Electromagnetic Properties of Hadrons in a Covariant Quark Model*†

TIMM GUDEHUS‡
Center for Theoretical Studies, University of Miami, Coral Gables, Florida 33124
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Among others, formulas for the $\omega\pi\gamma$, the $\rho\pi\gamma$, and the $N^*N\gamma$ transition moments, and relations between the electric, magnetic, and transition form factors of the $\frac{1}{2}^+$ baryons and the $\frac{3}{2}^+$ baryon resonances, are derived within the framework of a covariant quark model.

PREDICTIONS about the electromagnetic properties of the lowest hadron states which have been calculated from the nonrelativistic quark model,[1] $SU(3)$,[2] and $SU(6)$[3] may also be derived from the covariant quark model described elsewhere[4–6]; however, the last-named model gives information in addition to that gained from the first three. The most important result seems to be that in this model the magnetic moments μ_H of the hadrons H turn out to be of the order $e/2M_H$, where M_H is the hadron mass, so that these moments can be large in comparison to the quark magnetic moment $e/2m$, even if the quark mass m is large compared with M_H. Besides this, other new predictions can be derived; for example, an absolute value of the $\omega\pi\gamma$ transition moment leads to the strikingly good prediction $T_{\omega\to\pi\gamma}=1.22$ MeV. However, in this model the calculation of the correct absolute value of the proton magnetic moment still remains an unsolved problem.

In the covariant quark model, the quark-antiquark structure of a hadron state $|H\rangle$ is described by a Bethe-Salpeter amplitude

$$\chi^{(H)}(x_1\cdots x_n)=(2\pi)^{3/2}\langle 0|\,T[\psi(x_1)\cdots\psi(x_k)\bar\psi(x_{k+1})\cdots\bar\psi(x_n)]|H\rangle \quad (1)$$

with a definite number of quark and antiquark fields $\psi(x)$, $\bar\psi(x)$. For the Fourier transforms $\chi^{(H)}(p_1\cdots p_n)$ of these amplitudes, certain ansätze are made which correspond to assumptions about the quark wave functions of the hadrons in the nonrelativistic quark model. As has been shown in Ref. 4, the possible ansätze for the Bethe-Salpeter amplitudes are strongly restricted by their general decomposition into covariants.

The relevant ansätze for the Bethe-Salpeter amplitudes of the pseudoscalar-meson nonet P, the vector-meson nonet V, the $\frac{1}{2}^+$-baryon octet O, and the $\frac{3}{2}^+$-baryon decuplet D are

$$\chi^{(P)}(p,P)=(1/\sqrt{M_P})\gamma_5(\gamma\cdot P-M_P)T^{(P)}\chi(p)\,, \quad (2)$$

$$\chi^{(V)}(p,P,e)=(1/\sqrt{M_V})\gamma\cdot e(\gamma\cdot P-M_V)T^{(V)}\chi(p)\,, \quad (3)$$

$$\begin{aligned}\chi^{(O)}{}_{\nu_1a_1,\nu_2a_2,\nu_3a_3}(p,p',P,u)&=(1/\sqrt{3}M_O)[u_{\nu_1}(\gamma_5(\gamma\cdot P-M_O)C)_{\nu_2\nu_3}T^{(O)}{}_{a_1a_2a_3}\\&\quad+\text{cycl.}(1,2,3)]\chi(p,p')\,,\end{aligned} \quad (4)$$

and

$$\begin{aligned}\chi^{(D)}{}_{\nu_1a_1,\nu_2a_2,\nu_3a_3}(p,p',P,u)&=(1/\sqrt{3}M_D)[u_{\nu_1}{}^{\mu}(\gamma_\mu(\gamma\cdot P-M_D)C)_{\nu_2\nu_3}T^{(D)}{}_{a_1a_2a_3}\\&\quad+\text{cycl.}(1,2,3)]\chi(p,p')\,,\end{aligned} \quad (5)$$

respectively.[7] P_μ is the total momentum of the hadron state $|H\rangle$. p [$=\frac{1}{2}(p_1-p_2)$ in Eqs. (2) and (3) and $=p_1-p_3$ in Eqs. (4) and (5)] and p' [$=p_2-p_3$ in Eqs. (4) and (5)] are the relative momenta of the quarks. $e_\mu(P)$, $u(P)$, and $u_\mu(P)$ are, respectively, the polarization vectors and spinors characterizing the spin state of the hadron. The 3×3 matrices $T^{(P)}$ and $T^{(V)}$ and the tensors $T^{(O)}{}_{a_1a_2a_3}=T^{(O)}{}_{a_1k}\epsilon_{ka_2a_3}$ and $T^{(D)}{}_{a_1a_2a_3}$ are given in Refs. 4 and 5. The invariant functions $\chi(p)$ and $\chi(p,p')$ are assumed to be essentially different from zero only for relative momenta $|p_\mu/m|\ll1$, for $\mu=0$, 1, 2, 3, where m is the quark mass. Furthermore, the invariant functions associated with pseudoscalar and vector mesons are assumed to be approximately equal, as are those of the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons. The mass factors $1/\sqrt{M_M}$ and $1/M_B$ turn out to be necessary for the proper normalization of the electric form factors, as we shall see later. We emphasize that we do not claim that the ansätze (2)–(5) are the exact Bethe-Salpeter amplitudes. They should rather be considered as approximations of the exact amplitudes which cannot be calculated at present.

The ansätze (3)–(5) show some similarity to the expressions used for the description of these hadrons in the $\tilde U(12)$ scheme proposed by Delbourgo, Salam, and Strathdee.[8] This may indicate that there is a relationship between $\tilde U(12)$ and the covariant quark model similar to that between $SU(6)$ and the nonrelativistic quark model.

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‡ Present address: 2 Hamburg 55, Strandweg 53, Germany.

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[3] F. Gursey, A. Pais, and L. A. Radicati, Phys. Rev. Letters 13, 299 (1964).

[4] T. Gudehus, DESY Report No. 68/11, 1968 (unpublished).

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[6] T. Gudehus, Phys. Rev. (to be published).

[7] The Bethe-Salpeter amplitudes (2)–(5) and the vertex functions which we introduce later are reduced functions, i.e., a trivial factor $\delta(P-\sum p_i)$ has been split off from the functions as defined by (1).

A further basic assumption of the covariant quark model is that all interactions of the hadrons are mediated by a definite number of quarks. The transition amplitudes for hadron processes are approximated by all Feynman diagrams of lowest order in the coupling of the hadrons to a definite number of quarks. The internal structure of the hadrons is taken into account by the insertion of the vertex function for each hadron-quark vertex. The vertex function is determined by the Bethe-Salpeter amplitude. The meson-quark-antiquark vertex function is given by

$$\Gamma^{(M)}(p,P,e)=-(2\pi)^4 i(\gamma\cdot p+\tfrac{1}{2}\gamma\cdot P-m)\times\chi^{(M)}(p,P,e)(\gamma\cdot p-\tfrac{1}{2}\cdot\gamma P-m),$$

and the baryon-three-quark vertex function by

$$\begin{aligned}\Gamma^{(B)}&(p,p',P,u)\\ &=-(2\pi)^8[\tfrac{1}{3}(2\gamma\cdot p-\gamma\cdot p'+\gamma\cdot P)-m]\\ &\quad\times[\tfrac{1}{3}(2\gamma\cdot p'-\gamma\cdot p+\gamma\cdot P)-m]\\ &\quad\times[\tfrac{1}{3}(\gamma\cdot P-\beta\cdot p-\gamma\cdot p')-m]\chi^{(B)}(p,p',P,u),\end{aligned}$$

wherein $\chi^{(M)}$ and $\chi^{(B)}$ are Bethe-Salpeter amplitudes, and the notation is the same as in formulas (2)–(5).[7]

For a justification of these "Feynman rules for the quark model," we can argue following Molpurgo[9] that, owing to the small kinetic energy of the quarks in the lowest hadron states, the probability for the production of virtual quark-antiquark pairs is very small. Because of this, diagrams with a higher number of quark lines are suppressed relative to the lowest-order diagrams.

According to these Feynman rules for the quark model, we approximate the meson-photon interaction by the diagrams shown in Fig. 1 and the baryon-photon interaction by those shown in Fig. 2. The resulting expressions for the expectation values of the electromagnetic current $j_\mu(x)$ are

$$\begin{aligned}(2\pi)^3\langle P',e',M'|\,j_\mu(0)\,|P,e,M\rangle&\\ =-(2\pi)^4 im\int dp\; Q_\mu{}^{AA'}&\\ \times[\bar{\chi}^{(M')}{}_{A'B}(p-\tfrac{1}{2}P')\,\chi^{(M)}{}_{BA}(p-\tfrac{1}{2}P)&\\ +\chi^{(M)}{}_{A'B}(p-\tfrac{1}{2}P)\,\bar{\chi}^{(M')}{}_{BA}(p-\tfrac{1}{2}P')]&\quad(6)\end{aligned}$$

and

$$\begin{aligned}(2\pi)^3\langle P',u',B'|\,j_\mu(0)\,|P,u,B\rangle&\\ =(2\pi)^8 m^2\int\int dp\,dp'&\\ \times Q_\mu{}^{A'A}\,\bar{\chi}^{(B')}{}_{A_1A_2A'}(P'-2p-p',P'-2p'-p,P',u')&\\ \times\chi^{(B)}{}_{A_1A_2A}(P-2p-p',P-2p'-p,P,u)&\quad(7)\end{aligned}$$

for the mesons and the baryons, respectively. Equations (6) and (7) have to be summed over repeated indices $A,B,\ldots.$ [$A=(a,\nu)$ denotes a pair consisting of a quark index a and a Dirac index ν.] In the derivation of expressions (6) and (7), the relative momenta of the quarks and the momentum transfer $q_\mu=P_\mu'-P_\mu$ have been neglected relative to the quark mass m. This is justified by the previously mentioned assumption that the χ's are essentially different from zero only for $|p_\mu/m|\ll1$ and for momentum transfers $|q_\mu/m|\ll1$. The $\bar{\chi}$'s are given by

$$\bar{\chi}^{(M)}(p,P,e)=-\chi^{(\bar{M})}(p,-P,\bar{e})$$

and (8)

$$\bar{\chi}^{(B)}(p,p',P,u)=C^{-1}\cdot C^{-1}\cdot C^{-1}\cdot\chi^{(B)}(p,p',-P,v),$$

where $\bar{M}$ denotes the antiparticle of M, $v=C\bar{u}^T$, and C is the charge-conjugation matrix.

The 12×12 matrix Q_μ depends on the specific interaction of the quarks with the electromagnetic field. Q_μ has in general the form

$$Q_\mu=Q\cdot\Gamma_\mu,\qquad(9)$$

where Q is the charge matrix

$$Q=\tfrac{1}{3}e\begin{bmatrix}2&0&0\\0&-1&0\\0&0&-1\end{bmatrix}\qquad(10)$$

and Γ_μ is built up out of Dirac matrices and functions of the momentum transfer q_μ. If the electromagnetic interaction of the quark is minimal, we have

$$Q_\mu=Q\cdot\gamma_\mu.\qquad(11)$$

It should be noted that one can also find the result (6) by using a formula derived by Mandelstam[10] for the expectation value of the electromagnetic current for two-particle bound states.

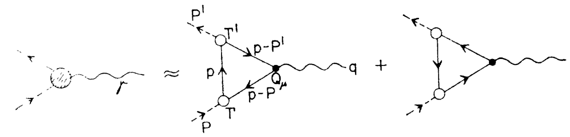

FIG. 1. Feynman diagrams approximating the meson-photon coupling. Thin lines with arrows denote quarks and antiquarks, respectively; dashed lines denote the mesons; and the wavy line denotes the photon. Γ and Γ′ abbreviate the corresponding vertex functions.

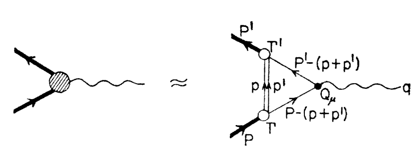

FIG. 2. Feynman diagram for the electromagnetic current of the baryons, which are denoted by heavy lines.

[8] R. Delbourgo, A. Salam, and T. Strathdee, Proc. Roy. Soc. (London) **A278**, 146 (1965).

[9] G. Molpurgo, Physics, **2**, 95 (1965).

[10] S. Mandelstam, Proc. Roy. Soc. (London) **A233**, 248 (1955).

TABLE I. Constants a_{VP} calculated with the help of formula (24), and theoretical decay widths $\Gamma_{VP\gamma}{}^{\text{theoret}}$ resulting from the present model and, for comparison, experimental decay widths $\Gamma_{VP}{}^{\text{expt}}$.[a]

	$\rho\to\pi\gamma$	$\omega\to\pi\gamma$	$\phi\to\pi\gamma$	$\rho\to\eta\gamma$	$\omega\to\eta\gamma$	$\phi\to\eta\gamma$	$X^0\to\rho\gamma$	$X^0\to\omega\gamma$	$\phi\to X^0\gamma$	Units
a_{VP}	1/3	1	0	$1/\sqrt{3}$	$1/3\sqrt{3}$	$2\sqrt{2}/3\sqrt{3}$	$\sqrt{2}/\sqrt{3}$	$\sqrt{2}/3\sqrt{2}$	$-2/3\sqrt{3}$	e
$\Gamma_{VP\gamma}{}^{\text{theoret}}$	0.13	1.22	0	13.5×10^{-3}	1.7×10^{-3}	61.5×10^{-3}	35×10^{-3}	3×10^{-3}	7×10^{-6}	MeV
$\Gamma_{VP\gamma}{}^{\text{expt}}$	<0.4	1.2	...	...	...	...	...	...	...	MeV

[a] A. H. Rosenfeld *et al.*, University of California Lawrence Radiation Laboratory Report No. UCRL-8030, 1968 (unpublished).

Assuming the minimal interaction (11), inserting the Bethe-Salpeter amplitudes (2)–(5) into expressions (6) and (7), and making no further assumptions or approximations, we arrive at the final results

$$(2\pi)^3\langle P',P'|j_\mu(0)|P,P\rangle=e_PF(q^2)(P_\mu'+P_\mu),\quad(12)$$

$$-(2\pi)^3\langle P',e',V'|j_\mu(0)|P,e,V\rangle =e_VF(q^2)\bar{e}'\cdot e\ (P_\mu'+P_\mu) +e_VF(q^2)(\bar{e}'\cdot q\ e_\mu-\ e\cdot q\ \bar{e}_\mu'),\quad(13)$$

$$-(2\pi)^3\langle P',P|j_\mu(0)|P,e,V\rangle =[a_{PV}/(M_PM_V)^{1/2}]F(q^2)\epsilon_{\mu\nu\rho\sigma}P'^\nu P^\rho e^\sigma\quad(14)$$

and

$$(2\pi)^3\langle P',u',O'|j_\mu(0)|P,u,O\rangle =e_OG(q^2)[(1/2M_O)(P_\mu'+P_\mu)\bar{u}'u] +u_OG(q^2)[(1/4M_O{}^2)\bar{u}'r_\mu u],\quad(15)$$

$$-(2\pi)^3\langle P,u',D|j_\mu(0)|P,u,D\rangle =e_DG(0)[(1/2M_D)(P_\mu+P_\mu)\bar{u}'^\nu u_\nu],\quad(16)$$

$$-(2\pi)^3\langle P',u',O|j_\mu(0)|P,u,D\rangle =(a_{OD}/M_OM_D)G(q^2)\epsilon_{\mu\nu\rho\sigma}P'^\nu P^\rho(\bar{u}'u^\sigma),\quad(17)$$

where $r_\mu=\frac{1}{2}[\gamma_\mu(\gamma\cdot P'+\gamma\cdot P)\gamma\cdot q-\lambda\cdot q(\gamma\cdot P'+\lambda\cdot P)\gamma_\mu]$. For the decuplet expectation value (16) we have given only the $q=0$ result, because the general $q\neq0$ result is somewhat lengthy and not very interesting at present. The universal form factors $F(q^2)$ and $G(q^2)$ are given by

$$F(q^2)=-4(2\pi)^4im\int dp\ \chi(p)\chi(p-\tfrac{1}{2}q),\quad(18)$$

$$G(q^2)=-\frac{8}{3^4}(2\pi)^8m^2\int dp\,dp'\ \chi(p,p')\ \chi(p-q,\ p'-q)\quad(19)$$

and the constants e, u, and a by

$$e_M=\mathrm{Tr}[Q(T^{(M)}T^{(\bar{M})}-T^{(\bar{M})}T^{(M)})],\quad(20)$$

$$e_O=\mathrm{Tr}[Q(T^{(O)}T^{(\bar{O})}-T^{(\bar{O})}T^{(O)})],\quad(21)$$

$$e_D=3\ Q_{aa'}T^{(D)}{}_{a_1a_2a}T^{(D)}{}_{a_1a_2a'},\quad(22)$$

$$u_O=\mathrm{Tr}[Q(T^{(O)}T^{(\bar{O})}+T^{(\bar{O})}T^{(O)})],\quad(23)$$

$$a_{PV}=-\mathrm{Tr}[Q(T^{(V)}T^{(\bar{P})}+T^{(\bar{P})}T^{(V)})],\quad(24)$$

and

$$a_{OD}=Q_{aa'}T^{(O)}{}_{a_1a_2a}T^{(D)}{}_{a_1a_3a'}.\quad(25)$$

The constants e_H are the charges of the hadrons. Therefore the normalization condition $G_E{}^H(0)=e_H$ for the electric form factors of the hadrons gives the conditions

$$F(0)=1\quad\text{and}\quad G(0)=1\quad(26)$$

for the universal form factors. From Eqs. (18) and (19) we see that these are actually normalization conditions for the reduced Bethe-Salpeter amplitudes $\chi(p)$ and $\chi(p,p')$, respectively. Owing solely to the factorization of the right mass factors $1/\sqrt{M_M}$ and $1/M_B$ in Eqs. (2)–(5), we arrive at a normalization condition which is the same for the amplitudes of all particles of the same $SU(6)$ multiplet. Another mass-dependent normalization of the χ's would not be consistent with the assumption that the χ's are approximately the same for all particles of one multiplet.

Expressions (12)–(14) contain the following results and predictions for the mesons:

(a) The electric and magnetic form factors of all the vector and pseudoscalar mesons as well as the transition form factors are equal to the one function $F(q^2)$ defined by (18).

(b) The magnetic moments μ_V of the vector mesons are independent of the quark mass m, and

$$\mu_V=e_V/2M_V.\quad(27)$$

(c) The quadrupole moments of the vector mesons are zero.

(d) The VP-transition moments are given by

$$\mu_{VP\gamma}=a_{VP}/(M_VM_P)^{1/2}.\quad(28)$$

With the help of the formula

$$\Gamma_{V\to P\gamma}=\frac{\mu_{VP\gamma}{}^2}{4\pi}\frac{1}{3}\left(\frac{M_V{}^2-M_P{}^2}{2M_V}\right)^3,\quad(29)$$

we calculate the widths as given and compare them with the experimental data in Table I.

For the baryons expressions (15)–(17) contain the following predictions:

(a) The electric form factors $G_E{}^O(q^2)$ and the magnetic form factors $G_M{}^O(q^2)$ of the octet baryons O[11]

[11] The form factors G_E and G_M are defined by the decomposition $(2\pi)^3\langle P',u'|j_\mu(0)|P,u\rangle=(1-q^2/4M^2)^{-1}\times\{G_E[(P_\mu'+P_\mu)/2M](\bar{u}'u)+2MG_M(\bar{u}'r_\mu u)\}$ and are the usual Sachs form factors.

TABLE II. Constants u_O as calculated with formula (23), the resulting theoretical baryon moments μ_O^{theoret}, and the known experimental moments μ_O^{expt} (see Ref. a in Table I).

	p	N	Σ^+	Σ^0	Σ^-	Λ^0	Ξ^0	Ξ^-	Units
u_O	1	$-\frac{2}{3}$	1	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	e
μ_O^{theoret}	2.79	−1.86	2.58	0.86	−0.86	−0.88	−1.67	−0.84	$e/2M_p$
μ_O^{expt}	2.79	−1.91	2.4±0.6	...	...	−0.7±0.3	...	...	$e/2M_p$

are related by

$$G_E{}^O(q^2)=(e_O/\mu_O)G_M{}^O(q^2)=e_O(1-q^2/4M_O{}^2)G(q^2). \quad (30)$$

It is well known that this relation is in good agreement with the experimental data for the proton and neutron form factors.

(b) The decuplet-octet transition form factors[12] are given by

$$G^{DB}(q^2)=\mu_{DB}{}^*\ G(q^2), \quad (31)$$

where $\mu_{DB}{}^*$ is the transition moment. From this, using Eq. (30), we find that, in particular,

$$G^{N^*N}(q^2)=(1-q^2/4M_p{}^2)^{-1}(\mu^*/\mu)G_M{}^p(q^2). \quad (32)$$

This relation is in not too bad agreement with the experimental slope of the N^*N-transition form factor recently measured at DESY[13] (see Fig. 3). We notice that relation (32) has been derived by the author already in Ref. 4.

(c) The magnetic moments of the octet baryons are given by

$$\mu_O=u_O/2M_O. \quad (33)$$

The values of the constants u_O are given in Table II. Relation (33) contains (up to the mass differences) the well-known $SU(6)$ relation for the baryon magnetic moments. But, since $u_p=e_p$, formula (33) predicts only the normal magnetic moment for the proton p, in disagreement with the experimental value. It seems to be difficult to remove this discrepancy within the framework of this model. However, a phenomenological correction of this discrepancy can be achieved by using the vertex operator

$$Q_\mu=Q\cdot(\gamma_\mu+\kappa i\sigma_{\mu\nu}q^\nu) \quad (34)$$

for the baryons instead of (11). Using this Q_μ, we find that the baryon magnetic moments are given by

$$\mu_O=(u_O/2M_O)(1+2M_O\,\kappa) \quad (35)$$

instead of by Eq. (33). To fit the proton moment we have to choose $\kappa=1.79/2M_p$. The predictions resulting from Eq. (35) for the other baryon moments are given in Table II.

(d) With the same vertex operator (34) we find that the DO-transition moments are given by

$$\mu_{DO}{}^*=-(a_{DO}/M_DM_O)[1+(M_D+M_O)\,\kappa]. \quad (36)$$

The constants a_{DO} and the moments $\mu_{DO}{}^*$ are listed in Table III. The connections between the moments $\mu_{DO}{}^*$ (which differ in dimension from a magnetic moment by a factor $1/M$) and the Gourdin-Salin coupling constants[14] C_3, C_4, and C_5 are given by

$$C_3=(M_\pi/e)M_D\mu_{DO}{}^*, \quad C_4=-(M_\pi{}^2/e)\mu_{DO}{}^*,$$

and

$$C_5=0. \quad (37)$$

In particular, with the help of the formula

$$\Gamma_{D\to O\gamma}=\frac{\mu_{DO}{}^{*2}}{4\pi}\tfrac{1}{3}(M_D+M_O)^2\left(\frac{M_D{}^2-M_O{}^2}{2M_D}\right)^3, \quad (38)$$

we calculate from Eq. (36) that $\Gamma_{N^*\to N\gamma}=0.5$ MeV. The present experimental value of this quantity is 0.65 ± 0.07 MeV.[15]

It should be emphasized that the introduction of the phenomenological coupling (34) is by no means an explanation of the above-mentioned wrong prediction

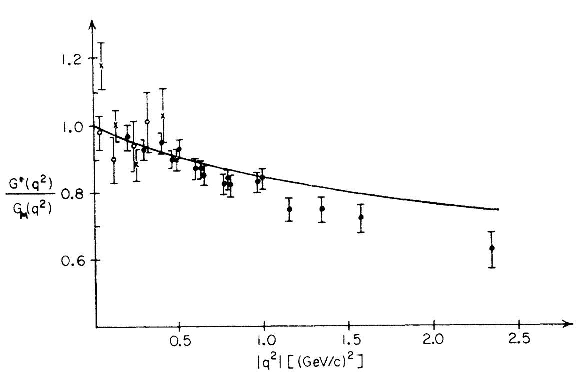

FIG. 3. Comparison of the ratio of the N^*N form factor G^* to the proton magnetic form factor G_M as measured by Bartel *et al.* (Ref. 12) and by W. W. Ash *et al.*, Phys. Letters **24B**, 165 (1967) with the predictions of the present model. The ratio G^*/G has been normalized to 1 at $q^2=1$. The error bars do not include systematical errors which may result from an analysis of the experimental data with a momentum-dependent width in the Breit-Wigner formula (Ref. 12).

[12] The connection between the transition form factors $G^{DO}(q^2)$ defined by (17) and the form factors introduced elsewhere—for example, in Ref. 14, J. D. Bjorken and J. D. Walecka, Ann. Phys. (N. Y.) **38**, 35 (1966), and W. W. Ash *et al.*, Phys. Letters **24B**, 165 (1967)—can easily be established by using relations (37). In particular, the form factors $G_M{}^*$ and $G_E{}^*$, which are used by Ash *et al.* for the $N^*N\gamma$ coupling are related to our G^{N^*N} by

$$G_M{}^*(q^2)^2+G_E{}^*(q^2)^2=\tfrac{2}{3}M[(M+M^*)^2-q^2]G^{N^*N}(q^2)^2,$$

where $G_M{}^{*2}+G_E{}^{*2}$ is the form factor which is actually measured in the type of experiment described in Ref. 12 and by Ash *et al.*

[13] W. Bartel *et. al.*, Phys. Letters **28B**, 148 (1968).

[14] M. Gourdin and P. Salin, Nuovo Cimento **27**, 309 (1963).

[15] R. H. Dalitz and D. G. Sutherland, Phys. Rev. **146**, 1180 (1966); F. Gutbrod, DESY (private communication).

TABLE III. Constants a_{DO} as calculated with formula (25) and the resulting theoretical transition moments μ_{DO}^*.

	$N^{+*}p$	$N^{0*}N$	Σ^+Y^+	Σ^0Y^0	Σ^-Y^-	Λ^0Y^0	$\Xi^0\Xi^{0*}$	$\Xi^-\Xi^{-*}$	Units
a_{DO}	$-1/\sqrt{3}$	$-1/\sqrt{3}$	$1/\sqrt{3}$	$-1/2\sqrt{3}$	0	$1/2$	$1/\sqrt{3}$	0	e
μ_{DO}^*	$1.78/M_{N^*}M_N$	$1.78/M_{N^*}M_N$	$-2/M_YM_\Sigma$	$1/M_YM_\Sigma$	0	$1.69/M_YM_\Lambda$	$2.15/M_\Xi M_{\Xi^*}$	0	e

for the absolute magnetic moment of the proton. Several of the approximations and assumptions made may be the cause of this wrong result. For example, an additional term in the Bethe-Salpeter amplitude (4) of the baryons cannot be excluded. Such a term would correspond to an addition of an $L=2$ state to the $L=0$ state in the nonrelativistic limit of (5). (L is the sum of the relative angular momenta of the quarks.) Another possibility may be that one has to take into account additional diagrams in which the photon couples, via a vector meson, to the baryons. Such contributions seem to be important, as the vector-dominance model[16] shows. It is difficult, however, to modify the present model in a consistent way to take into account these contributions. In any event, the introduction of at least one new parameter seems to be inevitable.

Finally, we want to calculate the coupling constants $g_{V\gamma}$. These are defined by $(2\pi)^{3/2}\langle 0|j_\mu(0)|P,e,V\rangle = eg_{V\gamma}e_\mu$ and are important in the vector-dominance model. The $g_{V\gamma}$ are connected with the constants γ_V, often preferred in the vector-dominance model[16] and given by $\gamma_V = M_V^2/2g_{V\gamma}$. According to the Feynman rules for the quark model, we find from the diagrams in Fig. 4 and with $Q_\mu = Q\cdot\gamma_\mu$ the results

$$g_{\rho\gamma} = (\sqrt{\tfrac{9}{2}})M_\rho^{1/2}\,C\,,\quad g_{\omega\gamma} = (\sqrt{\tfrac{1}{2}})M_\omega^{1/2}\,C\,,$$

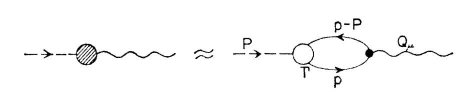

FIG. 4. Feynman diagram approximating the coupling of vector mesons to the photon.

[16] See, for instance, H. Joos, Acta Phys. Austriaca Suppl. 4, 320 (1967).

and (39)

$$g_{\phi\gamma} = -M_\phi^{1/2}\,C\,.$$

C is a new constant given by

$$C = 4\int dp\,\chi(p)\,. \tag{40}$$

By elimination of this C, we find from Eq. (39) that

$$g_{\rho\gamma}:g_{\omega\gamma}:g_{\phi\gamma} = 3\sqrt{M_\rho}:\sqrt{M_\omega}:-\sqrt{2}\sqrt{M_\phi}\,. \tag{41}$$

This result differs from the corresponding $SU(6)$ relation by the mass factors. The prediction for the decay width of $\phi \to e^+e^-$ resulting from our formula (41) is in better agreement with the recent experimental results[17] than the $SU(6)$ prediction, which gives too small a value.

We wish to point out that in recent months two similar approaches to a relativistic quark model have been published.[18,19] These articles contain some calculations for the mesons, but no calculation of the properties of baryons. Furthermore, the results of Kitazoe and Teshima[18] differ from our results, since they use a slightly different ansatz for the Bethe-Salpeter amplitude of the mesons.

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