

## Concerning the $n$ - $p$ Scattering Cross Section at 24 MeV\*

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Recent measurements of the  $n$ - $p$  scattering cross section at 24 MeV are compared with the later Yale phenomenological multienergy phase-parameter analysis, (Y-IV) $_{pp+n_p}$ . Good agreement is found for both relative and absolute values. Lack of symmetry in the angular distribution of the differential cross section  $\sigma(\theta)$ , expected from the Y-IV phases, fits in with the total cross section and the new relative  $\sigma(\theta)$  measurements. The newer data support the YLAN3M, YLAN4M, Y-IV type of phase-parameter set at energies near 25 MeV, with the rather large absolute value of the negative  ${}^1P_1$  phase shift  $K_1$  and a positive value of the  ${}^3S_1$ - ${}^3D_1$  coupling parameter  $\rho_1$ . No claim is made that there exist no other phase-parameter sets acceptable on the basis of scattering data alone; but since the positive sign of  $\rho_1$  fits in naturally with the positive sign of the quadrupole moment of the deuteron, and since the low-energy anchor of  $K_1$  has been obtained by means of a reasonable potential, the new measurements do not indicate a need for discarding these elementary considerations as approximate guides in phase-parameter analysis. The relative importance of contributions of a few phase parameters is pointed out, with special reference to the fore-aft asymmetry of the angular distribution expected from the Y-IV phase parameters.

### I. INTRODUCTION

VARIOUS types of neutron-proton ( $n$ - $p$ ) phase-parameter fits<sup>1-4</sup> to experimental data are available in the vicinity of 25 MeV. Some of the newer fits<sup>3,4</sup> give a negative value of the  ${}^3S_1$ - ${}^3D_1$  coupling parameter  $\rho_1$ . Such a value, when used with simple potential models, yields a negative value for  $Q$ , the quadrupole moment of the deuteron, in contradiction with experiment.<sup>5</sup> It appears unlikely<sup>6</sup> that these models are so poor as to give the wrong sign of  $Q$ . In addition to the considerations in footnote 15 of Ref. 6, it may be mentioned that  $Q$  is sensitive to the properties of the deuteron-ground-state wave function at large internucleon-separation distances  $r$  for which the charge distributions of the nucleons are well approximated by points or by small spheres to which the considerations quoted are directly applicable. On the

other hand, some of the older<sup>1</sup> and newer<sup>2</sup> fits give a positive  $\rho_1$  and, if they are interpreted in terms of a potential, they are in approximate agreement with the experimental  $Q$ . It is characteristic of the latter fits that for the energies here considered the negative  ${}^1P_1$ -state phase shift  $K_1$  is rather large in absolute value. They differ in this respect from the other type of fit.

A recently performed experiment of Rothenberg and Masterson<sup>7</sup> provides differential cross-section values  $[\sigma(\theta)]$  for  $n$ - $p$  scattering at c.m. angles  $\theta$  mostly in the backward direction ( $\pi/2 < \theta < \pi$ ). It will be seen from the present note that the measured relative values of the differential cross section are in very good agreement with expectation from the newer fits<sup>2</sup> for which  $\rho_1 > 0$ , and that taken together with some recent total cross-section,  $\sigma_{\text{tot}}$ , measurements<sup>8</sup> as well as the general run of older observations, they speak for relatively large values of  $|K_1|$ , approximately, such as in Ref. 2.

A dominant share of the effect of  $K_1$  on  $\sigma(\theta)$  arises from the contribution of singlet scattering in the approximation of taking into account only the  ${}^1S_0$  phase shift  $K_0$  as well as  $K_1$ . This contribution is<sup>9</sup>

$$\frac{1}{4} {}^1\sigma(\theta) = \frac{1}{4} (\Lambda/2\pi)^2 [\sin^2 K_0 + 6 \sin K_0 \cos(K_0 - K_1) \times \sin K_1 \cos \theta + 9 \sin^2 K_1 \cos^2 \theta]. \quad (1)$$

Here,  ${}^1\sigma(\theta)$  is the differential singlet-scattering cross section and  $\Lambda$  is the wavelength. The second term in square brackets contributes to the dissymmetry of the angular distribution about  $\theta = 90^\circ$ . For the phenomenological phase-parameter fit (Y-IV) $_{pp+n_p}$  and the similar fits (Y-IV) $_{n-p}$  and (Y-IV) $_{n-p}M$ , it gives a major contribution to the expected dissymmetry. Experimental evidence regarding differences in the fore and aft properties of the angular distribution of  $\sigma(\theta)$  in the

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<sup>1</sup> M. H. Hull, Jr., K. E. Lassila, H. M. Ruppel, F. A. McDonald, and G. Breit, *Phys. Rev.* **122**, 1606 (1961); **128**, 830 (1963); G. Breit, A. N. Christakis, M. H. Hull, Jr., H. M. Ruppel, and R. E. Seamon, in *Proceedings of the Twelfth Annual Conference on High-Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965), p. 17; G. Breit and R. D. Haracz, in *High Energy Physics* (Academic Press Inc., New York, 1967), Vol. 1.

<sup>2</sup> G. Breit, *Rev. Mod. Phys.* **39**, 560 (1967). This report was based primarily on work done in collaboration with Friedman, Haracz, Holt, Prakash, and Seamon. R. E. Seamon, K. A. Friedman, G. Breit, R. D. Haracz, J. M. Holt, and A. Prakash, *Phys. Rev.* **165**, 1579 (1968). The last of these papers is referred to in the text as SFBHHP.

<sup>3</sup> M. H. MacGregor, R. A. Arndt, and R. M. Wright, *Phys. Rev.* **169**, 1128 (1968); **169**, 1149 (1968); **173**, 1272 (1968).

<sup>4</sup> Yu. M. Kazarinov, in *Proceedings of the Twelfth Annual Conference on High-Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965), p. 70; S. I. Bilenkaya, Z. Janout, Yu. M. Kazarinov, and F. Lehar, Joint Institute for Nuclear Research Report No. E-2609, 1966 (unpublished); Yu. M. Kazarinov, F. Lehar, and Z. Janout, *Rev. Mod. Phys.* **39**, 571 (1967).

<sup>5</sup> J. M. B. Kellogg, I. I. Rabi, N. F. Ramsey, Jr., and J. R. Zacharias, *Phys. Rev.* **56**, 728 (1939); **57**, 677 (1940); A. Nordsieck, *ibid.* **58**, 310 (1940); H. G. Kolsky, T. E. Phipps, N. F. Ramsey, and H. B. Silsbee, *ibid.* **87**, 395 (1952); G. P. Auffray, *Phys. Rev. Letters* **6**, 120 (1961).

<sup>6</sup> Last paper in Ref. 1.

<sup>7</sup> L. N. Rothenberg and T. G. Masterson, *Bull. Am. Phys. Soc.* **14**, 511 (1969). The writers are grateful to Rothenberg and Masterson for communicating their results and for permission to use them before publication.

<sup>8</sup> D. E. Groce and B. D. Sowerby, *Nucl. Phys.* **83**, 199 (1966).

<sup>9</sup> C. Kittel and G. Breit, *Phys. Rev.* **56**, 744 (1939).

Y-IV interpretation gives, therefore, reasonably direct evidence regarding an important feature of singlet scattering. Crudely speaking, the analysis of data on observables involving nucleon spins obtained from  $p-p$  and  $n-p$  scattering supplies information on other phase parameters, which limits somewhat the alternatives for contributions to the dissymmetry from other causes than  $K_1$ . Through this slightly circuitous relationship, the sign and magnitude of  $\rho_1$  become connected with the absolute value of  $K_1$ . As will be seen more concretely in Sec. III, however, the differential cross section itself is insensitive to  $\rho_1$  if the other phase parameters have their (Y-IV) $_{pp+n_p}$  values. Since in the Yale work the low-energy start of  $K_1$  has been guided by a potential which accounts approximately for phase-parameter phenomenology at other energies than 24 MeV and since  $\rho_1 > 0$  is likely from  $Q > 0$ , the fits here tested have some inherent plausibility. On the other hand, a lack of satisfactory agreement would imply a need for modification of the semitheoretical guides just mentioned. Section II of this paper contains a description of the calculations and Sec. III a discussion of the results and a crude analysis of the main contributions to the differential cross section in terms of phase shifts.

## II. CALCULATIONS

The numerical work reported here has been preceded by the receipt of a copy of an abstract from the authors of Ref. 7 which contained sufficient information to conclude that their data are in fairly satisfactory agreement with (Y-IV) $_{pp+n_p}$ . Later correspondence showed the procedure used by them for obtaining absolute values. The value of the parameter  $B$  used in Gammel's representation<sup>10</sup>

$$\sigma(\theta) = A(1 + B \cos^2\theta) \quad (2)$$

mentioned in Ref. 7 presupposes fore and aft symmetry as implied by the last equation. The (Y-IV) $_{pp+n_p}$  fit gives an angular distribution slightly peaked toward  $180^\circ$ . Complete consistency with (Y-IV) $_{pp+n_p}$  cannot be expected, therefore, if Eq. (2) is used literally with  $B$  determined from the experimental data of Refs. 7 and 8, the procedure used in Ref. 7. Such a procedure is reasonably justifiable, however, for the latest published Livermore-type fit which does not give such peaking and partly for the Dubna 23.1-MeV single-energy analysis which gives a slight peaking in the opposite direction. On the other hand, (Y-IV) $_{pp+n_p}$  is responsible for the values supplied by the University of Wisconsin workers on the basis of their measurements, and those of Ref. 8, being systematically lower than the (Y-IV) $_{pp+n_p}$  values by practically the experi-

mental uncertainty resulting from counting statistics alone at  $\theta = 89.0^\circ$ ,  $146.4^\circ$ , and  $164.7^\circ$ , while for  $\theta = 118.3^\circ$ , it is lower than the (Y-IV) $_{pp+n_p}$  by about 1.6 times that uncertainty. If the dissymmetry is crudely taken into account making use of one value of  $B$  between  $\pi/2$  and  $\pi$  and of another between 0 and  $\pi/2$ , employing for the former the Wisconsin workers' fits to their data and for the latter the ratio of the two  $B$  values estimated from the values of  $\sigma(0)$  and  $\sigma(\pi)$  expected from (Y-IV) $_{pp+n_p}$ , then the systematic disagreement largely disappears. This readjustment corresponds to increasing the experimental values by 1.7%. If values determined from the graph for (Y-IV) $_{pp+n_p}$  are used throughout, then the correction changes to 1.5%, a relatively insignificant modification. The relationship of the  $\sigma(\theta)$  thus obtained to the (Y-IV) $_{pp+n_p}$  values is illustrated in Fig. 1, employing the first of the two adjustments just mentioned.

The implications of the preceding paragraph, regarding the unlikelihood of a symmetric angular distribution, are weakened by the uncertainty of the phase-parameter-fit predictions. It is impossible to estimate the effect of such uncertainties for all fits that may be attempted, as would be necessary for a complete answer, but an approximate estimate may be made by means of the (Y-IV) $_{pp+n_p}$  parallel-shift uncertainties. The centers of the error bars are consistently below the error belt corresponding to parallel shifts in the 0-70-MeV energy range used in standard deviation convention. The top of the  $118.3^\circ$  error bar misses the error belt by more than the latter's width. The tops of the  $89.0^\circ$  and  $146.4^\circ$  error bars are barely caught by the error belt. For this error belt, the evidence is thus reasonably definite against a symmetric angular

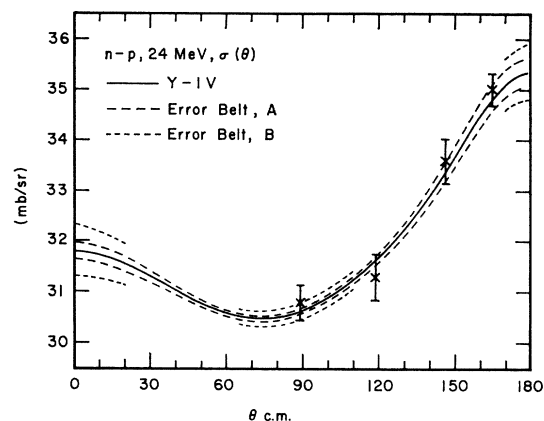


FIG. 1. Comparison of measurements described in Refs. 7 and 8 with phenomenological phase-parameter fit (Y-IV) $_{pp+n_p}$  employing normalization discussed in connection with Eq. (3) which gives results graphically indistinguishable from those obtained by the cruder procedure described soon after Eq. (2). Error-belt lines A and B correspond, respectively, to parallel-shift uncertainties of Table I, viz., through 0-69 MeV and 14.90-32.98 MeV, respectively.

<sup>10</sup> J. L. Gammel, *Fast Neutron Physics* (Interscience Publishers, Inc., New York, 1960), Part II, Chap. VI, p. 2185.

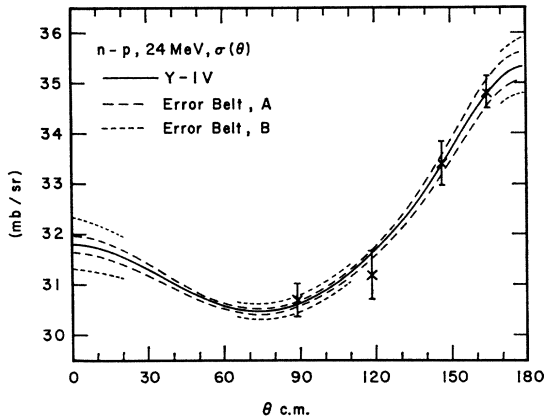


FIG. 2. Comparison of relative cross-section values of Ref. 7 with phenomenological phase-parameter fit (Y-IV)<sub>pp+n<sub>p</sub></sub> employing least-squares adjustment with weights determined by counting statistics. Conventions as in Fig. 1.

distribution, but since parallel shifts in smaller energy intervals give somewhat wider error belts and since phase-parameter fits giving symmetric distributions may have different error-belt widths from those for the (Y-IV)<sub>pp+n<sub>p</sub></sub> fit, judgment regarding exclusion of a symmetric angular distribution of  $\sigma(\theta)$  must be reserved.

Considered as a measurement, the work of Ref. 7 is concerned only with relative values of  $\sigma(\theta)$  at different angles. A least-squares adjustment of their values to the (Y-IV)<sub>pp+n<sub>p</sub></sub> angular distribution employing relative weights corresponding to counting-statistics uncertainties gives the points shown in Fig. 2. The  $\chi^2$  value for deviations from the (Y-IV)<sub>pp+n<sub>p</sub></sub> expectation is 0.75, a very satisfactorily low value.

The  $\sigma_{\text{tot}}$  of Ref. 8 is slightly above the  $\sigma_{\text{tot}}$  corresponding to (Y-IV)<sub>pp+n<sub>p</sub></sub>. Since these are believed to be very good measurements, it appeared desirable to let them influence the comparison by a more careful procedure than that used for Fig. 1. To do so, the following viewpoint was taken. If all information regarding the value of an adjustable factor for  $\sigma(\theta)$  having a common value at all  $\theta$  contained in the multienergy fit (Y-IV)<sub>pp+n<sub>p</sub></sub> were disregarded and the Groce-Sowerby  $\sigma_{\text{tot}}$  at 23.95 MeV were used instead, a readjustment of the fit could be made. With sufficient care, it could conceivably result in reproducing arbitrarily assigned values of  $\sigma_{\text{tot}}$  at 24.0 MeV in the vicinity of that corresponding to Ref. 8 and the relative angular distribution of the present fit. That this could be done without making the multienergy fit unreasonable in some respects has not been demonstrated but nevertheless appears very probable. The minimization problem is then that of finding the best values of  $A_0$  and  $B$  for

$$\chi^2 = w_0(A_0\eta_0 - y_0)^2 + \sum_{i=1}^n (w_i/B^2)(A_0\eta_i - By_i)^2, \quad (3)$$

where

- $\eta_0, y_0$  = phase-parameter (Y-IV)<sub>pp+n<sub>p</sub></sub> and measured value of  $\sigma_{\text{tot}}$ ,
- $w_0 = 1/(\Delta y_0)^2$ ,
- $\eta_i$  ( $i = 1, 2, \dots, n$ ) = (Y-IV)<sub>pp+n<sub>p</sub></sub> values of  $\sigma(\theta)$  at  $n$  values of  $\theta$ ,
- $y_i$  ( $i = 1, 2, \dots, n$ ) = corresponding measured relative values of  $\sigma(\theta)$  at the same values of  $\theta$  employing arbitrary normalization,
- $\Delta y_0$  = experimental uncertainty of  $y_0$ ,
- $\Delta y_i/y_i$  ( $i = 1, 2, \dots, n$ ) = fractional statistical uncertainty of  $y_i$ ,
- $w_i = 1/(\Delta y_i)^2$ ,
- $B$  = normalization adjustment factor for  $\sigma(\theta)$ .

The factor  $A_0$  takes care of the assumed possibility of changing all the  $\sigma(\theta)$  by the same factor employing modified phase shifts, which is then also the factor giving the change in  $\sigma_0$ . Minimization of  $\chi^2$  gives

$$A_0 = y_0/\eta_0, \quad A_0/B = \sum_{i=1}^n w_i \eta_i y_i / \sum_{i=1}^n w_i \eta_i^2 \equiv A'. \quad (4)$$

The first formula in Eq. (4) makes the first term on the right-hand side vanish. The normalization of the modified fit values of  $\sigma(\theta)$  therefore becomes adjusted so as to reproduce the measured  $\sigma_{\text{tot}}$  exactly. The form of  $A'$  is recognized to be that of the reciprocal of the normalization factor to be applied to the  $y_i$  as a result of a least-squares adjustment of the initially available relative values of  $\sigma(\theta)$  to reproduce the initial fit values  $\eta_i$ . That this should be the case is clear without calculation by considering the minimization of the second term alone for the special case  $A_0 = 1$  and from the fact that it may be written as

$$\sum_{i=1}^n w_i (A' \eta_i - y_i)^2. \quad (4')$$

Employment of  $A_0$  and  $B$  of Eq. (4) may be described as changing the (Y-IV)<sub>pp+n<sub>p</sub></sub> $\sigma_{\text{tot}}$  at 24 MeV to the value expected from Ref. 8, changing the  $\sigma(\theta)$  of (Y-IV)<sub>pp+n<sub>p</sub></sub> in the same ratio, and adjusting the relative values of  $\sigma(\theta)$  from Ref. 7 by least squares applying the same normalization factor at all angles so as to fit the  $\sigma(\theta)$  of the changed fit. In making these calculations, the value  $397.2 \pm 1.7$  mb for  $\sigma_{\text{tot}}$  given in Ref. 8 as applying at 23.951 MeV was corrected for the slope of the empirical  $\sigma_{\text{tot}}-E$  curve and was used at  $396.30 \pm 1.7$  mb at 24 MeV. The values of the  $By_i$  so obtained are so close to those plotted in Fig. 1 that the difference is practically undetectable to the eye even though the method described earlier was crude. The comparison in the figure is with the *original* (Y-IV)<sub>pp+n<sub>p</sub></sub> fit values  $\eta_i$  and not with the values adjusted by means of  $A_0$ . The deviations of the experimental points from the phenomenological fit values give  $(\chi^2)_{\text{ang. distr.}} = 1.36$ . If

$\sigma_{tot}$  is included, then there results  $(\chi^2)_{tot}=2.2$ . Since one of the angular distribution measurements is needed as a reference standard, the first of these two values is more properly speaking a  $\chi^2$  for three rather than four data and the second may be argued to be for four data. In these conventions, the  $\chi^2$  per datum has the values 0.45 and 0.55, respectively.

### III. DISCUSSION

Were the comparison made with the (Y-IV) $_{pp+np}$  fit subjected to the idealized adjustment described in introducing Eq. (3), i.e., employing  $A\eta_0$  in place of  $\eta_0$ , then the first term in Eq. (3) would be zero and the second would be the same as though the  $y_i$  were adjusted by least squares to the original  $\eta_i$ , employing the same adjustable normalization factor for the four angles, the final  $\chi^2$  depending on the numerical value of  $A'$  and not on the way it is introduced, as seen from Eq. (4'). The value of  $\chi^2$  per datum including  $\sigma_{tot}$  as a datum for the idealized readjusted fit is  $0.75/4=0.19$ . The readjusted fit may not be exactly obtainable but it would be surprising if some improvement without change in the main fit characteristics (large  $|K_1|$ , positive  $\rho_1$ ) could not be made. It is expected, therefore, that the five high-accuracy cross-section data considered here can be fit together with other data at neighboring energies with a local  $\chi^2$  per datum applying to the five measurements (counted as four) between 0.2 and 0.55. On the basis of data discussed here, there is, furthermore, no objection to the newer fits (Y-IV) $_{pp+np}$ , (Y-IV) $_{n-p}$ , (Y-IV) $_{n-p}M$ , the  $\chi^2$  values per datum of 0.45 and 0.55 being quite satisfactory. On the other hand, these fits tend to give a somewhat too low  $\sigma_{tot}$  in the general energy region under discussion. Some improvements will undoubtedly have to be made, as is the case with any phenomenological fit. The various  $\chi^2$  values quoted above do not consider the uncertainties of the values of the phase parameters of the phenomenological phase-parameter fit. By standard methods,<sup>6</sup> the effect of these on the prediction can be estimated; Figs. 1 and 2 show the effect of doing so by the "error-belt" curves. In each of the two cases, the phase-parameter uncertainties and the corresponding statistical correlation coefficients were obtained by the parallel-shift procedure. The two error-belt curves for each parallel-shift determination are plots of  $\sigma(\theta)\pm\Delta\sigma(\theta)$ , where  $\Delta\sigma$  is the standard deviation of the predicted  $\sigma$  as derived from the experimental errors of the data used in deriving the phase parameters. The narrower of the two belts is for parallel shifts 0-69 MeV, the wider for such shifts 14.90-32.98 MeV. For the former, the influence of some very accurate essentially zero-energy and very-low-energy data is perhaps overestimated and the width of the belt may be too small. For the narrower energy interval, the high-accuracy zero-energy data have no direct influence on the value of  $\Delta\sigma$ , although

they do influence the type of fit. In order not to confuse the figures, the wider belt is shown only at very low and very high angles and also from  $\theta=65^\circ$  to  $\theta=110^\circ$ . Consideration of the error belts improves the agreement of the predicted with the measured values.

Of the two semitheoretical guides that have been mentioned in support of the starting points for  $I=0$  phase parameters, the one appealing to the employment of a reasonable potential for the low-energy anchor may be argued to be the weaker because of the lack of rigor in theoretical justifications of static local potentials for the two-nucleon interaction. On the other hand, in the energy region definitely below the meson production threshold, which is the region within which a description of scattering in terms of a potential picture is not complicated by the introduction of an imaginary part, there is general agreement between investigators regarding  $K_1$  being negative. However, the internucleon repulsion indicated by this fact and by the energy dependence of the phase shift decreases the relative importance of small nucleon-nucleon separations for which the objections to static potentials are especially cogent. The employment of a potential model for a low-energy anchor is therefore not as questionable a procedure in this case as might appear at first sight.

The  $I=1$  phase parameters, obtained mainly with the aid of  $p-p$  scattering data, enter the analysis and contribute noticeably to the fore-aft asymmetry. For example,  ${}^3\delta P_0$  and  ${}^3\delta P_1$  contribute about  $-1.5$  and  $+1.5$  mb/sr, respectively, to  $\sigma(\pi)-\sigma(0)$ , largely canceling each other's effects. These contributions and those mentioned below were obtained by removing the phase shifts in question from the complete set of the Y-IV fit and noting the decrease in  $\sigma(\pi)-\sigma(0)$ . In other words, the contribution of a phase shift is meant to be that obtained when the phase shift in question is the last one added to obtain the complete set. The contribution of the whole odd-parity coupled state,  $J=2^-$ , composed of  ${}^3P_2$  and  ${}^3F_2$ , to this asymmetry measure is about  $-2.4$  mb/sr. Since the asymmetry of the (Y-IV) $_{pp+np}$  fit is about 3.5 mb/sr, inaccuracies in the values of the triplet  $L=1$  phases could affect the influence of the  $\sigma(\theta)$  data on the final phase-parameter fit quite markedly. The uncertainties in the values of the  $I=1$  phase parameters are of three types: (a) the uncertainties caused by the data; (b) those caused by the type of  $I=1$  fit used, i.e., those arising from the possible existence of several minima of  $\chi^2$  in the multi-dimensional space of the phase parameters; (c) those arising from the inaccuracy of short-range charge independence. Of these three causes, the easiest to consider is (a). Quoting from Table VI of SFBHHP,<sup>2</sup> the most relevant uncertainties and corresponding phase parameters are as in Table I. The uncertainties appearing in this table do not invalidate the considerations regarding the effect of the triplet  $P$  waves on

TABLE I.  $I=1$  phase parameters in radians and their uncertainties obtained by the parallel-shift procedure in (A) the 0-69-MeV and (B) 14.90-32.98-MeV energy regions.

State	${}^1S_0$	${}^3P_0$	${}^3P_1$	${}^3P_2$	${}^3F_2$	${}^1D_2$
Phase parameter	0.8919	0.1391	-0.0847	0.0429	0.0015	0.0148
Uncertainty A	0.0002	0.0030	0.0010	0.0009	0.0017	0.0003
Uncertainty B	0.0035	0.0094	0.0045	0.0018	...	0.0013

the fore-aft asymmetry. The parallel-shift energy interval 0-69 MeV (uncertainty A) is rather large. Its employment puts much faith into the correctness of the general course of phase shifts considered as functions of the energy. The smaller energy interval (uncertainty B) gives larger uncertainties as is seen from the error belts sketched in Figs. 1 and 2 and from Table I. They are not large enough, however, to destroy the qualitative validity of the considerations regarding the asymmetry. Probable effects of causes (b) and (c) are much harder to evaluate. The (Y-IV) $_{pp+np}$  fit took the Coulomb effects into account for all phases employing a potential model. In the case of  $K_0$ , the corrections for the apparent violation of short-range charge independence have been made on the basis of experimental evidence and are probably reasonably reliable. A further discussion of the rather difficult questions involved in causes (b) and (c) is outside of the immediate scope of this paper.

The uncertainties of  $K_1$  obtained by the parallel-shift procedure in the 0-69-MeV and 14.90-32.98-MeV energy ranges, respectively, are about 10 and 28% of the value of this phase shift.

The portion of the fore-aft asymmetry expected to be caused by  $K_1$  cannot be specified exactly, therefore. In view of this, future changes by 10 or even 30% from the value expected on the basis of the present (Y-IV) fit for this contribution to the asymmetry would not be surprising. The contributions of the  ${}^3P$  states to the asymmetry that have been discussed above are also subject to appreciable uncertainties, as seen in Table I.

The differential scattering cross section is not sensitive to changes of the phase parameter  $\rho_1$ . Even if  $\rho_1$  is made to have the value 0 or to change sign, the changes in  $\sigma(\theta)$  are much smaller than those caused by removing  $P$  waves individually or all together. Thus, the contribution of  $\rho_1$  to  $\sigma(\theta)$ , judged by the effect of its removal from the calculation, is 0.11

mb/sr at  $\theta=5^\circ$ . It decreases monotonically from this value to  $-0.06$  mb/sr at  $\theta=125^\circ$  and then increases slightly as  $\theta$  goes up to  $180^\circ$ . The effect of  $\rho_1$  on the angular distribution is strongly nonlinear between  $|\rho_1|$  and  $-|\rho_1|$ , but the maximum change of  $|\sigma(\theta)|$  at a fixed angle with a change of  $2|\rho_1|$  in  $|\rho_1|$  is only about 0.2 mb/sr. Thus, when the other phase parameters are kept fixed, the selection of  $\rho_1 > 0$  is not markedly influenced by measurements of  $\sigma(\theta)$ . The spin correlation coefficient  $C_{nn}(\theta)$  is sensitive to  $\rho_1$ , however, and a measurement of it at 23 MeV is available. The contribution to  $C_{nn}(\theta)$  due to  $\rho_1$  ranges from about 30% at small angles to more than 100% at the large ones. The measurements of  $\sigma(\theta)$  at these energies are, on the other hand, not directly informative concerning the value of  $\rho_1$ . Their connection with the  $P$  states and especially with the  ${}^1P_1$  phase shift  $K_1$  is rather direct. It is of interest that the low-energy anchor for this state had as its origin a potential and that the new data are not in disagreement with the fit. In a limited sense, it appears fair to claim that they speak for a qualitative agreement of the potential picture with the approximate magnitude of the  ${}^1P_1$  phase shift. Through the general consistency with a positive  $\rho_1$ , the data support the conservative viewpoint of the electromagnetic properties of the nucleons being approximately preserved in the deuteron.

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While the manuscript of this paper was in the final stages of typing, there arrived a copy of a letter from Dr. John C. Hopkins of the Los Alamos Scientific Laboratory to Mr. Lawrence N. Rothenberg (Ref. 7). Employing Yale phase shifts based essentially on those used here combined with unpublished Yale potential fitting, Dr. Hopkins represented the expected  $\sigma(\theta)$  by a quadratic in  $\cos\theta$  which he compared with the experimental values of Ref. 7 obtaining an average deviation of about 1.8%, about the magnitude of the experimental error. It is desired to thank Dr. Hopkins for this communication which corroborates the findings reported above.