Model of the Baryon Ground State*

E. GOLOWICH
University of Massachusetts, Amherst, Massachusetts 01002
(Received 13 January 1969)

We construct a bootstrap description of the lowest-mass states with baryon number one. A fundamental consideration in this approach is the selection of the most important two-particle channels coupled to these ground-state baryons. We suggest that S- and P-wave couplings to meson-baryon composites dominate, and that existing evidence from decays of resonant states supports our idea. A theoretical description which fits in naturally with this observation is the $SU(6)$-multiplet classification of baryon states first formulated by Capps and by Belinfante and Cutkosky. This symmetry of S- and P-wave baryon-interaction vertices may be viewed as an extension of the original Chew-Low static interaction. The dynamical basis for our model is a set of bootstrap equations implied by the Bethe-Salpeter equation. We assign positive- and negative-parity baryons to $\mathbf{56}^+$ and $\mathbf{70}^-$, respectively, and determine the $\mathbf{70}^- - \mathbf{56}^+$ mass difference, as well as the $\mathbf{56}^+\mathbf{56}^+\mathbf{35}^-$, $\mathbf{70}_F^-\mathbf{70}^-\mathbf{35}^-$, and $\mathbf{70}_D^-\mathbf{70}^-\mathbf{35}^-$ P-wave and the $\mathbf{70}^-\mathbf{56}^+\mathbf{35}^-$ S-wave coupling constants. We calculate a $\mathbf{70}^- - \mathbf{56}^+$ mass difference of 230 MeV, and a $\mathbf{70}^-$ F/D ratio of -1.1. The relation of the model to ρ-universality and also to the parity-doublet conjecture for baryons is discussed.

I. INTRODUCTION

HIGH-ENERGY scattering reactions so far have failed to reveal fundamental particles with exotic properties such as noninteger baryon number of noninteger hypercharge. This fact, coupled with the existence of a large spectrum of hadrons, has strongly influenced many attempts to gain a theoretical understanding of the strong interactions. It is natural to suspect that the observed hadrons are bound states or resonances of virtual-particle constituents which interact with single-particle-exchange (Yukawa) forces. This collective or "bootstrap" approach has given many successful insights into the spectroscopy of mesons and baryons, both with the classification of particles into multiplets[1] and with the existence and structure of excited states.[2] A conventional assumption in such calculations entails the use of the particles themselves as basic hadron coordinates. To make this kind of model practical, one must truncate the number of channels coupled to a given particle. Physically, one is assuming that the properties of a hadron are qualitatively determined by a certain subset of two-particle composites, the selection of which depends upon dynamical and kinematical considerations such as spin, mass, isospin, etc. This "local" point of view, in which the entire hadron spectrum is built up piece by piece, has the grave disadvantage of being very complex. Indeed, the over-all difficulty of the problem has caused many to question the wisdom of using particles themselves as the fundamental hadron variables. Rather, it is possible that the underlying laws of hadron dynamics are most elegantly described in terms of a Lie algebra generated by commutation relations of a suitably defined field theory, e.g., current algebra,[3] or in terms of some simple global property of the S matrix. Just how far this direction of thought will proceed remains to be seen, but it raises the interesting question of whether different formulations of the problem lead to equivalent predictions in areas of overlap. In particular, it has recently been suggested[4] that baryon Regge trajectories obey the approximate relations $\mathrm{Re}J \propto W^2$, where J gives the angular momentum value of the trajectory evaluated at center-of-mass energy W. This relation, augmented by the MacDowell symmetry, implies that baryons occur in nature as parity doublets unless certain of the residue functions vanish at integer values of J. Such a property seems surprising from the viewpoint of conventional bootstrap theory, in which positive- and negative-parity baryons have been predicted in various model calculations to have entirely distinct properties.[5]

It is our purpose in this paper to investigate a model of the ground-state baryons (a "ground-state" baryon is one having the lowest mass in its particular channel) using standard bootstrap assumptions. However, concentrating on a possible conflict between such an approach and the baryon parity-doublet scheme, we pay particular attention to the truncation approximation and try to include as many important coupled channels as possible. In particular, Sec. II contains a phenomenological analysis of baryon decays, from which we infer the most important couplings of two-particle composites to a given baryon. The empirical evidence indicates the dominance of those composites having S- and P-wave orbital angular momentum. This result is shown to be consistent with the over-all structure of the baryon spectrum. Given this S- and P-wave dominance, in Sec. III we consider a mathematical model, the $SU(6)$ symmetry of Capps, and of Belinfante and Cutkosky, which describes large multi-

* Supported in part by the National Science Foundation.

[1] For instance, see A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963).

[2] P. A. Carruthers, Phys. Rev. **133**, B497 (1964); E. Golowich, *ibid.* **168**, 1745 (1968).

[3] M. Gell-Mann, Physics **1**, 63 (1964).

[4] V. Barger and D. Cline, Phys. Letters **26B**, 85 (1967).

[5] For instance, L. F. Cook and B. Lee, Phys. Rev. **127**, 283 (1962); P. Auvil and J. J. Brehm, *ibid.* **145**, 1152 (1966); P. Carruthers and M. M. Nieto, *ibid.* **163**, 1646 (1967); R. H. Capps, *ibid.* **158**, 1343 (1967).

plets of particles interacting with S- and P-wave vertices.[6] As explained there, we may interpret the symmetry most simply as an extension of the static Chew-Low interaction.[7] We stress that our use of this symmetry is dynamical in the sense that we examine the magnitudes of masses and coupling constants that $SU(6)$ multiplets must have in order to interact with particle-exchange forces. In particular, our model determines whether assignment of positive-parity ground-state baryons to **56** and negative-parity ground-state baryons to **70** is consistent with bootstrap dynamics. The paper concludes with a discussion of results in Sec. IV. In the Appendix we discuss certain group-theoretic details, including a formula based on a factorization property of crossing coefficients for higher symmetries which facilitates their calculation.

II. BARYON SPECTROSCOPY—ANALYSIS OF COUPLING CONSTANTS

Strictly speaking, a given hadron is coupled to an infinite number of channels. It appears likely that the coupling to two-particle composites predominates, as is implied, for example, in production processes where multiparticle final states typically cluster into a few resonant states. This still leaves the problem of choosing which subset of the two-particle channels, if any, ap-approximately determines the physics of a hadron. We consider in the rest of this section experimental and theoretical approaches to answering this question.

We begin with an analysis of the experimental information in the resonance region derived from measurements of decay widths. The experimental data are very rich in content, especially for baryon decays, where transitions of the type (see Fig. 1)

$$\tfrac{1}{2}^{\pm},\ \tfrac{3}{2}^{\pm},\ \tfrac{5}{2}^{\pm},\ \tfrac{7}{2}^{\pm} \to \tfrac{1}{2}^{+}\otimes 0^{-},$$
$$\tfrac{3}{2}^{-} \to \tfrac{1}{2}^{-}\otimes 0^{-},$$
$$\tfrac{3}{2}^{-},\ \tfrac{5}{2}^{\pm} \to \tfrac{3}{2}^{+}\otimes 0^{-},$$
$$\tfrac{5}{2}^{-} \to \tfrac{3}{2}^{-}\otimes 0^{-}$$

have been measured.[8] These data have been widely examined (see, for example, Refs. 9–13), especially with regard to extraction of coupling constants from the decay widths. As stressed in Refs. 9, 10, and 12, the extraction procedure is beset with difficulties, and no single method appears usable without ambiguity. In this paper, we consider an approach which is Lorentz-invariant and treats the high-spin kinematics correctly, although it ignores hadron structure.[14] We describe transitions in terms of field-theoretic interaction Lagrangians using the free field operators.[15] For couplings of a baryon of spin S, parity P_S (with field operator $\chi^{\mu_1,\cdots,\mu_{n-1}}$, $n=S+\frac{1}{2}$) to a $\frac{1}{2}^{+}$baryon–0^{-} meson composite (with field operators ψ and ϕ, respectively), we write

$$\mathcal{L}=(g/\mu^{(n-1)})\bar{\psi}\chi^{\eta_1,\cdots,\eta_{n-1}}\partial_{\eta_1},\ \cdots,\ \partial_{\eta_{n-1}}\phi \qquad (1)$$

if $(-)^n P_S=+1$, and we include a factor γ_5 if $(-)^n P_S=-1$. The former corresponds to transitions $\frac{1}{2}^{-}$, $\frac{3}{2}^{+}$, $\frac{5}{2}^{-}$, $\cdots,\to\frac{1}{2}^{+}\otimes 0^{-}$, the latter to $\frac{1}{2}^{+}$, $\frac{3}{2}^{-}$, $\frac{5}{2}^{+}$, $\cdots,\to\frac{1}{2}^{+}\otimes 0^{-}$. The arbitrary mass μ in Eq. (1) serves to make the coupling constant g dimensionless. A standard calculation[14] then yields the following relation between width Γ and coupling constant g:

$$\frac{g^2}{4\pi}=\frac{n}{2^n}\frac{(2n)!}{(n!)^2}\frac{M}{E\pm m}\frac{\mu^{2(n-1)}}{q^{2n-1}}\Gamma, \qquad (2)$$

where one takes the $+(-)$ for $(-)^n P_S=+1$ (-1), respectively. In Eq. (2), M is the mass of the spin-S baryon, E and m are the $\frac{1}{2}^{+}$ baryon energy and mass, and q is the decay momentum evaluated in the rest frame of the decay baryon. A similar relation holds for the transition of a spin-S baryon to a $\frac{3}{2}^{+}$ baryon–0^{-}-meson composite (we use the Rarita-Schwinger field operator ψ_μ to describe the $\frac{3}{2}^{+}$ baryon). If $(-)^{n+1}P_S=+1$, corresponding to transitions $\frac{1}{2}^{+}$, $\frac{3}{2}^{-}$, $\frac{5}{2}^{+}$, $\cdots,\to\frac{3}{2}^{+}\otimes 0^{-}$, then

$$\mathcal{L}=(g/\mu^{n-2})\bar{\psi}_{\eta_1}\chi^{\eta_1\eta_2,\cdots,\eta_{n-1}}\partial_{\eta_2},\ \cdots,\ \partial_{\eta_{n-1}}\phi, \quad (3)$$

where we restrict ourselves to the coupling involving the smallest number of derivatives on the meson field. If $(-)^{n+1}P_S=-1$, corresponding to $\frac{1}{2}^{-}$, $\frac{3}{2}^{+}$, $\frac{5}{2}^{-}$, $\cdots,\to\frac{3}{2}^{+}\otimes 0^{-}$, we must insert a factor γ_5 between the baryon field operators. The relation between coupling constant and width for this interaction is

$$\frac{g^2}{4\pi}=\frac{n}{2^n}\frac{(2n)!}{(n!)^2}\frac{M}{q^{2n-3}} \times\frac{\mu^{2(n-2)}}{[(2n-1)/(n-1)](E\pm m)+(2q^2/3m^2)(E\pm 2m)}, \quad (4)$$

where we follow the notation of Eq. (2) except that $+(-)$ is now chosen for $(-)^{n+1}P_S=+1$ (-1), respectively.

Presumably, from examination of the phenomenological coupling constants of Eqs. (2) and (4), one can determine with which of the baryon states the ground-state particles interact most strongly. However, the situation is not altogether straightforward, because the set of baryons $\frac{1}{2}^{-}$, $\frac{3}{2}^{+}$, $\cdots$, and $\frac{1}{2}^{+}$, $\frac{3}{2}^{-}$, $\cdots$, are not treated equivalently in Eqs. (1) and (3). We illustrate this with a simple example. The nucleon $N(938)$ and

[6] R. H. Capps, Phys. Rev. Letters **14**, 31 (1965); J. G. Belinfante and R. E. Cutkosky, *ibid.* **14**, 33 (1965).
[7] J. G. Belinfante and G. H. Renninger, Phys. Rev. **148**, 1573 (1966).
[8] N. Barash-Schmidt *et al.*, Rev. Mod. Phys. **41**, 109 (1969).
[9] P. Carruthers and J. Shapiro, Phys. Rev. **159**, 1465 (1967).
[10] M. Goldberg *et al.*, Nuovo Cimento **45**, 169 (1966).
[11] R. D. Tripp *et al.*, Nucl. Phys. **B3**, 10 (1967).
[12] N. Masuda and S. Mikamo, Phys. Rev. **162**, 1517 (1967).
[13] E. Golowich, Phys. Rev. **177**, 2295 (1969).
[14] J. G. Rushbrooke, Phys. Rev. **143**, 1345 (1966).
[15] P. Carruthers [Phys. Rev. **152**, 1345 (1966)] uses a different approach which yields equivalent results.

3-3 resonance $\Delta(1236)$ dominate the low-energy strangeness-zero channel. It is generally believed that these particles are dynamically equivalent (neither more fundamental than the other) and that their couplings to the πN channel are roughly comparable (as is the case in the Chew-Low static model[16]). However, Eq. (2), which reduces in this case to (we suppress isospin notation)

$$\mathcal{L}=g_{NN\pi}\bar{\psi}\gamma_5\psi\phi+(f_{\Delta N\pi}/\mu)\bar{\psi}\chi_\mu\partial^\mu\phi\,, \qquad (5)$$

implies $g^2(NN\pi^0)/4\pi\cong 14.5$ and $f^2(\Delta N\pi^0)/4\pi=0.24$. Clearly, the relative size of these numbers is not indicative of the true relative importance of the πN channel to N and Δ. A more meaningful classification for the $NN\pi$ coupling is the pseudovector vertex

$$\mathcal{L}=(f_{NN\pi}/\mu)\bar{\psi}\gamma_5\gamma_\mu\psi\partial^\mu\phi\,, \qquad (6)$$

where $f^2(NN\pi^0)/4\pi=0.08$, or extracting out the $SU(2)$ Clebsch-Gordan coefficients of spin and isospin, $f^2(NN\pi)/4\pi=0.24$. We generalize this procedure to arbitrary spin by making the transformation $\gamma_5\to(1/\mu)\gamma_5\gamma_\nu\partial^\nu$ in Lagrangians were a γ_5 occurs. It is not difficult to relate the new coupling constant f with the old coupling constants g of Eqs. (2) and (4). In momentum space, $i\partial^\mu\phi\to k^\mu$ is the meson four-momentum, which is related to the baryon four-momenta by the conservation law $k^\mu=p^\mu-p^{\mu\prime}$ for the transition baryon $(p^\mu)\to$ baryon $(p^{\mu\prime})+$meson (k^μ). Using the baryon wave equations[17] and the anticommutativity of γ_5 with γ_μ, we immediately find that

$$f=[\mu/(M+m)]g\,, \qquad (7)$$

where M and m are the parent and decay baryon masses. To summarize, we determine phenomenological coupling constants f by applying Eqs. (2) and (4) to experimental data, except when a factor of γ_5 appears in Eq. (1) or (3). In that case, we use the redefinition Eq. (7), which supplies a set of coupling constants suitable for comparison. Numerical results for various transitions are presented in Table I. Since the size of any given coupling depends on internal symmetries such as $SU(3)$ in addition to spin kinematics, we exhibit only the largest couplings characteristically seen for a given value of L, the orbital angular momentum of the decay particles. We see in Table I that S- and P-wave transitions dominate the higher partial waves.[18] *That is, the couplings are strongest in the lowest L states consistent with parity conservation.*

Thus, our study of the resonance region implies an S- and P-wave dominance of baryon dynamics. Clearly, the omission of higher partial waves puts a limitation on the accuracy of our description of ground state baryons and, in fact, must ultimately be included if we are to have a full understanding of excited states. However, as a starting point, our approach is consistent with the data. Actually, there are reasons for believing that the S- and P-wave dominance approximation holds for the entire spectrum of baryons, excited as well as ground states. Existing Chew-Frautschi plots imply a baryon mass-spin relation $M^2\propto J$. In the conventional single-particle-exchange dynamics for hadrons, the range of a force does not get much larger than the pion-Compton wavelength. Now consider the relation $l\cong kr$ which gives angular momentum states l, reached by a two-particle composite of momentum k with relative separation r. If $l\sim M^2$ whereas $k\sim M$, as is indicated for the baryon spectrum, it is impossible to generate the high-spin states observed in nature with forces of fixed range and high orbital angular momentum. That is, even high-spin states apparently couple most strongly to low-orbital-angular-momentum composites. In fact, it is possible to construct a bootstrap model with just this property as a basis for understanding excited states.[2] This point of view appears to be also demanded in the Regge-pole description of baryons.[19]

TABLE I. Phenomenological baryon coupling constants. All coupling constants are determined from Eqs. (2) and (4) unless a γ_5 factor is required in the interaction Lagrangian. Then the modification in Eq. (7) is made. Data are taken from Ref. 8.

Initial	Final	Baryon spin-parity quantum numbers	Γ (MeV)	Orbital angular momentum	$g/(4\pi)^{1/2}$
$N^*(\frac{1}{2}^-,1550)$	$N\pi$	$\frac{1}{2}^-\to\frac{1}{2}^+$	39	S	0.25
$N^*(\frac{1}{2}^-,1550)$	$N\eta$	$\frac{1}{2}^-\to\frac{1}{2}^+$	91	S	0.59
$Y_0^*(\frac{1}{2}^-,1405)$	$\Sigma\pi$	$\frac{1}{2}^-\to\frac{1}{2}^+$	50	S	0.44
$N^*(\frac{3}{2}^-,1525)$	$\Delta\pi$	$\frac{3}{2}^-\to\frac{3}{2}^+$	46	S	0.35
$\Delta(\frac{3}{2}^+,1236)$	$N\pi$	$\frac{3}{2}^+\to\frac{1}{2}^+$	120	P	0.60
$Y_1^*(\frac{3}{2}^+,1385)$	$\Lambda\pi$	$\frac{3}{2}^+\to\frac{1}{2}^+$	32.8	P	0.36
$N^*(\frac{1}{2}^+,1470)$	$N\pi$	$\frac{1}{2}^+\to\frac{1}{2}^+$	135	P	0.13
$Y_1^*(\frac{3}{2}^-,1660)$	$Y_0^*(1450)\pi$	$\frac{3}{2}^-\to\frac{1}{2}^-$	18.5	P	0.30
$Y_1^*(\frac{5}{2}^-,1767)$	$Y_0^*(1520)\pi$	$\frac{5}{2}^-\to\frac{3}{2}^-$	14.3	P	0.26
$Y_0^*(\frac{3}{2}^-,1520)$	$N\bar{K}$	$\frac{3}{2}^-\to\frac{1}{2}^+$	7.2	D	0.25
$Y_0^*(\frac{3}{2}^-,1690)$	$N\bar{K}$	$\frac{3}{2}^-\to\frac{1}{2}^+$	9.0	D	0.064
$N^*(\frac{3}{2}^-,1525)$	$N\pi$	$\frac{3}{2}^-\to\frac{1}{2}^+$	63.3	D	0.041
$Y_1^*(\frac{3}{2}^-,1660)$	$N\bar{K}$	$\frac{3}{2}^-\to\frac{1}{2}^+$	9.0	D	0.064
$Y_1^*(\frac{5}{2}^-,1767)$	$N\bar{K}$	$\frac{5}{2}^-\to\frac{1}{2}^+$	43.7	D	0.058
$Y_0^*(\frac{5}{2}^-,1830)$	$\Sigma\pi$	$\frac{5}{2}^-\to\frac{1}{2}^+$	33.6	D	0.044
$N^*(\frac{5}{2}^-,1680)$	$N\pi$	$\frac{5}{2}^-\to\frac{1}{2}^+$	68	D	0.05
$Y_1^*(\frac{5}{2}^-,1767)$	$Y_1^*(1385)\pi$	$\frac{5}{2}^-\to\frac{3}{2}^+$	13.3	D	0.069
$N^*(\frac{5}{2}^+,1668)$	$N\pi$	$\frac{5}{2}^+\to\frac{1}{2}^+$	84.5	F	0.010
$Y_0^*(\frac{5}{2}^+,1815)$	$N\bar{K}$	$\frac{5}{2}^+\to\frac{1}{2}^+$	52.5	F	0.036
$N^*(\frac{7}{2}^+,1930)$	$N\pi$	$\frac{7}{2}^+\to\frac{1}{2}^+$	88	F	0.090
$Y_1^*(\frac{7}{2}^-,2100)$	$N\bar{K}$	$\frac{7}{2}^-\to\frac{1}{2}^+$	42	G	0.003

III. BOOTSTRAP MODEL OF BARYON GROUND STATE

We have argued that the data imply a dominance of S- and P-wave couplings of a baryon to two-particle

[16] G. F. Chew, Phys. Rev. Letters 9, 233 (1962).

[17] C. Fronsdal, Nuovo Cimento Suppl. 9, 416 (1958).

[18] The sole exception is the $Y_0^*(1520)\to N\bar{K}$ D-wave coupling. It is possible that a good model for this state must include the D-wave channels.

[19] J. Harte and R. C. Brouwer, Phys. Rev. 164, 1841 (1967).

composites. Having limited ourselves to these channels, we want to include as many of them as possible in the calculation while keeping the problem from becoming prohibitively difficult. To satisfy these dual constraints, we shall use group theory to provide relations among the many channels we consider. A natural-symmetry scheme to use is the $SU(6)$ of Capps[6] and Belinfante and Cutkosky.[6] The baryon couplings described by this theory may be viewed as an extension of the Chew-Low static model interaction as follows. The isospin-invariant [$SU(2)$-invariant] vertex describing the πNN coupling is

$$[f(\pi NN)/\mu_\pi]\bar{\psi}_N \sigma \boldsymbol{\tau} \psi_N \cdot \nabla \boldsymbol{\pi}\,, \quad (8)$$

where μ_π is the pion mass and where we have suppressed the source form factor. For $SU(3)$, we generalize this to

$$(1/\mu_P)\bar{\psi}_B \sigma [f(BBP)\mathbf{F}+d(BBP)\mathbf{D}]\psi_B \cdot \nabla \mathbf{P}\,, \quad (9)$$

where $\mathbf{P}$ is the 0^--meson octet, μ_P is its mass, and $f(BBP)$, $d(BBP)$ are the antisymmetric and symmetric couplings of two octets to a third octet. Our extension to $SU(6)$ is based on the static model vertex

$$[f(B'BM)/\mu]\bar{\psi}_{B'} G_{B'B}{}^M \psi_B P^M\,, \quad (10)$$

where B,B' are $SU(6)$ baryon fields, P is an $SU(6)$ meson field with mass μ, and $G_{B'B}{}^M$ is an appropriate $SU(6)$ operator. We assign the ground-state positive- and negative-parity baryons to $\mathbf{56}^+$ and $\mathbf{70}^-$, respectively (superscript indicates parity). The $SU(3)$ and $SU(2)$ content of these multiplets is $\mathbf{56}^+=\mathbf{8}^2\oplus\mathbf{10}^4$, $\mathbf{70}^-=\mathbf{8}^2\oplus\mathbf{8}^4\oplus\mathbf{10}^2\oplus\mathbf{1}^2$. It is natural to assign the $SU(2)$ angular momentum label to the baryon *spins*, e.g., $\mathbf{56}^+$ contains $\Delta(1236)\subset\mathbf{10}^4$, $N(939)\subset\mathbf{8}^2$, etc., and $\mathbf{70}^-$ contains $N^*(1525)\subset\mathbf{8}^4$, $Y_0^*(1405)\subset\mathbf{1}^2$, etc. The mesons belong to $\mathbf{35}^-$ (with content $\mathbf{8}^3\oplus\mathbf{8}^1\oplus\mathbf{1}^3$), although their individual $SU(6)$ assignments depend on the way they couple to the baryons. The assignments are based on j-j coupling and thus depend on the relative intrinsic parity of the two baryons. In a vertex, for instance, coupling a spin-$\frac{1}{2}$ baryon to a pseudoscalar meson and a spin-$\frac{1}{2}$ baryon of opposite parity, conservation of parity limits allowed angular momenta to $l=0, 2, 4, \cdots$. However, conservation of angular momentum implies $l=0$ or 1. Therefore, the meson occurs in an $l=0$ state, and since for j-j coupling, $\mathbf{j}(\text{meson})=\mathbf{l}+\mathbf{s}(\text{meson})$, we have $\mathbf{j}=\mathbf{s}$ here. This is generally true in the same way for all $\mathbf{70}^-\to\mathbf{56}^+\otimes\mathbf{35}^-$ couplings. Thus, this set of couplings describes S-wave composites and since $\mathbf{j}(\text{meson})=\mathbf{s}(\text{meson})$, we make meson assignments according to spin: vector-meson octet $\mathbf{8}^3$, pseudoscalar octet $\mathbf{8}^1$, and vector singlet $\mathbf{1}^3$. Similarly, we can show baryons having the same parity must couple to P-wave ($l=1$) mesons. However, assignments are not unique here since we can form $j=1$ states from $l=1$, $s=0$ and from $l=1$, $s=1$. For instance, we can form $j=1$ states from $\nabla\phi$ and $\nabla\times\mathbf{V}$ where ϕ and $\mathbf{V}$ are pseudoscalar and vector-meson states, respectively. In general, we must write $\mathbf{8}^3=\alpha_8\nabla\phi_8+\beta_8\nabla\times\mathbf{V}_8$, where $\alpha_8{}^2+\beta_8{}^2=1$, and similarly for $\mathbf{1}^3=\alpha_1\nabla\phi_1+\beta_1\nabla\times\mathbf{P}_1$, with $\alpha_1{}^2+\beta_1{}^2=1$. The values of α_1, α_8, β_1, β_8 are based on considerations lying outside the model. Cutkosky and Jacobs[20] have shown that an $SU(6)$ Fermi-Yang model of the mesons implies $\alpha_1=\alpha_8=\sqrt{\frac{1}{3}}$. We use these values here. In summary, our model contains the following baryon couplings: S-wave $f(\mathbf{70}^-,\mathbf{56}^+,\mathbf{35}^-)$ and P-wave $f(\mathbf{56}^+\mathbf{56}^+\mathbf{35}^-)$, $f_F(\mathbf{70}^-\mathbf{70}^-\mathbf{35}^-)$, and $f_D(\mathbf{70}^-\mathbf{70}^-\mathbf{35}^-)$, the latter two corresponding to the fact that the Clebsch-Gordan series for $\mathbf{70}^-\otimes\mathbf{35}^-$ contains $\mathbf{70}^-$ twice. (See the Appendix for further details.)

Were we to restrict ourselves to employing only baryon-exchange forces, there would be no further vertices entering the calculation. However, we feel that since meson-exchange forces are important to the physics of baryons, they should be included. (This point has repeatedly been emphasized by Capps.[5]). Thus, we consider finally a $\mathbf{35}^-\to\mathbf{35}^-\otimes\mathbf{35}^-{}_F$ vertex,[21] limiting ourselves to the F-type vertex because of its adherence to the constraints of Bose statistics. The size of the coupling constant $f(\mathbf{35}^-\mathbf{35}^-\mathbf{35}^-)$ is not included as output in our calculation because we make no statement about internal meson dynamics. We determine this number from experiment, e.g., from the $\rho\pi\pi$-decay width.

One of the motivations for our use of the $SU(6)$ symmetry is to include as many of the ground-state baryons as possible in our calculation. Inclusion of the $\mathbf{56}^+$ and $\mathbf{70}^-$ multiplets does a very good job in this respect, the only notable omissions being the $SU(3)$ multiplets containing the Roper resonance $N^*(\frac{1}{2}^+,1470)$, the πN D-wave $N^*(\frac{5}{2}^-,1680)$, and the $Y_0^*(1520)$ unitary singlet. It is possible to extend our ideas to include these particles, say, by including other multiplets like $\mathbf{20}^+$ for the Roper resonance or by extending the symmetry to $SU(6)\otimes 0(3)$ for the latter two particles. [The $N^*(\frac{5}{2}^-,1680)$ trajectory would have a zero at $J^P=\frac{1}{2}^-$.] However, we feel the model satisfies the criteria of content and simplicity well enough to warrant study on its own. Aside from particles with large negative strangeness in $\mathbf{70}^-$, most of the states in $\mathbf{56}^+$ and $\mathbf{70}^-$ have been seen experimentally. We now calculate the empirical masses of these multiplets:

$$\begin{aligned} M_{56}&=(40/56)\bar{M}(\mathbf{10}^4)+(16/56)\bar{M}(\mathbf{8}^2)\\ &=1317\text{ MeV},\\ M_{70}&=(2/70)\bar{M}(\mathbf{1}^2)+(20/70)\bar{M}(\mathbf{10}^2)+(32/70)\bar{M}(\mathbf{8}^4)\\ &\qquad+(16/70)\bar{M}(\mathbf{8}^2)\\ &=1700\text{ MeV}, \end{aligned} \quad (11)$$

where we have estimated the mass of the $\mathbf{10}^2$ submultiplet of $\mathbf{70}^-$ as 1780 MeV from knowledge of its strangeness-zero member at 1640 MeV. Although the assignment of mesons depends on whether they appear in S or P waves, the average mass of $\mathbf{35}^-$ is practically

[20] R. E. Cutkosky and M. Jacobs, Phys. Rev. **162**, 1416 (1967).
[21] R. H. Capps, Phys. Rev. **148**, 1332 (1966).

the same for each case:

S wave:

$$\mu_{35}^2=(24/35)\bar{\mu}^2(8,1^-)+(8/35)\bar{\mu}^2(8,0^-)+(3/35)\bar{\mu}^2(1,1^-),$$
$$\mu_{35}=776\text{ MeV};$$

P wave:

$$\mu_{35}^2=(24/35)\bar{\mu}^2(8,1^-)+(8/35)\bar{\mu}^2(8,0^-)+(2/35)\bar{\mu}^2(1,1^-)+(1/35)\bar{\mu}^2(1,0^-),\quad(12)$$
$$\mu_{35}=779\text{ MeV}.$$

We take an average of these, $\mu_{35}=778$ MeV.

We now turn to the problem of baryon dynamics. A convenient calculation scheme is given by an approximation to the Bethe-Salpeter (BS) equation as formulated by Cutkosky and collaborators.[22] Starting with a BS vertex equation

$$\phi=K^{-1}I\phi,\quad(13)$$

where ϕ is the BS amplitude, K^{-1} is a two-particle Green's function, and I is the interaction kernel, we limit ourselves to single-particle-exchange processes, evaluated with static-model kinematics,

$$\phi_{b\alpha}{}^a=\sum_{e,f,\beta}\int\frac{\Gamma_{f\alpha}{}^e\Gamma_{b\beta}{}^e\phi_{f\beta}{}^a}{\Delta}+\sum_{c,\beta,\gamma}\int\frac{\Gamma_{\alpha\beta}{}^\gamma\Gamma_{cb}{}^\gamma\phi_{c\beta}{}^a}{\Delta'}.\quad(14)$$

The first and second terms represent baryon- and meson-exchange processes, respectively; Γ are the vertex functions, and Δ,Δ' are energy denominators. The labeling describes baryon a as the bound state of baryon b and meson α. Our dynamical equations are obtained by replacing the amplitudes ϕ and vertex functions Γ by coupling constants f and form factors v. The latter correspond physically to hadron structure and serve numerically to cut off the large momentum virtual processes. In this way we generate a set of vertex equations for coupling constants,

$$f_{b\alpha}{}^a=\sum_{e,f,\beta}f_{f\alpha}{}^e f_{b\beta}{}^e f_{f\beta}{}^a D_{ab}{}^{ef}+\sum_{c,\beta,\gamma}f_{\alpha\beta}{}^\gamma f_{ab}{}^\gamma f_{c\beta}{}^a E_{acb},\quad(15)$$

where $D_{ab}{}^{ef}$ and E_{abc} are baryon- and meson-exchange dynamical factors, respectively. They depend both on the masses of the virtual particles and their quantum numbers (and thus contain the relevant crossing coefficients). One may think of these equations as producing a set of solutions for the coupling constants f as a function of the baryon-mass difference ΔM. Further dynamical information comes from the normalization condition[22]

$$N_a=\sum_{b,e,f,\alpha,\beta}f_{f\alpha}{}^e f_{b\beta}{}^e f_{b\alpha}{}^a f_{f\beta}{}^a W_{bf}{}^{ae}+\sum_{b,c,\alpha,\beta,\gamma}f_{b\alpha}{}^a f_{\alpha\beta}{}^\gamma f_{bc}{}^\gamma f_{c\beta}{}^a Y_{acb},\quad(16)$$

where the first and second terms again represent internal baryon- and meson-exchange dynamics, respectively; the replacement of vertex functions by coupling constants and form factors has been made; and W and Y are normalization dynamical factors, defined by

$$W_{bf}{}^{ae}=K_a{}^b\partial D_{ab}{}^{ef}/\partial M_a,$$
$$Y_{acb}=K_a{}^b\partial E_{acb}/\partial M_a.\quad(17)$$

There are two such normalization equations in this calculation, one each for **56**$^+$ and **70**$^-$.

For baryon-exchange processes with *S*- and *P*-wave internal mesons, the dynamical factors are

$$D_{ab}{}^{ef}\frac{v(q)}{(2w_q)^{1/2}}=\frac{C_{ab}{}^{ef}}{4\pi^2}\times\int_{\mu_{35}}^{\infty}\frac{kv^2(k)dw}{(w_k+\Delta_{fa})(w_k+\Delta_{eb})}\frac{v(q)}{(2w_q)^{1/2}}\quad(18a)$$

for *S* wave, and

$$D_{ab}{}^{ef}\frac{v(q)}{(2w_q)^{1/2}}=\frac{C_{ab}{}^{ef}}{12\pi^2}\times\int_{\mu_{35}}^{\infty}\frac{k^3v^2(k)dw}{(w_k+\Delta_{fa})(w_k+\Delta_{eb})}\frac{v(q)}{(2w_q)^{1/2}}\quad(18b)$$

for *P* wave. In the above, $C_{ab}{}^{ef}$ is proportional to a crossing coefficient, Δ_{ij} is the mass difference between baryons i and j, v is a form factor, and w, k and μ_{35} are the internal meson's energy, momentum, and mass, respectively. Of interest is whether in addition to coupling constant and mass-difference information, the model has further theoretical content. For instance, in the nonrelativistic domain, the Schrödinger equation yields information both about energy eigenvalues *and* about wave functions. Recent work on the determination of form factors in self-consistent dynamical theories, using simple model calculations, implies that exponential dependence occurs in the momentum transfer variable.[23] If our model is restricted to only baryon-exchange processes, no indication about the qualitative behavior of form factors is obtained because the external momentum dependence factors out of the integration over internal virtual momentum. [See Eqs. (18a) and (18b).] This is no longer true when meson-exchange processes are included in the calculation. To first order in the baryon-mass difference,

$$E_{acb}\frac{v(q)}{(2w_q)^{1/2}}=\frac{1}{(2w_q)^{1/2}}\int\frac{d^3k}{(2\pi)^3}\frac{P_{acb}}{2w_k{}^2w_{k-q}{}^2}\times v(k)v^2(\mathbf{k}+\mathbf{q})\left(1+\frac{3}{4}\frac{M_a+M_b-2M_c}{w_k}\right),\quad(19)$$

[22] R. E. Cutkosky and M. Leon, Phys. Rev. **135**, B1445 (1964); **138**, B667 (1965); K. Y. Lin and R. E. Cutkosky, *ibid.* **140**, B205 (1965).

[23] J. D. Stack, Phys. Rev. **164**, 1904 (1967); J. Harte, *ibid.* **165**, 1557 (1968); R. Brout and F. Englert, Phys. Letters **27B**, 647 (1968).

where P_{acb} contains various powers of the meson momentum, depending on the particular process.[24] Performing the angular integration, we find that

$$E_{acb}\frac{v(q)}{(2w_q)^{1/2}}=\frac{1}{(2w_q)^{1/2}}\times\int_{\mu_{35}}^{\infty}\frac{v(k)v^2(\mathbf{k}+\mathbf{q})}{w}N_{acb}\frac{Q_0(y)}{q}dw, \quad (20)$$

where $y=(k^2+q^2)/2kq$ and N_{acb} contains powers of the meson momentum.[24] Thus, in particular, the sets of equations (15) and (16) do contain information about the momentum transfer dependence of the structure functions $v(q)$. At the level of our calculation, with the use of staticlike kinematics and neglect of short-range forces, we feel it unwise to take too seriously the form of $v(q)$ implied by the bootstrap model. Instead, we find that self-consistency can approximately hold for an exponential form $v(q)=\exp(-\phi q)$, the precise value of the cutoff ϕ being determined upon pinning down the scale of the model by requiring that one of the coupling constants have its empirical value.

The final form of our bootstrap equations is given in terms of dimensionless variables [generated by working with Eqs. (15) and (16) to first order in the baryon mass difference] $g_0=D_{32}^{1/2}f(\mathbf{56^+56^+35^-})$, $g_1=D_{12}^{1/2}\times f(\mathbf{70^-56^+35^-})$, $g_F=D_{32}^{1/2}f(\mathbf{70}_F\mathbf{^-70^-35^-})$, $g_D=D_{32}^{1/2}\times f(\mathbf{70}_D\mathbf{^-70^-35^-})$, $g_\mu=D_{12}^{1/2}f(\mathbf{35}_F\mathbf{^-35^-35^-})$, and

$$x=[M(\mathbf{70^-})-M(\mathbf{56^+})]D_{13}/D_{12}.$$

The quantities D_{ij} are defined by

$$D_{3n}=\frac{1}{12\pi^2}\int_{\mu_{35}}^{\infty}e^{-2\phi k}\frac{k^3}{w^n}dw,\qquad D_{1n}=\frac{1}{4\pi^2}\int_{\mu_{35}}^{\infty}e^{-2\phi k}\frac{k}{w^n}dw. \quad (21)$$

We then have for the vertex bootstrap equations

$$\begin{aligned}
g_0={}&(11/15)g_0^3+(9/4)(5/33)^{1/2}g_1^2g_FC(-2x)\\
&-\tfrac{1}{2}(15/22)^{1/2}g_1^2g_DC(-2x)+\tfrac{1}{2}(8/15)^{1/2}\alpha_1g_0^2g_\mu\\
&+\tfrac{3}{8}(10/3)^{1/2}\alpha_2g_1^2g_\mu C(-\tfrac{3}{2}\alpha_3x),\\
g_1={}&g_1[\tfrac{1}{4}g_1^2+(9/5)(5/33)^{1/2}g_Fg_0-\tfrac{2}{5}(15/22)^{1/2}g_Dg_0\\
&+\tfrac{3}{4}(8/15)^{1/2}\alpha_1g_0g_\mu C(\tfrac{3}{4}\alpha_4x)+\tfrac{1}{2}(2/11)^{1/2}\alpha_1g_Fg_\mu\\
&\times C(-\tfrac{3}{4}\alpha_4x)+\tfrac{3}{2}(1/11)^{1/2}\alpha_1g_Dg_\mu C(-\tfrac{3}{4}\alpha_4x)], \qquad (22)\\
g_F={}&7/11g_F^3-1/11g_D^2g_F-(3/44\sqrt{2})g_D^3\\
&+9/5(5/33)^{1/2}g_1^2g_0C(2x)+\tfrac{1}{2}(2/11)^{1/2}\alpha_2g_1^2g_\mu\\
&\times C(\tfrac{3}{2}\alpha_3x)+(2/11)^{1/2}\alpha_1g_\mu(g_F^2+g_D^2),\\
g_D={}&-(111/352)g_D^3-(9/44\sqrt{2})g_D^2g_F-1/11g_F^2g_D\\
&-\tfrac{2}{5}(15/22)^{1/2}g_1^2g_0C(2x)+\tfrac{3}{2}(1/11)^{1/2}\alpha_2g_1^2g_\mu C(\tfrac{3}{2}\alpha_3x)\\
&-3/11(11)^{1/2}\alpha_1g_D^2g_\mu+2(2/11)^{1/2}\alpha_1g_Dg_Fg_\mu.
\end{aligned}$$

In the above, the function C serves as a good approximation to the exact dynamical factors defined in (15) and (18):

$$\begin{aligned}C(x)&=1+x &&\text{if } x>0,\\ &=1/(1-x) &&\text{if } x<0.\end{aligned} \quad (23)$$

The constants α are ratios of meson-exchange and baryon-exchange dynamical factors which appear naturally when we use the dimensionless variables previously discussed. With the definitions

$$E_{0n}=\frac{1}{2\pi^2}\int_{\mu_{35}}^{\infty}\frac{v(k)v^2(\mathbf{k}+\mathbf{q})}{w^n}\frac{Q_0(y)}{q}dw,\qquad E_{2n}=\frac{1}{6\pi^2}\int_{\mu_{35}}^{\infty}\frac{k^2v(k)v^2(\mathbf{k}+\mathbf{q})}{w^n}\frac{Q_0(y)}{q}dw, \quad (24)$$

the quantities α are $\alpha_1=E_{21}/(D_{12}D_{32})^{1/2}$, $\alpha_2=E_{01}D_{32}^{1/2}/D_{12}^{3/2}$, $\alpha_3=D_{12}E_{02}/(D_{13}E_{01})$, and $D_{12}E_{22}/(D_{13}E_{21})$. The $SU(6)$ crossing coefficients can be calculated using methods described in Ref. 25 and in the Appendix. The normalization equations have the form

$$\begin{aligned}
&(11/15)g_0^4+g_1^2[(9/4)(5/33)^{1/2}g_Fg_0C(-3\eta_1x)\eta_2-\tfrac{1}{2}(15/22)^{1/2}g_Dg_0C(-3\eta_1x)\eta_2+\tfrac{5}{16}g_1^2C(-\eta_1x-\eta_3x)(\beta_7/\eta_3)\\
&\quad+(9/4)(5/33)^{1/2}g_Fg_0C(-\eta_3x)\beta_1-\tfrac{1}{2}(15/22)^{1/2}g_Dg_0C(-\eta_3x)\beta_1]+\tfrac{3}{8}(8/15)^{1/2}g_0^3g_\mu\beta_2\\
&\quad+g_1^2\{(9/32)(10/3)^{1/2}g_0g_\mu C(-\beta_3x)\beta_4+\tfrac{9}{32}(10/3)^{1/2}C(-\eta_3x)g_0g_\mu\beta_5+\tfrac{15}{32}(2/11)^{1/2}g_Fg_\mu C[-\eta_3x-(8/3)\beta_6x]\beta_5\\
&\quad-[45/32(11)^{1/2}]g_Dg_\mu C(-\eta_3x-\tfrac{5}{2}\beta_6x)\beta_5\}=(7/11)g_F^4-(111/352)g_D^2-(2/11)g_D^2g_F^2-(3/11\sqrt{2})g_D^3g_F\\
&\quad+g_1^2[\tfrac{1}{4}g_1^2C(\eta_3x+\eta_1x)(\beta_7/\eta_3)+(9/5)(5/33)^{1/2}g_Fg_0C(\eta_3x)\beta_1-\tfrac{2}{5}(15/22)^{1/2}g_Dg_0C(\eta_3x)\beta_1\\
&\quad+(9/5)(5/33)^{1/2}g_Fg_0(3\eta_1x)\eta_2-\tfrac{2}{5}(15/22)^{1/2}g_Dg_0C(3\eta_1x)\eta_2]+\tfrac{3}{4}(2/11)^{1/2}g_F^3g_\mu\beta_2+(9/4)(2/11)^{1/2}g_0^2g_Fg_\mu\beta_2\\
&\quad-(9/64\sqrt{11})g_D^3g_\mu\beta_2+g_1^2[\tfrac{9}{16}(8/15)^{1/2}g_0g_\mu C(\eta_3x+\tfrac{5}{2}\beta_6x)\beta_5+\tfrac{3}{8}(2/11)^{1/2}g_Fg_\mu C(\beta_3x)\beta_4\\
&\qquad-(9/8\sqrt{11})g_Dg_\mu C(\beta_3x)\beta_4+\tfrac{3}{8}(2/11)^{1/2}g_Fg_\mu C(\eta_3x)\beta_5-(9/8\sqrt{11})g_Dg_\mu C(\eta_3x)\beta_5]. \qquad (25)
\end{aligned}$$

The dimensionless constants η and β are ratios of dynamical factors and Green's functions appearing naturally in the normalization equations when we define the coupling constant and mass-difference variables. They are defined by $\eta_1=D_{14}D_{12}/D_{13}^2$, $\eta_2=D_{32}D_{13}/(D_{33}D_{12})$, $\eta_3=D_{12}^2/(D_{11}D_{13})$, $\beta_1=D_{11}D_{32}/(D_{12}D_{31})$, $\beta_2=E_{22}D_{32}^{1/2}(D_{33}D_{12}^{1/2})$, $\beta_3=E_{03}D_{12}/(E_{02}D_{13})$, $\beta_4=E_{02}D_{32}^{3/2}/(D_{33}D_{12}^{3/2})$, $\beta_5=D_{11}E_{22}D_{32}^{3/2}/(D_{33}D_{31}D_{12}^{3/2})$,

[24] For details in this type of calculation, see E. Golowich, Phys. Rev. **164**, 1912 (1967).

[25] For a restricted version of this model, see E. Golowich, Phys. Rev. **153**, 1466 (1967).

$\beta_6 = E_{23}D_{12}/(E_{22}D_{13})$, and $\beta_7 = D_{32}^2/D_{33}D_{31}$, and are all of the order of unity.

The nonlinear algebraic equations (23) and (25) were solved with the aid of a computer. We found only one solution: $g_0 = 0.84$, $g_1 = 0.58$, $g_F = 0.66$, $g_D = -0.61$, and $x = 0.22$. The cutoff variable ϕ was determined by requiring that the value of g_0 give the correct $NN\pi$ pseudovector coupling constant. This implies that $\phi = 1.05\ \mu_{35}^{-1}$ or a cutoff around 750 MeV, which corresponds approximately to cutoffs found in previous models employing $SU(2)$ and $SU(3)$ symmetry.[22,24] Dependence of the solutions on the cutoff is very weak, with magnitudes varying about 15% for the cutoff range $0.7 < \phi < 1.1\ \mu_{35}^{-1}$. As mentioned previously, we determined the trimeson coupling constant g_μ from the experimental $\rho \to \pi\pi$ decay width. A value of $\Gamma_{\rho\pi\pi} = 100$ MeV inserted into

$$f_\rho^2/4\pi = \tfrac{3}{2}\Gamma M_\rho^2/P_{\pi\pi}^3 \qquad (26)$$

gives $f_\rho = 4.9$, from which, by means of an $SU(6)$ Clebsch-Gordan coefficient, we find $f(\mathbf{35}_F\mathbf{35}^-\mathbf{35}^-) = 16.9$. Since $g_\mu = D_{12}^{1/2} f(\mathbf{35}_F\mathbf{35}^-\mathbf{35}^-)$, this implies that $g_\mu = 0.75$. We did the calculation with several values of g_μ because the $\rho\pi\pi$ width is a matter of some dispute. We found the solutions qualitatively unchanged, although having a larger numerical dependence on g_μ than on the cutoff ϕ.

We consider next the set of predictions implied by the solutions.[25] The mass difference between multiplets $\mathbf{56}^+$ and $\mathbf{70}^-$ is given by $M(\mathbf{70}^-) - M(\mathbf{56}^+) = xD_{12}/D_{13} = 230$ MeV as compared with the empirical value of 380 MeV. [See Eq. (11).] Our result has the correct feature that $\mathbf{70}^-$ lies above $\mathbf{56}^+$ in mass, but is smaller in magnitude than the actual value. The scale of the P-wave transitions in which two negative-parity baryons couple is determined by the quantities g_F and g_D. These transitions are difficult to detect, but there has been recent work[26] done on the decay $Y_1^*(1660) \to Y_0^*(1405)\pi$ with the result that $\Gamma(Y_1^*Y_0^*\pi) \cong 22$ MeV. As shown in Table I, this imples a pseudovector coupling constant $f(Y_1^*Y_0^*\pi) = 1.1\ \mu_\pi^{-1}$ or in different units, $f(Y_1^*Y_0^*\pi) = 6.1\ \mu_{35}^{-1}$. This number can be compared to the $SU(6)$ couplings f_F, f_D by means of Clebsch-Gordan coefficients $f(Y_1^*Y_0^*\pi) = (11)^{-1/2}(\frac{1}{3}f_F - \frac{3}{16}f_D)$ or, in terms of the dimensionless variables, $D_{32}^{1/2} f(Y_1^*Y_0^*\pi) = (11)^{-1/2}(\frac{1}{3}f_F - \frac{3}{16}g_D)$. Since $D_{32}^{1/2} = 0.022\ \mu_{35}$ for a cutoff $\phi = 1.05\ \mu_{35}^{-1}$, we have $\frac{1}{3}g_F - \frac{3}{16}g_D = 0.42$ from the experimental data. From the calculation, we find $\frac{1}{3}g_F - \frac{3}{16}g_D = 0.34$, in reasonable agreement. Unfortunately, we cannot further test the $\mathbf{70}^-$ F/D ratio because of the paucity of data on the $\mathbf{70}^- \to \mathbf{70}^- \otimes \mathbf{35}^-$ transitions. Finally, there are the S-wave transitions in which two baryons of opposite parity couple. From the decay width of the transition $Y_0^*(1405)\Sigma\pi$ we infer a coupling constant $f/(4\pi)^{1/2} = 0.44$ (see Table I),

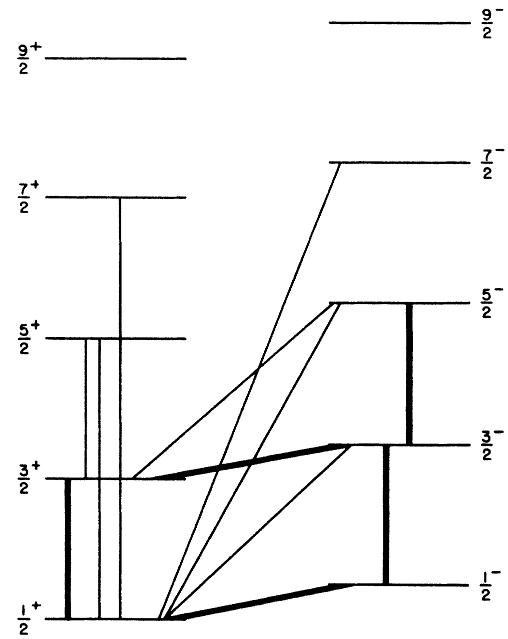

FIG. 1. Energy-level diagram depicting baryon decays. Levels are identified by spin and parity (J^P). All transitions involve emission of a pseudoscalar meson. Heavy lines indicate the dominant transition modes.

which implies $f(\mathbf{70}^-\mathbf{56}^+\mathbf{35}^-) = 5.1$ and thus $g_1 = D_{12}^{1/2} \times f(\mathbf{70}^-\mathbf{56}^+\mathbf{35}^-) = 0.24$, roughly a factor of 2 smaller than the value $g_1 = 0.58$ from the model. This result for g_1, within the rough accuracy expected of the calculation, is probably the most model-dependent of our predictions because the ratio of S- and P-wave dynamical factors is more sensitive than the solutions themselves to the type of cutoff used.

The sizes of various terms in the vertex equations (22) indicate how they compare with previous bootstrap calculations. For instance, the largest coupling constant $f(\mathbf{56}^+\mathbf{56}^+\mathbf{35}^-)$ is dominated by two processes, namely, $\mathbf{56}^+$ and $\mathbf{35}^-$ exchange in elastic $\mathbf{56}^+ \otimes \mathbf{35}^-$ scattering. This explains the success of earlier calculations which sought to explain the physics of the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryon states using the most obvious of the elastic channels. However, the couplings of the $\mathbf{70}^-$ multiplet, $\mathbf{70}^-\mathbf{56}^+\mathbf{35}^-$ and $\mathbf{70}_{F,D}^-\mathbf{70}^-\mathbf{35}^-$, are not dominated in our model by any particular process. From this we must conclude that a comprehensive understanding of the negative-parity baryons is a true many-body problem, probably beyond the scope of the conventional bootstrap approach unless some large group such as $SU(6)$ is employed.

IV. SUMMARY AND CONCLUSIONS

The work described in this paper divides into essentially two parts: first a phenomenology of the baryon resonance region, then a related bootstrap calculation.

[26] M. Primer *et al.*, Phys. Rev. Letters **20**, 610 (1968); J. Button-Shafer, *ibid.* **21**, 1123 (1968).

The reason for our phenomenological study of baryon decay widths is that in bootstrap theory the very existence of hadrons is related to the couplings they have with various composites. To make any approximation scheme viable, one must have an idea of the dominant couplings. The procedure we follow for extracting coupling constants from decay widths is given by Eqs. (2) and (4), except where a factor of γ_5 appears in the phenomenological Lagrangians (1) and (3). In that case, we make the substitution $\gamma_5 \rightarrow (1/\mu)\gamma_5\gamma_\nu\partial^\nu$ in order to compare different baryon transitions in an equivalent manner. The coupling constants in Table I imply that the most important composites are those with the smallest orbital angular momentum. This empirical evidence from the resonance region reinforces the same kind of conclusion reached by observing that Regge trajectories appear to be linear in the square of the center-of-mass energy.[2,19] This outlook was not evident in the earliest attempts to understand higher-spin states such as $N^*(1688)$ and $N^*(1525)$, where F- and D-wave pion-nucleon channels appeared to be a good starting point.

Given that the truncation problem can be handled as indicated above and that the long-range part of the hadron potential is actually described by single-particle-exchange processes, it appears that, if done carefully, the foundations of bootstrap theory are reasonably secure. However, the situation is still complicated because in any given calculation the number of different spin and strangeness states required may be quite large. In our calculation we have attempted to overcome the many-body difficulties by concentrating on the ground state and by using group theory. The S- and P-wave $SU(6)$ symmetry fits in naturally with the kinematical constraints implied by our phenomenology and at the same time, makes a plausible statement about the nature of the appearance of $SU(6)$ in nature—as a limiting case characterized by the static model. Although the symmetry gives us a workable model, it clearly limits the accuracy of our calculation and forces us to take seriously mainly the qualitative features of our results. It may well be possible to improve on this by seeing how the model responds to perturbations, thus giving us insights into the nature of symmetry breaking.[27]

Our main conclusion is that an $SU(6)$ description of the baryon ground state in terms of $\mathbf{56}^+$ and $\mathbf{70}^-$ multiplets of positive- and negative-parity baryons appears to make sense. The baryon- and meson-exchange interactions imply that $\mathbf{70}^-$ is 230 MeV heavier than $\mathbf{56}^+$ in our model. The presence of meson-exchange forces in the calculation gives some information about the momentum transfer dependence of the baryon form factors. We find that self-consistency can be achieved with a form factor $v(q)=\exp(-\phi q)$ with $\phi \cong \mu_{35}^{-1}$ implying a cutoff of roughly 750 MeV. The least model-dependent of our numerical results is the relative size of the P- wave couplings $\mathbf{56}^+\mathbf{56}^+\mathbf{35}^-$ and $\mathbf{70}_{F,D}^-\mathbf{70}^-\mathbf{35}^-$, which, for example, are seen experimentally in the πNN coupling constant and the $Y_1^*(1660) \rightarrow Y_0^*(1405)\pi$ decay width, respectively. The agreement with experiment is good, although not enough data exist to test the prediction $f(\mathbf{70}_F^-\mathbf{70}^-\mathbf{35}^-)/f(\mathbf{70}_D^-\mathbf{70}^-\mathbf{35}^-) = -1.1$. The S-wave couplings $\mathbf{70}^-\mathbf{56}^+\mathbf{35}^-$ are predicted to be roughly a factor of 2 larger than one finds in nature, although this particular prediction is model-dependent relative to the type of cutoff used.

We conclude by comparing the results of our model with other approaches. For example, if ρ dominance holds precisely at small momentum transfers, then universality of electric charge implies that the longitudinal component of the ρ meson is coupled universally to the isospin current. This relation is almost, but not quite, a feature of our model. For the P-wave $\mathbf{56}^+\mathbf{56}^+\mathbf{35}^-$ vertex, the longitudinal part of the ρ is, in fact, coupled to the isospin of the baryons in $\mathbf{56}^+$. (The ρ is assigned to the $\mathbf{8}^1$ part of $\mathbf{35}^-$.) Since we normalize the $\mathbf{35}^-\mathbf{35}^-\mathbf{35}_F^-$ and $\mathbf{56}^+\mathbf{56}^+\mathbf{35}^-$ couplings to experiment (which in the equality of $f_{\rho NN}$ to $f_{\rho\pi\pi}$ is consistent with ρ universality), ρ universality is a feature of this sector of the model. However, the existence of a nonzero D-type coupling of $\mathbf{70}^-$ to $\mathbf{70}^-\otimes\mathbf{35}^-$ violates ρ universality [e.g., see the $SU(6)$ isoscalar factors for $7\theta_D^- \rightarrow \mathbf{70}^- \otimes \mathbf{35}^-$ in the Appendix], although this is almost certain to be experimentally undetectable. As for the parity-doublet conjecture, our model implies that positive- and negative-parity baryons do not appear to have a 1-1 correspondence. It is difficult to see how the parity-doublet scheme can coexist in a naturel way with bootstrap theory. If indeed, it turns out to be true, then the dynamics of Regge trajectories will most likely turn out not to depend in any important way on their couplings to external particles.

APPENDIX

We explain here some of the group-theoretic details employed in the paper. We begin by considering the calculation of $SU(6)$ crossing coefficients (see also Appendix A in Ref. 25). One can always proceed by carrying out an explicit sum over Clebsch-Gordan coefficients, in a way analogous to that described by de Swart[28] for the group $SU(3)$. For higher symmetries, such a sum is extremely tedious to calculate. However, it is possible to simplify the calculation by noting that the sum factors into a product of isoscalar factors for the higher symmetry and of crossing coefficients belonging to relevant lower symmetries. Since the reader is probably more familiar with the $SU(3)$ symmetry, we treat only that group here.

[27] Some of the many papers along this line include I. P. Gyuk and S. F. Tuan, Phys. Rev. **151**, 1253 (1966); J. G. Belinfante, *ibid.* **140**, B154 (1965); J. G. Koerner, *ibid.* **152**, 1389 (1966).

[28] J. J. de Swart, Nuovo Cimento **31**, 420 (1964).

We follow de Swart's notation for $SU(3)$ Clebsch-Gordan coefficients,

$$\begin{pmatrix} \mu_1 & \mu_2 & \mu_\gamma \\ \nu_1 & \nu_2 & \nu \end{pmatrix}, \tag{A1}$$

where μ_1, μ_2, and μ_γ give the multiplicities of the $SU(3)$ multiplets and ν_1, ν_2, and ν are the respective magnetic quantum numbers. Each $SU(3)$ Clebsch-Gordan coefficient factors into a product of an $SU(3)$ isoscalar factor and an isospin $SU(2)$ Clebsch-Gordan coefficient C:

$$\begin{pmatrix} \mu_1 & \mu_2 & \mu_\gamma \\ \nu_1 & \nu_2 & \nu \end{pmatrix} = \left(\begin{matrix} \mu_1 & \mu_2 \\ I_1Y_1 & I_2Y_2 \end{matrix}\middle|\begin{matrix} \mu_\gamma \\ IY \end{matrix}\right) C_{I_{z1}I_{z2}I_z}{}^{I_1I_2I}, \tag{A2}$$

again with de Swart's notation. If we define the s and t channels by (a bar over a symbol represents antiparticle)

$$\begin{aligned} s&:\ \mu_1+\mu_2 \to \mu_3+\mu_4, \\ t&:\ \mu_1+\bar{\mu}_3 \to \bar{\mu}_2+\mu_4, \end{aligned} \tag{A3}$$

then we have for the crossing matrix $\beta_{st}(\mu_1\mu_2\mu_3\mu_4)$

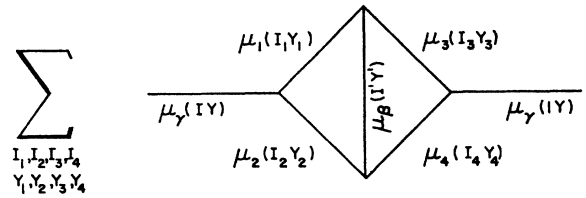

FIG. 2. Diagrammatic representation of Eq. (A7), in which a crossing coefficient involving the exchange of state μ_β projected into state μ_γ is given as a sum over the relevant isoscalar factors.

$$(\mu_{\beta\gamma}|\beta_{st}(\mu_1\mu_2\mu_3\mu_4)|\mu'_{\beta'\gamma'}) = \sum_{\nu_1\nu_2\nu_3\nu_4\nu'} (-)^{Q_3-Q_2} \begin{pmatrix} \mu_1 & \mu_2 & \mu_\gamma \\ \nu_1 & \nu_2 & \nu \end{pmatrix} \begin{pmatrix} \mu_3 & \mu_4 & \mu_\beta \\ \nu_3 & \nu_4 & \nu \end{pmatrix} \begin{pmatrix} \bar{\mu}_2 & \mu_4 & \mu_{\beta'}' \\ -\nu_2 & \nu_4 & \nu' \end{pmatrix} \begin{pmatrix} \mu_1 & \bar{\mu}_3 & \mu_{\beta'}' \\ \nu_1 & -\nu_3 & \nu' \end{pmatrix}, \tag{A4}$$

where Q_i is the charge of particle i and $\mu_{\beta\gamma}$, and $\mu'_{\beta'\gamma'}$ represent the s,t channel $SU(3)$ multiplets under consideration. Using Eq. (A2) and the Gell-Mann-Okubo formula $Q_i = T_{zi}+\frac{1}{2}Y_i$, Eq. (A4) becomes

$$\begin{aligned} (\mu_{\beta\gamma}|\beta_{st}(\mu_1\mu_2\mu_3\mu_4)|\mu'_{\beta'\gamma'}) = &\sum_{Y_1Y_2Y_3Y_4Y'} \sum_{I_1I_2I_3I_4I'} (-)^{(Y_3-Y_2)/2} \left(\begin{matrix} \mu_1 & \mu_2 \\ I_1Y_1 & I_2Y_2 \end{matrix}\middle|\begin{matrix} \mu_\gamma \\ IY \end{matrix}\right) \left(\begin{matrix} \mu_3 & \mu_4 \\ I_3Y_3 & I_4Y_4 \end{matrix}\middle|\begin{matrix} \mu_\beta \\ IY \end{matrix}\right) \\ &\times \left(\begin{matrix} \bar{\mu}_2 & \mu_4 \\ I_2,-Y_2 & I_4Y_4 \end{matrix}\middle|\begin{matrix} \mu'_{\beta'} \\ I'Y' \end{matrix}\right) \left(\begin{matrix} \mu_1 & \bar{\mu}_3 \\ I_1Y_1 & I_3,-Y_3 \end{matrix}\middle|\begin{matrix} \mu'_{\gamma'} \\ I'Y' \end{matrix}\right) \sum_{I_{z1}I_{z2}I_{z3}I_{z4}I_{z'}} C_{I_{z1}I_{z2}I_z}{}^{I_1I_2I} \\ &\times C_{I_{z3}I_{z4}I_z}{}^{I_3I_4I} C_{I_{z2}I_{z4}I_{z'}}{}^{I_2I_4I'} C_{I_{z1}-I_{z3}I_{z'}}{}^{I_1I_3I'} (-)^{I_{z3}-I_{z2}}, \end{aligned} \tag{A5}$$

but an $SU(2)$ Racah coefficient is defined by

$$(2I'+1)W(I_1I'II_4;\,I_3I_2) = (-)^{I_1+I_4} \sum (-)^{I_{z3}-I_{z2}} C_{I_{z1}I_{z2}I_z}{}^{I_1I_2I} C_{I_{z3}I_{z4}I_z}{}^{I_3I_4I} C_{I_{z1}-I_{z3}I_{z'}}{}^{I_1I_3I'} C_{I_{z2}-I_{z4}I_{z'}}{}^{I_2I_4I'}, \tag{A6}$$

and substituting (A5) into (A4), we have the result

$$\begin{aligned} (\mu_{\beta\gamma}|\beta_{st}(\mu_1\mu_2\mu_3\mu_4)|\mu'_{\beta'\gamma'}) = &\sum_{Y_i,I_i} (-)^{Y_4/2-I_4}(-)^{Y_1/2+I_1}(2I'+1)W(I_1I'II_4;\,I_3I_2) \\ &\times \left(\begin{matrix} \mu_1 & \mu_2 \\ I_1Y_1 & I_2Y_2 \end{matrix}\middle|\begin{matrix} \mu_\gamma \\ IY \end{matrix}\right) \left(\begin{matrix} \mu_3 & \mu_4 \\ I_3Y_3 & I_4Y_4 \end{matrix}\middle|\begin{matrix} \mu_\beta \\ IY \end{matrix}\right) \left(\begin{matrix} \bar{\mu}_2 & \mu_4 \\ I_2-Y_2 & I_4-Y_4 \end{matrix}\middle|\begin{matrix} \mu'_{\beta'} \\ I'Y' \end{matrix}\right) \left(\begin{matrix} \mu_1 & \bar{\mu}_3 \\ I_1Y_1 & I_3-Y_3 \end{matrix}\middle|\begin{matrix} \mu'_{\gamma'} \\ I'Y' \end{matrix}\right). \end{aligned} \tag{A7}$$

The formula (A7) is viewed diagrammatically as in Fig. 2. The simplification of (A7) over (A4) becomes quite substantial for a higher symmetry because the lengthy sum over Clebsch-Gordan coefficients is replaced by a much shorter sum over isoscalar factors, etc. For $SU(6)$, a crossing matrix element is given in terms of $SU(6)$ isoscalar factors and $SU(3)$ and $SU(2)$ crossing coefficients. Several $SU(6)$ crossing coefficients are given in Table II.

In order to compute certain $SU(6)$ crossing coefficients and also to compare our theoretical results with experiment, it is necessary that we have $SU(6)$ Clebsch-Gordan coefficients. Most of the useful ones, namely, $\mathbf{35}\otimes\mathbf{56}\to\mathbf{56}$, $\mathbf{70}$ and $\mathbf{35}\otimes\mathbf{35}\to\mathbf{35}_F$, are already tabulated.[29] A tabulation of the $\mathbf{35}\otimes\mathbf{70}$ isoscalar factors exists,[30] but neither of the $\mathbf{70}$ multiplets in the Clebsch-Gordan series agrees with generator matrix elements. Since we wish to classify $\mathbf{70}_F$ in this way (along with a relative orthogonal $\mathbf{70}_D$), we have performed the calculation using the appropriate

[29] C. L. Cook and G. Murtaza, Nuovo Cimento 39, 531 (1965).
[30] J. C. Carter (private communication).

TABLE II. Certain SU (6) crossing coefficients. Notation is standard except in part (a), where f denotes the antisymmetric coupling $\mathbf{35}\otimes\mathbf{35}\rightarrow\mathbf{35}_f$. We use F and D throughout to denote the couplings $\mathbf{70}\otimes\mathbf{35}\rightarrow\mathbf{70}_F$, $\mathbf{70}_D$.

(a) $\mathbf{70}\otimes\mathbf{35}\rightarrow\mathbf{70}\otimes\mathbf{35}$ (s-t)

	Direct								
Exchange	**20**	**56**	$\mathbf{70}_{FF}$	$\mathbf{70}_{DD}$	$\mathbf{70}_{FD}$	$\mathbf{70}_{DF}$	**540**	**560**	**1134**
1	$(10)^{1/2}/35$	$(28)^{1/2}/35$	$1/(35)^{1/2}$	$1/(35)^{1/2}$	0	0	$(270)^{1/2}/35$	$(280)^{1/2}/35$	$(567)^{1/2}/35$
$\mathbf{35}_{fF}$	$3/2(11)^{1/2}$	$1/2(11)^{1/2}$	$1/(11)^{1/2}$	$1/(11)^{1/2}$	0	0	$1/3(11)^{1/2}$	0	$-1/3(11)^{1/2}$
$\mathbf{35}_{fD}$	$-\frac{1}{2}(2/11)^{1/2}$	$3/2(22)^{1/2}$	0	$-\frac{3}{32}(2/11)^{1/2}$	$1/(11)^{1/2}$	$1/(11)^{1/2}$	$\frac{5}{32}(2/11)^{1/2}$	$-\frac{11}{32}(2/11)^{1/2}$	$(7/96)(2/11)^{1/2}$

(b) $\mathbf{56}\otimes\mathbf{35}\rightarrow\mathbf{70}\otimes\mathbf{35}$ (s-u)

	Direct			
Exchange	**56**	$\mathbf{70}_F$	$\mathbf{70}_D$	**1134**
56	$\frac{2}{5}$	$18/5(33)^{1/2}$	$-\frac{4}{5}(3/22)^{1/2}$	$-(4/45)(\frac{3}{2})^{1/2}$
$\mathbf{70}_F$	$9/2(33)^{1/2}$	$2/11$	$6/11(2)^{1/2}$	$\frac{1}{3}(2/11)^{1/2}$
$\mathbf{70}_D$	$-(3/22)^{1/2}$	$6/11(2)^{1/2}$	$9/11$	$-2(11)^{1/2}/99$
1134	$-(9/5)(\frac{3}{2})^{1/2}$	$(27/5)(2/11)^{1/2}$	$-(18/55)(11)^{1/2}$	$\frac{3}{5}$

three-quark wave functions. [See Eq. (4.2) of Ref. 31.] Our results are the following:

$\mathbf{70}_F$

$$\mathbf{8}^4=(1/\sqrt{33})[(\sqrt{10})\mathbf{8}^3\otimes\mathbf{8}_A{}^4+(\sqrt{5})\mathbf{8}^3\otimes\mathbf{8}_S{}^2-\mathbf{8}^3\otimes\mathbf{8}_A{}^2+(\sqrt{5})\mathbf{8}^2\otimes\mathbf{10}^2-\mathbf{8}^3\otimes\mathbf{1}^2+(\sqrt{5})\mathbf{1}^3\otimes\mathbf{8}^4+(\sqrt{6})\mathbf{8}^1\otimes\mathbf{8}_A{}^4],$$

$$\mathbf{8}^2=(1/\sqrt{33})[-(\sqrt{10})\mathbf{8}^3\otimes\mathbf{8}_S{}^4+\sqrt{2}\,\mathbf{8}^3\otimes\mathbf{8}_A{}^4+\sqrt{2}\,\mathbf{8}^3\otimes\mathbf{8}_A{}^2-(\sqrt{10})\mathbf{8}^3\otimes\mathbf{10}^2-\sqrt{2}\,\mathbf{8}^3\otimes\mathbf{1}^2+\mathbf{1}^3\otimes\mathbf{8}^2+\sqrt{6}\,\mathbf{8}^1\otimes\mathbf{8}_A{}^2],$$

$$\mathbf{10}^2=(1/\sqrt{33})[2\sqrt{2}\,\mathbf{8}^3\otimes\mathbf{8}^4+2\sqrt{2}\,\mathbf{8}^3\otimes\mathbf{8}^2-2\,\mathbf{8}^3\otimes\mathbf{10}^2-\mathbf{1}^3\otimes\mathbf{10}^2-(\sqrt{12})\mathbf{8}^1\otimes\mathbf{10}^2],$$

$$\mathbf{1}^2=(1/\sqrt{33})[-4\,\mathbf{8}^3\otimes\mathbf{8}^4+\mathbf{8}^3\otimes\mathbf{8}^2+\mathbf{1}^3\otimes\mathbf{1}^2],$$

$\mathbf{70}_D$

$$\mathbf{8}^4=(1/16\sqrt{165})[(55\sqrt{5})\mathbf{8}^3\otimes\mathbf{8}_S{}^4-35\,\mathbf{8}^3\otimes\mathbf{8}_A{}^4+10\sqrt{2}\,\mathbf{8}^3\otimes\mathbf{8}_S{}^2+20(\sqrt{10})\mathbf{8}^3\otimes\mathbf{8}_A{}^2$$
$$+65\sqrt{2}\,\mathbf{8}^3\otimes\mathbf{10}^2+9(\sqrt{10})\mathbf{8}^3\otimes\mathbf{1}^2+10\sqrt{2}\,\mathbf{1}^3\otimes\mathbf{8}^4+22(\sqrt{5})\mathbf{1}^3\otimes\mathbf{8}^2+55\sqrt{3}\,\mathbf{8}^1\otimes\mathbf{8}_S{}^4-7(\sqrt{15})\mathbf{8}^1\otimes\mathbf{8}_A{}^4],$$

$$\mathbf{8}^2=(1/16\sqrt{33})[4(\sqrt{5})\mathbf{8}^3\otimes\mathbf{8}_S{}^4+40\,\mathbf{8}^3\otimes\mathbf{8}_A{}^4+22(\sqrt{5})\mathbf{8}^3\otimes\mathbf{8}_S{}^2-26\,\mathbf{8}^3\otimes\mathbf{8}_A{}^2-7(\sqrt{5})\mathbf{8}^3\otimes\mathbf{10}^2+15\,\mathbf{8}^3\otimes\mathbf{1}^2$$
$$+22\sqrt{2}\,\mathbf{1}^3\otimes\mathbf{8}^4-2\sqrt{2}\,\mathbf{1}^3\otimes\mathbf{8}^2-4\sqrt{3}\,\mathbf{8}^1\otimes\mathbf{8}_A{}^2-11(\sqrt{15})\mathbf{8}^1\otimes\mathbf{10}^2+11\sqrt{3}\,\mathbf{8}^1\otimes\mathbf{1}^2],$$

$$\mathbf{10}^2=(1/8\sqrt{33})[26\sqrt{2}\,\mathbf{8}^3\otimes\mathbf{8}^4-7\sqrt{2}\,\mathbf{8}^3\otimes\mathbf{8}^2+40\,\mathbf{8}^3\otimes\mathbf{10}^2+20\,\mathbf{1}^3\times\mathbf{10}^2-11(\sqrt{6})\mathbf{8}^1\otimes\mathbf{8}^2-4\sqrt{3}\,\mathbf{8}^1\otimes\mathbf{10}^2],$$

$$\mathbf{1}^2=\tfrac{1}{4}(3/22)^{1/2}[6\,\mathbf{8}^3\otimes\mathbf{8}^4+5\,\mathbf{8}^3\otimes\mathbf{8}^2+4\,\mathbf{1}^3\otimes\mathbf{1}^2+(11/\sqrt{3})\mathbf{8}^1\otimes\mathbf{8}^2].$$

In the above the subscripts A and S represent the antisymmetric and symmetric couplings of two $SU(3)$ octets to a third $SU(3)$ octet.

---

[31] M. A. B. Bég and A. Pais, Phys. Rev. 138, B692 (1965).