} \approx m_\rho$, then the sum rule becomes

$$m_{K^*}{}^2=\tfrac{1}{4}m_\rho{}^2+\tfrac{3}{8}(m_\omega{}^2+m_\phi{}^2). \quad (9)$$

In terms of the mean experimental masses, the left-hand side gives 0.797 GeV^2 and the right-hand side 0.765 GeV^2, a difference of 4%. The equality $m_{10}{}^2 \simeq m_{20}{}^2$ is satisfied in this case to within 20%, and the situation in the case of the $J^P=1^-$ octets is, therefore, similar to the case of the $J^P=2^+$ and 1^+ octets. This result suggests that the ρ, ρ', and K^*, $K^{*\prime}$ mesons are degenerate (or almost degenerate) in mass and can be considered to form double poles even in the presence of $SU(3)$ breaking. It is clear that the ρ' could be at a higher mass, e.g., if the ρ' mass were at ≈ 1.6 GeV,[10] then (8) would suggest that the $K^{*\prime}$ mass was at ≈ 1.1 GeV.

We have looked at the predictions of this model for the decay widths of the $J^P=1^-$ mesons into two pseudoscalar mesons and have found that the decays can be described consistently although no useful predictions can be made in view of the number of parameters that appear in the calculations.

We have proposed a simple $SU(3)$ model describing two octets having $J^P=2^+$, assuming that both of the peaks in the split A_2 mesons have $J^P=2^+$ and $I^G=1^-$. The model predicts a K_N' around 1300 MeV and predicts the partial decay widths. It will be interesting to obtain more accurate data on the branching ratios of both A_{2L} and A_{2H} in order to test the predictions of the model. The result that $m_{10}{}^2 \approx m_{20}{}^2$ raises the interesting possibility that the $J^P=2^+$ octets have a common origin in an octet of fundamental double poles.[11] We have also proposed a double octet model of the $J^P=1^+$ and $J^P=1^-$ mesons. A double pole model of the ρ meson[12] has been shown to lead to good agreement with both electromagnetic form factor data and πp charge-exchange scattering and polarization data.

Our results for the meson octets suggest perhaps that there is a higher symmetry incorporating double octets in terms of some new quantum number. If this higher symmetry is broken, then $m_{10}{}^2$ will not equal $m_{20}{}^2$ identically.

[9] The relation between g and g' is determined by requiring that $\Gamma(f^{0\prime} \to 2\pi)=0$. It is interesting to note that in our model this also implies $\Gamma(f^0 \to 2\eta)\equiv 0$. If the coupling of $f^0 \to 2\eta$ is of the same order as $f^{0\prime} \to 2\eta$, then $\Gamma(f^0 \to 2\eta) \simeq 1$ MeV.

[10] J. W. Moffat, Phys. Rev. Letters **20**, 620 (1968).

[11] T. J. Gajdicar and J. W. Moffat, Phys. Rev. **181**, 1875 (1969), and references contained therein.

[12] R. E. Kreps and J. W. Moffat, Phys. Rev. **175**, 1942 (1968); **175**, 1945 (1968).