PHYSICAL REVIEW VOLUME 184, NUMBER 5 25 AUGUST 1969

Massive Gauge Theory and Weak Form Factors

T. W. CHEN
Department of Physics, New Mexico State University, Las Cruces, New Mexico 88001
(Received 9 May 1969)

Closely following the theory of Yang and Mills, massive gauge theories are formulated for a four-vector field which is a mixture of spin-1 and spin-0 particles. The gauge invariance of theories can be maintained within the present framework even for *massive* spin-1 particles. Furthermore, interactions can also be generated for spin-0 particles. The usual gauge transformation for massive electrodynamics is discussed, as well as the chiral gauge transformations. The interactions so generated are shown to lead to the modified Goldberger-Treiman relation.

I. INTRODUCTION

IT is well known that electrodynamics can be generated by gauge transformations.[1] It is also believed that the Yang-Mills trick[2] is the best way to explore the mystery of interactions among various fields. The purpose of the present work is to consider the theory of Yang and Mills through a four-component vector field which is a mixture of a spin-1 particle and a spin-0 particle.

Recently, a number of phenomenological Lagrangian theories[3–5] have been proposed within the framework of mixing A_1 mesons and pions. Various sum rules of current algebra are obtained in these theories in a much simpler fashion. The idea behind this method is simply that the interactions obtained for A_1 mesons imply interactions for pions in a unique way.

The idea of interpreting gauge invariance through the spin-0 part of a vector field is not new. Feldman and Matthews,[6] for example, have successfully presented it. Although the spin-0 particle responsible for the gauge transformation is not physical in their theory, it nevertheless provides us with a new look at the gauge theory —in particular, with the idea that exact *massive* gauge fields can also be established within gauge-invariant theories.

In the present work, the Yang-Mills trick is straightforwardly generalized to (axial-) vector fields as well as (pseudo) scalar fields. The invariance of theories in the weaker sense[7] that the equations of motion are invariant under gauge transformations is discussed. In this way, a gauge-invariant theory for *massive* electrodynamics is developed through the help of a spin-0 field without inconsistency. More interestingly, interactions are generated for spin-1 particles as well as for spin-0 particles. In the next section, the Lagrangian theory for a spin-1–spin-0 mixing field is formulated within the framework of elementary Lagrangian theory. The gauge theory for massive electrodynamics is presented in Sec. III. The chiral gauge transformations are discussed in Sec. IV. It is shown that broken chiral symmetry enables us to generate interactions for pseudoscalar particles as well. Finally, the modified Goldberger-Treiman relation is derived in Sec. V using the interactions obtained in Sec. IV. The paper concludes with a brief summary and discussion (Sec. VI).

II. LAGRANGIAN THEORY

We shall begin with the formulation of a Lagrangian theory for a four-component vector (or axial-vector field) $\phi^\mu(x)$, which consists of a spin-1 meson $a^\mu(x)$ of mass m and a spin-0 meson $b(x)$ of mass μ. We shall assume that ϕ^μ has the natural local form

$$\phi^\mu = a^\mu + \lambda\partial^\mu b\,, \tag{2.1}$$

where λ is a constant. In the absence of interactions, a^μ satisfies the usual free Proca equation of mass m, and b satisfies the usual free Klein-Gordon equation of mass μ. The Lagrangians for free a^μ and b are then the usual ones:

$$\mathcal{L}_0(a) = -\tfrac{1}{4}(a_{\mu\nu})^2 - \tfrac{1}{2}m^2(a_\mu)^2 \tag{2.2}$$

and

$$\mathcal{L}_0(b) = -\tfrac{1}{2}(\partial^\mu b)^2 - \tfrac{1}{2}\mu^2 b^2\,, \tag{2.3}$$

where $a_{\mu\nu} \equiv \partial_\mu a_\nu - \partial_\nu a_\mu$. Substituting Eq. (2.1) in $\mathcal{L}_0(a)$, we have

$$\mathcal{L}_0(a) = \mathcal{L}_0(\phi) = -\tfrac{1}{4}(\phi_{\mu\nu})^2 - \tfrac{1}{2}m^2(\phi_\mu)^2 - \tfrac{1}{2}(\lambda m)^2(\partial^\mu b)^2 + \lambda m^2\phi_\mu\partial^\mu b\,. \tag{2.2$'$}$$

Thus the free total Lagrangian for ϕ^μ and b is given by[8]

$$\mathcal{L}_0(\phi,b) = -\tfrac{1}{4}(\phi_{\mu\nu})^2 - \tfrac{1}{2}m^2(\phi_\mu)^2 - \tfrac{1}{2}[1+(\lambda m)^2] \times(\partial^\mu b)^2 - \tfrac{1}{2}\mu^2 b^2 + \lambda m^2\phi^\mu\partial_\mu b\,. \tag{2.4}$$

It should be noted that the Lagrangian (2.4) is achieved merely by redefining the spin-1 field a^μ. The last term alone, which is responsible for the mixing property of ϕ^μ, does not give rise to interactions between spin-1 and spin-0 fields. The quantizations can

[1] See, for example, J. Bernstein, *Elementary Particles and Their Currents* (W. H. Freeman and Co., San Francisco, 1968), for various views of gauge transformations.
[2] C. N. Yang and R. L. Mills, Phys. Rev. **96**, 191 (1954); R. Utiyama, *ibid.* **101**, 1957 (1956); S. L. Glashow and M. Gell-Mann, Ann. Phys. (N. Y.) **15**, 437 (1961).
[3] J. Schwinger, Phys. Letters **24B**, 473 (1967).
[4] J. Wess and B. Zumino, Phys. Rev. **163**, 1727 (1967).
[5] L. M. Brown and H. Munczek, Phys. Rev. Letters **20**, 680 (1968); T. W. Chen and R. E. Pugh, *ibid.* **20**, 880 (1968); A. Burnel and H. Caprasse, Nucl. Phys. **B8**, 65 (1968); M. J. Sweig and W. W. Wada, Phys. Rev. Letters **21**, 441 (1968).
[6] G. Feldman and P. T. Matthews, Phys. Rev. **130**, 1633 (1963).
[7] For detailed discussions on this aspect, see Refs. 1 and 6.
[8] The same Lagrangian as a model has been studied in Ref. 5.

be carried out consistently either through ϕ^μ (and b) or a^μ (and b). To see this, the Euler-Lagrange equations for ϕ^μ and b can be obtained directly from Eq. (2.4) by varying ϕ^μ and b. That is,

$$[(\Box - m^2)g^{\mu\nu} - \partial^\mu\partial^\nu]\phi_\nu + \lambda m^2\partial^\mu b = 0\,, \tag{2.5}$$

$$[1+(\lambda m)^2]\Box b - \mu^2 b - \lambda m^2 \partial^\mu\phi_\mu = 0\,. \tag{2.6}$$

The coupled equations above are obviously equivalent to the Proca equation for a^μ and the Klein-Gordon equation for b, which follow immediately from the Lagrangians (2.2) and (2.3).

Thus, we see that in the absence of interactions the fields ϕ^μ and b describe a spin-1 and a spin-0 field as well as the fields a^μ and b do. The field ϕ^μ, however, provides us with a different view of the theory of interactions. In the next sections we consider gauge transformations in terms of ϕ^μ and the interactions so generated. Here we merely note that the equations of motion for ϕ^μa nd b in the presence of the interactions $\mathcal{L}_I(\phi^\mu,b)$ are

$$[(\Box - m^2)g^{\mu\nu} - \partial^\mu\partial^\nu]\phi_\nu + \lambda m^2\partial^\mu b = j_\phi{}^\mu \tag{2.7}$$

and

$$[1+(\lambda m)^2]\Box b - \mu^2 b - \lambda m^2 \partial^\mu\phi_\mu = j_b\,, \tag{2.8}$$

where

$$j_\phi{}^\mu \equiv \partial_\nu \frac{\partial\mathcal{L}_I}{\partial\partial_\nu\phi_\mu} - \frac{\partial\mathcal{L}_I}{\partial\phi_\mu} \tag{2.9}$$

and

$$j_b \equiv \partial_\nu \frac{\partial\mathcal{L}_I}{\partial\partial_\nu b} - \frac{\partial\mathcal{L}_I}{\partial b}\,. \tag{2.10}$$

Equation (2.8) can be further reduced if Eq. (2.7) is used. We have

$$(\Box - \mu^2)b = j_b - \lambda\partial^\mu j_\mu\,. \tag{2.11}$$

III. GAUGE TRANSFORMATIONS

We consider one-parameter gauge transformations, closely following the well-known theory of Yang and Mills. We begin with a consideration of the gauge theory of massive electrodynamics.

Our gauge transformations are the usual local infinitesimal transformations: For the matter field $\psi(x)$ (i.e., spin-$\frac{1}{2}$ charged field),

$$\psi(x) \to [1+ig\Lambda(x)]\psi(x)\,; \tag{3.1}$$

for the vector field $\phi^\mu(x)$,

$$\phi^\mu(x) \to \phi^\mu(x) + \partial^\mu\Lambda(x)\,; \tag{3.2}$$

and, in addition, the transformation[9] for the scalar field $b(x)$,

$$b(x) \to b(x) + (1/\lambda)\Lambda(x)\,, \tag{3.3}$$

where $\Lambda(x)$ satisfies $\Box\Lambda(x)=0$.

The purposes of the additional transformation of $b(x)$ are twofold: The invariance of the theory can be maintained even in the presence of the mass term of the spin-1 particle, and interactions can be generated for the scalar field as well as for the vector field. The invariance of the theory can be seen easily from the free equations of motion (2.5) and (2.6). We see that if $\mu=0$, then they are invariant under the changes (3.1)–(3.3) even though $m\neq 0$. Thus the invariance of the theory (in the sense that the equations of motion are invariant) is preserved with the help of the massless scalar field b.

The interactions follow as usual from the replacement (with a constant α) of

$$\partial_\mu \to D_\mu = \partial_\mu - ig[\alpha\phi_\mu + \lambda(1-\alpha)\partial_\mu b] \tag{3.4}$$

in the free Lagrangian of the charged particles. Clearly, the replacement (3.4) is a straightforward generalization of the Yang-Mills trick. The Lagrangian $\mathcal{L}'(\psi, L^\mu\psi)$ is invariant under the transformations (3.1)–(3.3) if

$$ig\left(\frac{\partial\mathcal{L}'}{\partial\psi}\psi + \frac{\partial\mathcal{L}'}{\partial\partial_\mu\psi}\partial_\mu\psi - \bar\psi\frac{\partial\mathcal{L}'}{\partial\bar\psi} - \partial_\mu\bar\psi\frac{\partial\mathcal{L}'}{\partial\partial_\mu\bar\psi}\right) = 0 \tag{3.5}$$

and

$$ig\left(\frac{\partial\mathcal{L}'}{\partial\partial_\mu\psi}\psi - \psi\frac{\partial\mathcal{L}'}{\partial\partial_\mu\bar\psi}\right) + \frac{\partial\mathcal{L}'}{\partial\phi_\mu} + \frac{1}{\lambda}\frac{\partial\mathcal{L}'}{\partial\partial_\mu b} = 0\,. \tag{3.6}$$

The second condition [Eq. (3.6)] is obviously satisfied by $\mathcal{L}'$, which is generated according to the replacement (3.4). The first condition, which is actually the gauge condition of the first kind, is also satisfied if $\mathcal{L}'$ is generated by the usual free Lagrangian for the matter field, viz.,

$$\mathcal{L}_0(\psi) = -\bar\psi(\gamma^\mu\partial_\mu + m)\psi\,. \tag{3.7}$$

The interaction so generated is then

$$\mathcal{L}_1 = i\alpha g\bar\psi\gamma^\mu\psi\phi_\mu + i\lambda(1-\alpha)g\bar\psi\gamma^\mu\psi\partial_\mu b\,. \tag{3.8}$$

With the interaction Lagrangian above, the equations of motion for ϕ^μ and b now become

$$[(\Box - m^2)g^{\mu\nu} - \partial^\mu\partial^\nu]\phi_\nu + \lambda m^2\partial^\mu b = -i\alpha g\bar\psi\gamma^\mu\psi \tag{3.9}$$

and

$$\Box b = i\lambda g\partial_\mu(\bar\psi\gamma^\mu\psi) = 0\,. \tag{3.10}$$

Because conditions (3.5) and (3.6) are exactly satisfied the vector current $-i\alpha g\bar\psi\gamma^\mu\psi$ is strictly conserved (CVC). Consequently, the scalar field b is left noninteracting. The parameters λ and α are not of interest in this case, since αg is just the usual coupling constant between the vector field and the matter field. Thus we are back to the usual massive electrodynamics with exact CVC. The scalar field b is there only to maintain the gauge invariance of the theory of the *massive* gauge field.

IV. CHIRAL GAUGE TRANSFORMATIONS

The chiral group is believed to exist only in a broken form. The interactions generated by the chiral trans-

[9] See the pion gauge transformation in M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960); G. Kramer and W. F. Palmer, Phys. Rev. **182**, 1490 (1969).

formation are therefore not unique but are more interesting, since interactions for pseudoscalar field can also be generated.

Here we consider only the simplest chiral gauge group, which is exact only when nucleon-mass and pion-mass terms are all absent. Our gauge transformations are now

$$\psi(x)\to[1+ig\gamma_5\Lambda(x)]\psi(x) \tag{4.1}$$

for the nucleon field,

$$b(x)\to b(x)+(1/\lambda)\Lambda(x) \tag{4.2}$$

for the pseudoscalar field, and

$$\phi^\mu(x)\to\phi^\mu(x)+\partial^\mu\Lambda(x) \tag{4.3}$$

for ϕ^μ, which is now a combination of an axial-vector field of mass m (i.e., A_1 meson) and a pseudoscalar pion field b of mass μ. Here $\Lambda(x)$ is a pseudoscalar c number function satisfying

$$\Box\Lambda(x)=0.$$

Neglecting the mass terms for ψ and b, exactly the same procedure as in Sec. III can be followed. The interactions are

$$\mathcal{L}_I=i\alpha g\bar{\psi}\gamma^\mu\gamma_5\psi\phi_\mu+i\lambda(1-\alpha)g\bar{\psi}\gamma^\mu\gamma_5\psi\partial_\mu b. \tag{4.4}$$

After reestablishing the mass terms, the equations of motion for b and ϕ^μ are now given by

$$[(\Box-m^2)g^{\mu\nu}-\partial^\mu\partial^\nu]\phi_\nu+\lambda m^2\partial^\mu b=-i\alpha g\bar{\psi}\gamma^\mu\gamma_5\psi \tag{4.5}$$

and

$$(\Box-\mu^2)b=i\lambda g\partial^\mu(\psi\gamma_\mu\gamma_5\psi). \tag{4.6}$$

This, of course, is essentially the gradient coupling of a pion to nucleons. We note that the interactions for b become meaningful only after the chiral group is broken. The axial-vector current responsible for the chiral gauge symmetry is

$$A^\mu=\left(\frac{\partial\mathcal{L}}{\partial\partial_\mu\Lambda}\right)_{\delta\phi^\mu=0}$$
$$=m^2\phi^\mu-(1/\lambda)[1+(\lambda m)^2]\partial^\mu b-i\alpha g\bar{\psi}\gamma^\mu\gamma_5\psi. \tag{4.7}$$

Its divergence is

$$\partial^\mu A_\mu=-(1/\lambda)\Box b, \tag{4.8}$$

which vanishes only if the pion and the nucleon masses are all absent.

V. GOLDBERGER-TREIMAN RELATION

The invariance of the Lagrangian under the transformations (4.1)–(4.3) is badly broken after reintroducing the nucleon and pion mass terms. It is interesting to observe, however, that the Lagrangian so generated is invariant under the following transformations, if only the pion mass is absent:

$$\begin{aligned}b&\to b+(F_A/\lambda m^2)v,\\ \psi&\to\omega,\\ \phi^\mu&\to\phi^\mu,\end{aligned} \tag{5.1}$$

where v is a constant and F_A, as will be clearer later, is the leptonic decay constant of the A_1 meson. The current generated by transformation (5.1) is then[10]

$$J_A{}^\mu=\frac{\partial\mathcal{L}}{\partial\partial_\mu v}=\frac{F_A}{m^2}\{m^2\phi^\mu$$
$$-(1/\lambda)[1+(\lambda m)^2]\partial^\mu b+ig(1-\alpha)\psi\gamma^\mu\gamma_5\psi\}. \tag{5.2}$$

This current has, first of all, the favorable properties of the usual gradient-coupling model. In particular, it has the notion of partially conserved axial-vector current (PCAC),[9] viz.,

$$\partial^\mu J_\mu{}^A=\frac{F_A}{m^2}\Big[-(1/\lambda)\Box b$$
$$+ig\partial^\mu(\psi\gamma_\mu\gamma_5\psi)\Big]=\frac{-F_A}{\lambda m^2}\mu^2 b. \tag{5.3}$$

Note that the equations of motion (4.5) and (4.6) are used in arriving at Eq. (5.3). Furthermore, the current (5.2) also allows an interesting role for the axial-vector A_1 meson: The parameter λ can be determined, if $J_A{}^\mu$ is identified as the weak current. F_A is then clearly the leptonic decay constant of the A_1 meson. From Eq. (5.3) we see that the leptonic decay constant for a pion is

$$F_\pi=F_A/\lambda m^2. \tag{5.4}$$

Thus, from the well-known sum rule[11]

$$F_\pi=F_A/m,$$

we conclude that[12]

$$\lambda=1/m. \tag{5.5}$$

The current now becomes

$$\begin{aligned}J_A{}^\mu&=F_A[\phi^\mu-(2/m)\partial^\mu b+ig(1-\alpha)\bar{\psi}\gamma^\mu\gamma_5\psi]\\ &=F_A[a^\mu-(1/m)+ig(1-\alpha)\bar{\psi}\gamma^\mu\gamma_5\psi],\end{aligned} \tag{5.6}$$

where $a^\mu\equiv\phi^\mu-(1/m)\partial^\mu b$ is the A_1 field satisfying

$$[(\Box-m^2)g^{\mu\nu}-\partial^\mu\partial^\nu]a_\nu=-i\alpha g\bar{\psi}\gamma^\mu\gamma_5\psi. \tag{5.7}$$

We note that the interaction reduces to the usual gradient model for a pion only if $\alpha=0$, since the A_1

[10] For the method of obtaining currents, see Bernstein (Ref. 1) or Gell-Mann and Lévy (Ref. 9).

[11] This sum rule can be derived from the sum rule of S. Weinberg [Phys. Rev. Letters **18**, 507 (1967)], together with that of K. Kawarabayashi and M. Suzuki [*ibid.* **16**, 255 (1966)] and Riazuddin and Fayyazuddin [Phys. Rev. **147**, 1071 (1966)]. See Ref. 5 for the details.

[12] The value $1/m$ for λ, which is essentially the A_1-π coupling constant, is widely used to produce many successful results. See Ref. 5.

meson is left noninteracting in this case, as is apparent from Eq. (5.7). In the following we shall restrict ourself to the choice $\alpha=1$, i.e., the case in which the interaction Lagrangian $\mathcal{L}_I$ is a function of ϕ^μ only. With $\alpha=1$, we have the current

$$J_A{}^\mu = F_A[a^\mu - (1/m)\partial^\mu b], \quad (5.8)$$

as well as the equations of motion

$$[(\Box - m^2)g^{\mu\nu} - \partial^\mu\partial^\nu]a_\nu = -ig\bar\psi\gamma^\mu\gamma_5\psi, \quad (5.9)$$

$$(\Box - \mu^2)b = i(g/m)\partial^\mu(\bar\psi\gamma^\mu\gamma_5\psi). \quad (5.10)$$

It should be noted that the current (5.8) also turns out to be a linear combination of a^μ and $\partial^\mu b$, just as ϕ^μ, but interestingly it differs from ϕ^μ in sign. This difference in sign becomes significant in the form factor for the weak axial-vector current below.

Consider the matrix element of the weak axial-vector current $J_A{}^\mu$ between one-nucleon states. The axial-vector part of the weak Hamiltonian is

$$H_\omega = i(G/\sqrt{2})_A{}^\mu L_\mu + \text{H.c.}, \quad (5.11)$$

where L^μ is the leptonic current. From Eqs. (5.8)–(5.11) it follows that

$$\langle \bar N(p)|J_A{}^\mu(0)|N(q)\rangle = F_A g\sqrt{2}\bar u(p) \times\left(\frac{g^{\mu\nu}+k^\mu k^\nu/m^2}{k^2+m^2-i\epsilon} - \frac{1}{m^2}\frac{k^\mu k^\nu}{k^2+\mu^2-i\epsilon}\right)\gamma_\nu\gamma_5 u(q), \quad (5.12)$$

where $k=p-q$, and where $g=g_{A_1NN}$, the A_1-nucleon coupling constant which by Eq. (5.10) is related to the π-nucleon coupling constant g_r by[13]

$$g=(m/2m_N)g_r. \quad (5.13)$$

Comparing with the conventional weak form factors defined by

$$\bar N\langle(p)|J_A{}^\mu(0)|N(q)\rangle = \bar u(p)[G_A(k^2)\gamma_\mu\gamma_5 + iH_A(k^2)k_\mu\gamma_5]u(q), \quad (5.14)$$

we have

$$G_A(k^2) = \frac{F_\pi g_r}{\sqrt{2}m_N}\frac{1}{1+k^2/m^2} \quad (5.15)$$

and

$$H_A(k^2) = -F_A g_{ANN}\frac{\sqrt{2}}{m^2}\left(\frac{1}{k^2+\mu^2} - \frac{1}{k^2+m^2}\right)2m_N; \quad (5.16)$$

i.e.,

$$H_A(k^2) = -\frac{F_\pi g_r}{\sqrt{2}m_N}\frac{m^2-\mu^2}{(k^2+\mu^2)(k^2+m^2)}2m_N \quad (5.17)$$

or

$$\frac{H_A(k^2)}{G_A(0)} = -2m_N\frac{1-(\mu/m)^2}{(k^2+\mu^2)(1+k^2/m^2)}. \quad (5.18)$$

Equations (5.15) and (5.18) are the modified Goldberger-Treiman relations[14] which reduce to the well-known Goldberger-Treiman relations[15] as $k^2 \to 0$. The different signs in front of the A_1-meson and pion contributions in Eqs. (5.12) and (5.16) should be noted.[16] The behavior at infinity of the pseudoscalar form factor H_A,[17] i.e.,

$$\lim_{k^2\to\infty} k^2 H_A(k^2) = 0, \quad (5.19)$$

is entirely due to this difference in signs.

VI. CONCLUSION

We have demonstrated that the gauge transformations through the mixture ϕ^μ have many attractive properties. The difficulty associated with the mass of a vector field in maintaining a gauge-invariant theory can be overcome through the help of the spin-0 field in ϕ^μ. Moreover, some useful interactions can be generated for spin-1 and spin-0 particles simultaneously. As an example, the modified Goldberger-Treiman relations are derived.

An extension of the idea presented in this paper to three-parameter gauge groups is worth investigation. Studies of this possibility, as well as of other applications, are in progress.

ACKNOWLEDGMENT

The author wishes to thank Professor R. L. Ingraham for several stimulating discussions.

[13] This is not new. See H. T. Nieh, Phys. Rev. **164**, 1780 (1967); Sweig and Wada (Ref. 5).

[14] Y. Nambu, Phys. Rev. Letters, **4**, 380 (1960); C. W. Kim and M. Ram, Phys. Rev. **162**, 1584 (1967); H. T. Nieh (Ref. 13).

[15] M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958); **111**, 354 (1958).

[16] One could make use of the field-current identity assertion and let the weak current be proportional to the field ϕ^μ, as was done by Sweig and Wada (Ref. 5). But then the form of $H_A(k^2)$ as given by Eq. (5.18) would not be obtained, although the Goldberger-Treiman relation for G_A could still be achieved.

[17] It is interesting to note that the Goldberger-Treiman relations were obtained in Ref. 14 under an assumption equivalent to Eq. (5.19).