The $\rho \to 4\pi$ Vertex in Chiral Dynamics

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The $\rho \to 4\pi$ vertex has been computed assuming vector-meson dominance, and the resulting interaction is described by a chiral-symmetric nonlinear Lagrangian plus a symmetry-breaking term with tensorial properties. The vertex function is used to compute an approximative analytical expression for the $e^+e^- \to 4\pi$ cross section which is valid from the threshold up to 850 MeV. For the four-pion decay of the ρ meson, we have obtained the following branching ratios: $\Gamma_{\rho^0\to 2\pi^+2\pi^-}/\Gamma_{\rho^0\to\pi^+\pi^-} \approx 10^{-4}$; $\Gamma_{\rho^0\to 2\pi^0\pi^+\pi^-}/\Gamma_{\rho^0\to 2\pi^+2\pi^-} = \frac{1}{4}$; $\Gamma_{\rho^+\to 2\pi^+\pi^-\pi^0}/\Gamma_{\rho^0\to 2\pi^+2\pi^-} = \frac{3}{4}$. Experiment indicates upper limits of the order 1.5×10^{-3} for the four-pion decay-mode branching ratios.

ONE of the most interesting results of the nonlinear chiral $SU_2\times SU_2$ effective-Lagrangian model is the prediction of various many-pion vertices. Recently Olsson and Turner[1] have shown that chiral $SU_2\times SU_2$ symmetry with $(\frac{1}{2},\frac{1}{2})$ tensorial breaking (σ model) gives fairly good results for the production reaction $\pi^- p \to \pi^+\pi^- n$ at the threshold. In order to test the model further, we might consider many-pion production in pion-nucleon collisions. This type of calculation is, however, hard to perform since the contributions of different resonances are important and their couplings are unknown; thus supplementary assumptions would be necessary. This is why we have considered the $\rho \to 4\pi$ vertex, which does not imply new coupling constants and which can be directly measured in the $\rho \to 4\pi$ decay and (in the vector-meson-dominance model) in the reaction $e^+e^- \to 4\pi$.

Choosing the nonlinear transformations of the fields as in the σ model and assuming vector-meson dominance, the chiral-symmetric Lagrangian[2] is

$$\begin{aligned}\mathfrak{L}^{\text{sym}} = & -\tfrac{1}{4}(\mathbf{f}_{\mu\nu}+g\mathbf{a}_\mu\times\mathbf{a}_\nu)^2-\tfrac{1}{4}(D_\mu\mathbf{a}_\nu-D_\nu\mathbf{a}_\mu)^2 \\ & -\tfrac{1}{2}m_\rho^2(\boldsymbol{\rho}_\mu^2+\mathbf{a}_\mu^2)-(\partial_\mu\sigma+g\boldsymbol{\pi}\mathbf{a}_\mu)^2-(D_\mu\boldsymbol{\pi}-g\sigma\mathbf{a}_\mu)^2 \\ & +\frac{g\delta}{m_\rho^2}[(\mathbf{f}_{\mu\nu}+g\mathbf{a}_\mu\times\mathbf{a}_\nu)\cdot(D_\mu\boldsymbol{\pi}-g\sigma\mathbf{a}_\mu)\times(D_\nu\boldsymbol{\pi}-g\sigma\mathbf{a}_\nu) \\ & +2(D_\mu\mathbf{a}_\nu-D_\nu\mathbf{a}_\mu)\cdot(D_\mu\boldsymbol{\pi}-g\sigma\mathbf{a}_\mu)(\partial_\nu\sigma+g\boldsymbol{\pi}\cdot\mathbf{a}_\nu)],\end{aligned}$$

where

$$\mathbf{f}_{\mu\nu}=\boldsymbol{\rho}_{\mu\nu}+g\boldsymbol{\rho}_\mu\times\boldsymbol{\rho}_\nu, \quad \mathbf{a}_\mu=\mathbf{A}_\mu+\frac{1}{m_A}D_\mu\boldsymbol{\pi}, \quad D_\mu=\partial_\mu+g\boldsymbol{\rho}_\mu\times,$$

$$\sigma=(f_\pi^2-\boldsymbol{\pi}^2)^{1/2}=f_\pi-\frac{1}{2f_\pi}\boldsymbol{\pi}^2-\frac{1}{8f_\pi^3}\boldsymbol{\pi}^4+\cdots,$$

$$f_\pi=\frac{m_A}{2g} \quad (m_A=m_\rho\sqrt{2}).$$

We consider that the symmetry-breaking part of the Lagrangian is composed of two parts: one depending only on the pion field, containing also the pion mass term, and the other containing the $\omega\rho\pi$ vertex. Both these parts can be written as $(N/2,N/2)$ tensors,[3–5] but the former will modify the partial conservation of axial-vector current (PCAC).[6] For our purpose it is enough to retain only the pion mass term, the four-pion interaction, and the $\omega\rho\pi$ interaction:

$$\mathfrak{L}^{\text{break}}=-\tfrac{1}{2}m_\pi^2\boldsymbol{\pi}^2-\xi\frac{m_\pi^2}{8f_\pi^2}\boldsymbol{\pi}^4+\frac{gh}{4m_\rho}\boldsymbol{\rho}_{\mu\nu}\boldsymbol{\pi}\omega_{\lambda\sigma}\epsilon_{\mu\nu\lambda\sigma}+\cdots,$$

where for an $(N/2,N/2)$ tensorial breaking

$$\xi=[8-N(N+2)]/5,$$

and for Schwinger's mass-term breaking $\xi=4$, while $h\approx 2$ from the $\omega\to\pi\gamma$ decay width.

With these Lagrangians, we have in the tree-diagram approximation eight Feynman graphs (see Fig. 1) to describe the effective $\rho\to 4\pi$ vertex. (However, the last two diagrams do not contribute to the most interesting $\rho^0\to 2\pi^+2\pi^-$ vertex.)

We define an effective ρ-4π Lagrangian by summing the contributions of all these diagrams. We shall work in the momentum space, using the notation

$$\int dx\,\mathfrak{L}(x)=(2\pi)^4\int dp_1\cdots dp_n\,\delta(\sum^n p_i)\,\mathfrak{L}'(p_1,\cdots p_n),$$

$$\mathfrak{L}'_{\rho 4\pi}=\frac{ig^3}{m_\rho^2}\Gamma_\mu{}^{i\alpha\beta\gamma\delta}(q,p_1,p_2,p_3,p_4)\times\rho_\mu{}^i(q)\pi^\alpha(p_1)\pi^\beta(p_2)\pi^\gamma(p_3)\pi^\delta(p_4).$$

Because we are not too far from the threshold (the mean kinetic energy per pion being 50 MeV), even for the physical mass of the ρ meson, we feel content to evaluate the form factors at the threshold. We believe that the error introduced in this way is not important, but it introduces essential simplifications especially when computing the phase-space integrals. At the threshold ($q^2=-16m_\pi^2$), we find

$$\Gamma_\mu{}^{i\alpha\beta\gamma\delta}=C\delta^{\gamma\delta}\epsilon_{i\alpha\beta}p_\mu{}^1,$$

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[6] D. Arnowitt, M. H. Friedman, and P. Nath, Phys. Letters **27B**, 657 (1968).

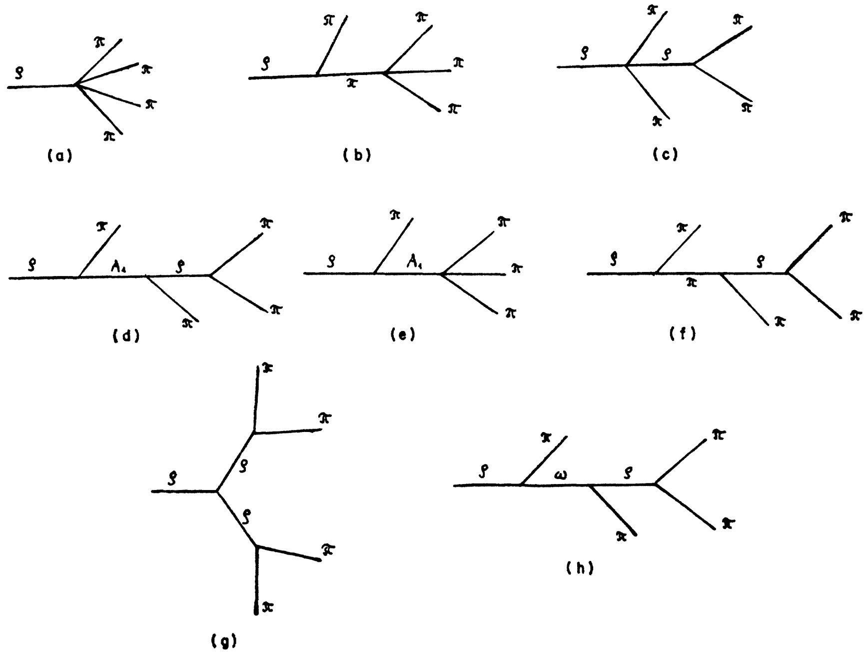

FIG. 1. Feynman graphs considered in the calculation of the effective $\rho \rightarrow 4\pi$ vertex.

where the constant C is equal to 1 for $m_\pi = 0$. For the actual pion mass its value is

$$C = 1.074 + 0.2162\xi + (0.1557 + 0.0333\xi)\delta - 0.0055\delta^2 - 0.0001\delta^3 .$$

For $0 < \delta < 1$ and N not too high, C is close to its soft-pion value.

It is interesting to remark here that the contributions of diagrams f, g, and h vanish at the threshold. The most important diagrams are a, b, and c, all being of the same order of magnitude, but the last two are of opposite sign, so that the contribution of the contact graph is dominant (in the soft-pion limit it alone gives $C = 1$).

Now, according to the vector-meson-dominance graphs of Fig. 2 the ratio of the cross sections for the processes $e^+e^- \rightarrow 4\pi$ and $e^+e^- \rightarrow 2\pi$ is

$$\frac{\sigma_{e^+e^- \rightarrow 2\pi^+2\pi^-}(W)}{\sigma_{e^+e^- \rightarrow \pi^+\pi^-}(W)} = \left(\frac{g^2}{4\pi}\right)^2 C^2 \frac{W}{[1-(W^2/4m_\rho^2)(1-\delta)]^2 (W^2-4m_\pi^2)^{3/2}} \frac{2}{\pi^5 m_\rho^4} I(W)$$

$$\frac{\sigma_{e^+e^- \rightarrow 2\pi^0\pi^+\pi^-}(W)}{\sigma_{e^+e^- \rightarrow 2\pi^+2\pi^-}(W)} = \tfrac{1}{4},$$

where $W = \sqrt{(-q^2)}$ is the c.m. energy, and

$$I(W) = \frac{\pi^3}{4W^2} \int_{4m_\pi^2}^{\infty} ds_1 \int_{4m_\pi^2}^{\infty} ds_2 \left\{ \frac{(s_1-4m_\pi^2)(s_2-4m_\pi^2)[W^2-(\sqrt{s_1}+\sqrt{s_2})^2][W^2-(\sqrt{s_1}-\sqrt{s_2})^2]}{s_1 s_2} \right\}^{1/2}$$
$$\times (s_1-4m_\pi^2)\left(1+\frac{s_1-s_2}{W^2}\right) \times \theta(W-\sqrt{s_1}-\sqrt{s_2}) .$$

In the vicinity of the threshold (first nonvanishing order in $W-4m_\pi$) the integral can be computed exactly and

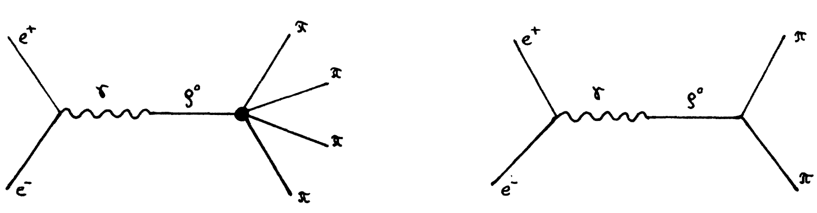

FIG. 2. ρ-meson dominance graphs for the processes $e^+e^- \rightarrow 4\pi$ and $e^+e^- \rightarrow 2\pi$.

gives

$$I(W)=\frac{\pi^4(W-4m_\pi)^{9/2}}{W}\frac{\sqrt{2}}{5}.$$

It can be shown that this formula is valid up to $W=6m_\pi$ with a precision of 20%.

Thus we finally have

$$\frac{\sigma_{e^+e^-\to2\pi^+2\pi^-}(W)}{\sigma_{e^+e^-\to\pi^+\pi^-}(W)}=\left(\frac{g^2}{4\pi}\right)^2C^2\frac{(W-4m_\pi)^{9/2}}{[1-(W^2/4m_\rho^2)(1-\delta)]^2(W^2-4m_\pi^2)^{3/2}m_\pi^{3/2}}2\times10^{-4}.$$

Of course, this ratio for $W^2=m_\rho^2$ also gives the branching ratio of the corresponding decay modes of the ρ^0 meson. For $g^2/4\pi\approx2$, $\delta=1$, and $\xi=1$ (σ model), we have

$$\Gamma_{\rho^0\to2\pi^+2\pi^-}/\Gamma_{\rho^0\to\pi^+\pi^-}\approx10^{-4},$$

to be compared with the experimental[7] upper limit 1.5×10^{-3}. For completeness we give here also the results for other decay modes:

$$\Gamma_{\rho^0\to2\pi^0\pi^+\pi^-}/\Gamma_{\rho^0\to2\pi^+2\pi^-}=\tfrac{1}{4},\quad \Gamma_{\rho^+\to2\pi^+\pi^-\pi^0}/\Gamma_{\rho^0\to2\pi^+2\pi^-}=\tfrac{3}{4}.$$

Thus, if chiral predictions are correct, the branching ratios for the 4π decay modes of the ρ meson are extremely small.

Note added in manuscript. Since completing this paper we have been informed that B. Renner (private communication) has done a similar calculation using hard-pion techniques.

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$SU(3)$ Model of Double Octets*

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On the assumption that both peaks of the A_2 have $J^P=2^+$, an $SU(3)$ double octet model is proposed. A sum rule predicts a K_N' at $\simeq1300$ MeV and implies that the two octets can be manifestations of an octet of fundamental double poles. Predictions of partial widths for the decays of the two octets are in reasonable agreement with experiment. The sum rule is also well satisfied for two octets of $J^P=1^+$ mesons and for two octets of $J^P=1^-$ mesons, provided the ρ' and $K^{*\prime}$ mesons are degenerate in mass with the $\rho(765)$ and $K^*(890)$.

THERE is increasing evidence for the splitting of the A_2 meson.[1,2] In the following, we shall explore the consequences for $SU(3)$ if both the A_2 peaks have $J^P=2^+$, as seems to be favored by the missing-mass experiments at CERN.[1,2] If indeed both A_{2L} and A_{2H} have $J^P=2^+$, then this would suggest that there are two $J^P=2^+$ octets, and we may anticipate that there is octet-octet mixing.[3] We shall adopt the position that the $f^0(1260)$ and $f^{0\prime}(1515)$ correspond to the isoscalar members of the two octets; this is in contrast to the usual picture,[4] which takes the f^0 and $f^{0\prime}$ to be mixtures of an octet and a singlet. In order to complete the two octets, we predict a partner for the $K_N(1420)$ which we call the K_N'.

We shall choose the following form for the mass Lagrangian:

$$L=L_0^M+L_B^M,\qquad(1)$$

where

$$L_0^M=m_{10}^2\,\mathrm{Tr}(M_1^\dagger M_1)+m_{20}^2\,\mathrm{Tr}(M_2^\dagger M_2)\qquad(2)$$

and

$$L_B^M=\tfrac{1}{2}m_{12}^2[(M_1^\dagger)_3{}^\alpha(M_2)_\alpha{}^3+(M_1^\dagger)_\alpha{}^3(M_2)_3{}^\alpha-\tfrac{2}{3}\,\mathrm{Tr}(M_1^\dagger M_2)]+\tfrac{1}{2}m_1^2[(M_1^\dagger)_3{}^\alpha(M_1)_\alpha{}^3+(M_1^\dagger)_\alpha{}^3(M_1)_3{}^\alpha-\tfrac{2}{3}\,\mathrm{Tr}(M_1^\dagger M_1)]+(1\leftrightarrow2).\qquad(3)$$

Here M_1 and M_2 denote the matrices corresponding to the two fundamental $J^P=2^+$ octets; m_{10} and m_{20} are the "bare" masses of the members of the octets, m_1^2 and m_2^2 are the usual Gell-Mann–Okubo mass-splitting

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[2] G. E. Chikovani *et al.*, Phys. Letters **28B**, 526 (1969).

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[4] S. L. Glashow and R. H. Socolow, Phys. Rev. Letters **15**, 329 (1965).