Final-State Interactions in Nonleptonic Hyperon Decay

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(Received 1 April 1969)

We discuss the consequences of including the final-state interactions in the analysis of Λ^0, Σ, and Ξ nonleptonic decays. Emphasis is on the role that the final-state interactions play in tests for T invariance, in tests of the $\Delta I=\frac{1}{2}$ rule (including the resolution of sign ambiguities), and in the determination of the decay amplitudes for these processes.

I. INTRODUCTION

THE principal reasons for current interest in nonleptonic hyperon decays are (1) to test time-reversal invariance, (2) to test the limit of validity of the $\Delta I=\frac{1}{2}$ rule, and (3) to provide data to distinguish between various theories of such weak-interaction processes. In addition to the weak interaction mediating these processes, there is a final-state strong interaction between the outgoing baryon and pion in these decays. This final-state interaction is small and has generally been ignored in the past analyses of these decays; now, however, experiments are being performed which are sensitive to it. Moreover, it has been realized[1,2] that additional knowledge of these processes can be obtained by including this final-state strong interaction in the analysis. In this article, we will discuss in detail the effects of including the final-state interactions in the analyses of these decays. In particular, we wish to emphasize the role they play in regard to the tests referred to above.

We will discuss the consequences of including the effects of the final-state interactions in the analysis of Λ^0, Σ, and Ξ nonleptonic decays. These decays are described by the usual decay parameters given by Lee and Yang[3] with the convention used here of

$$\alpha=\frac{2\,\mathrm{Re}S^*P}{|S|^2+|P|^2},\quad \beta=\frac{2\,\mathrm{Im}S^*P}{|S|^2+|P|^2},\quad \gamma=\frac{|S|^2-|P|^2}{|S|^2+|P|^2}.$$

Superscripts on the decay parameters refer to the sign of the decay pion for the particular hyperon being discussed. We use the S and P isospin amplitudes with subscript convention $S_{2\Delta I,2I}$. The effect of final-state interactions is given by the phase shifts δ_{2I} and $\delta_{2I,1}$, the appropriate pion-baryon isospin scattering phase shifts for S and P waves, respectively.

It should be pointed out that radiative corrections, which we ignore in this article, will also give contributions which should be included in the analysis comparing theoretical and experimental results. Little attention has been given to radiative corrections in these decays, except by Jarlskog,[4] who has estimated corrections to the Σ-decay parameters, and by Belavin and Narodetsky,[5] who have discussed radiative corrections to Λ^0-decay branching ratios. For our purpose, these radiative corrections are not relevant, since we are interested in deducing the effective amplitudes directly from experiment. They are important, however, when one wants to know the radiative corrections expected to a pure $\Delta I=\frac{1}{2}$ rule.

II. Λ^0 DECAY

The amplitudes in Λ^0 decay are, for $\Lambda^0\to p+\pi^-$,

$$S^-=-(\sqrt{\tfrac{2}{3}})S_{11}e^{i\delta_1}+(\sqrt{\tfrac{1}{3}})S_{33}e^{i\delta_3},$$
$$P^-=-(\sqrt{\tfrac{2}{3}})P_{11}e^{i\delta_{11}}+(\sqrt{\tfrac{1}{3}})P_{33}e^{i\delta_{31}}$$

and, for $\Lambda^0\to n+\pi^0$,

$$S^0=(\sqrt{\tfrac{1}{3}})S_{11}e^{i\delta_1}+(\sqrt{\tfrac{2}{3}})S_{33}e^{i\delta_3},$$
$$P^0=(\sqrt{\tfrac{1}{3}})P_{11}e^{i\delta_{11}}+(\sqrt{\tfrac{2}{3}})P_{33}e^{i\delta_{31}},$$

where δ_1, δ_3 are the pion-nucleon S-wave scattering phase shifts, and δ_{11}, δ_{31} the P-wave ones, for $I=\frac{1}{2}$ and $\frac{3}{2}$ at center-of-mass momentum of 100-MeV/c or 44-MeV incident pion kinetic energy.

From the phase-shift analysis of Roper *et al.*,[6] $\delta_1=6.0°$, $\delta_3=-3.8°$, $\delta_{11}=-1.1°$, and $\delta_{31}=-0.7°$. While it is difficult to assign errors to these values, typical uncertainties are $\sim\pm0.5°$. If time-reversal invariance is valid in this decay, S_{11}, S_{33}, P_{11}, and P_{33} are all real.

A. $\Delta I=\frac{1}{2}$ Rule

If the $\Delta I=\frac{1}{2}$ rule is valid in this decay, $S_{33}=P_{33}=0$, and

$$S^0/S^-=P^0/P^-=-1/\sqrt{2}.$$

* Work supported by U. S. Office of Naval Research, under Contract No. Nonr-1224(23).
† Work supported by U. S. Atomic Energy Commission.

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[2] O. E. Overseth, Phys. Rev. Letters 19, 395 (1967).
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This leads to the predictions that (1) the ratio of decay rates for the neutral mode Γ^0 to the charged mode Γ^- is

$$\Gamma^0(\Lambda^0\to n\pi^0)/\Gamma^-(\Lambda^0\to p\pi^-)=\tfrac{1}{2}$$

and (2) the decay parameters for the neutral and charged modes are equal, i.e., $\alpha^0=\alpha^-$, $\beta^0=\beta^-$, and $\gamma^0=\gamma^-$. These statements are also true when final-state interactions are included. Experimentally it is known that $\Gamma^0/\Gamma^-=0.53\pm0.02$,[7] $\alpha^0/\alpha^-=1.10\pm0.27$,[8] and $\gamma^0/\gamma^-=1.04_{-0.21}^{+0.33}$.[9,10] To find how much $\Delta I=\frac{3}{2}$ amplitude is allowed consistent with experiment, the ratio of decay rates is given by

$$\frac{\Gamma^0}{\Gamma^-}\cong\frac{1}{2}(1+3\sqrt{2})+\left(\frac{[S_{11}S_{33}\cos(\delta_1-\delta_3)+P_{11}P_{33}\cos(\delta_{11}-\delta_{31})]}{S_{11}^2+P_{11}^2}\right),$$

and the ratio of the α parameters by

$$\frac{\alpha^0}{\alpha^-}\cong\frac{1}{2}\frac{\Gamma^-}{\Gamma^0}\left(1+\frac{3}{\sqrt{2}}\frac{S_{33}\cos(\delta_{11}-\delta_3)}{S_{11}\cos(\delta_{11}-\delta_1)}+\frac{3}{\sqrt{2}}\frac{P_{33}\cos(\delta_{31}-\delta_1)}{P_{11}\cos(\delta_{11}-\delta_1)}\right),$$

where we neglect terms quadratic in the $\Delta I=\frac{3}{2}$ amplitudes. The experimental value for the ratio of decay rates implies the $\Delta I=\frac{3}{2}$ amplitudes are present in Λ^0 decay to $<2\%$, assuming time-reversal invariance is valid in these decays. As pointed out by Pondrom,[11] however, time-reversal violation could allow considerably larger $\Delta I=\frac{3}{2}$ amplitudes to be present, consistent with current experimental data on Λ^0 decay. Good determination of the decay parameters for $\Lambda^0\to n+\pi^0$ is needed to test the validity of the $\Delta I=\frac{1}{2}$ rule for Λ^0 decay.

B. Pseudo-$\Delta I=\frac{1}{2}$ Rule

There is a certain admixture of $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ amplitudes in Λ^0 decay which results in the same predictions as does the $\Delta I=\frac{1}{2}$ rule if final-state interactions are neglected,[12] namely, when

$$S^0/S^-=P^0/P^-=+1/\sqrt{2}.$$

However, as discussed in detail in Ref. 2, including the final-state interactions allows this pseudo-$\Delta I=\frac{1}{2}$ case to be experimentally distinguishable from the exact $\Delta I=\frac{1}{2}$ rule, and present experimental data argue against the occurrence of this particular admixture.

There is also the possibility that the $\Delta I=\frac{1}{2}$ rule applies for the S-wave but not the P-wave amplitudes, or vice versa. These possibilities can be ruled out, since it can be shown that both cases have the consequence that $\alpha^0/\alpha^-\approx-1$, contrary to experiment.

C. If $\Delta I\neq\frac{1}{2}$

Measurement of β^0, α^0, β^-, and α^- for Λ^0 decay allows complete determination of all decay amplitudes including relative sign if time-reversal invariance is valid.

D. Test of Time-Reversal Invariance

If time-reversal invariance is valid,

$$\frac{\beta^-}{\alpha^-}=\tan(\delta_{11}-\delta_1)\left(1+\sqrt{2}\frac{P_{33}\sin(\delta_{11}-\delta_{31})}{P_{11}\sin2(\delta_{11}-\delta_1)}+\sqrt{2}\frac{S_{33}}{S_{11}}\frac{\sin(\delta_3-\delta_1)}{\sin2(\delta_{11}-\delta_1)}\right)$$

to first order in ratio of $\Delta I=\frac{3}{2}$ to $\Delta I=\frac{1}{2}$ amplitudes. The test of time reversal is not very sensitive to the presence of $\Delta I=\frac{3}{2}$ amplitudes. For example, if they are present to 10%, the correction to $\delta_{11}-\delta_1$ is $\sim0.7°$. This is less than the error to which $\delta_{11}-\delta_1$ is presently known, i.e., experimentally, $\delta_{11}-\delta_1=-(6.5\pm1.5)°$. The experimental value[13] for the time-reversal test is $\beta^-/\alpha^-=-0.16\pm0.07=\tan(-9.0°\pm4.0°)$.

E. Tests for CP and CPT Invariance

Final-state interactions provide a distinction between tests for CP and CPT in hyperon decay. This has been discussed by Lee,[14] and only the results will be summarized here. If CP is valid, $S=-\bar{S}$ and $P=\bar{P}$, where $\bar{S}$ and $\bar{P}$ are the corresponding S and P amplitudes for anti-Λ^0 decay. This means $\alpha=-\bar{\alpha}$ and $\beta=-\bar{\beta}$, if CP is valid. Again the bars denote the parameters for anti-Λ^0 decay.

If CPT is valid, then

$$\frac{\bar{\alpha}}{\alpha}=-\frac{\cos(\delta_S-\delta_P-(\Delta_S-\Delta_P))}{\cos(\delta_S-\delta_P+(\Delta_S-\Delta_P))},$$

where δ_S, δ_P are the final-state interaction pion-nucleon phase shifts, and Δ_S, Δ_P are the T-violating phases of the S and P amplitudes. Under T invariance, $\Delta_S=\Delta_P=0$. Thus, if CPT is valid, (a) $\bar{\alpha}=-\alpha$ means both T and CP invariance are valid, and (b) $\bar{\alpha}\neq-\alpha$ means T and CP are both violated in this process. If there were

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[14] T. D. Lee, in *Preludes in Theoretical Physics*, edited by A. deShalit, H. Feshbach, and L. Van Hove (North-Holland Publishing Co., Amsterdam, 1966), p. 5.

no final-state interactions, *CPT* invariance would demand that $\alpha=-\bar{\alpha}$ regardless of the status of *CP*.

Thus in hyperon decay, $\bar{\alpha}\neq-\alpha$ implies *CP* violation in this process independent of the validity of the *CPT* theorem. This is also true if $\bar{\beta}\neq-\beta$.

Also, as usual, *CPT* invariance implies equality of Λ^0 and $\bar{\Lambda}^0$ lifetimes, whereas *CP* invariance implies equality of partial rates $\Gamma^0=\bar{\Gamma}^0$, and $\Gamma^-=\bar{\Gamma}^+$. This is also true when final-state interactions are included in the analysis.

III. Σ DECAY

The amplitudes for Σ decay are, for $\Sigma^+\to p\pi^0$,

$$S^0=\tfrac{1}{3}\sqrt{2}S_{11}e^{i\delta_1}+\tfrac{1}{3}\sqrt{2}S_{13}e^{i\delta_3}-(1/3\sqrt{2})S_{31}e^{i\delta_1}-(4/3\sqrt{5})S_{33}e^{i\delta_3}+\sqrt{(2/15)}S_{53}e^{i\delta_3},$$

$$P^0=\tfrac{1}{3}\sqrt{2}P_{11}e^{i\delta_{11}}+\tfrac{1}{3}\sqrt{2}P_{13}e^{i\delta_{31}}-(1/3\sqrt{2})P_{31}e^{i\delta_{11}}-(4/3\sqrt{5})P_{33}e^{i\delta_{31}}+\sqrt{(2/15)}P_{53}e^{i\delta_{31}};$$

for $\Sigma^-\to n\pi^-$,

$$S^-=S_{13}e^{i\delta_3}+(\sqrt{\tfrac{2}{5}})S_{33}e^{i\delta_3}+(1/\sqrt{15})S_{53}e^{i\delta_3},$$

$$P^-=P_{13}e^{i\delta_{31}}+(\sqrt{\tfrac{2}{5}})P_{33}e^{i\delta_{31}}+(1/\sqrt{15})P_{53}e^{i\delta_{31}};$$

and for $\Sigma^+\to n\pi^+$,

$$S^+=-\tfrac{2}{3}S_{11}e^{i\delta_1}+\tfrac{1}{3}S_{13}e^{i\delta_3}+\tfrac{1}{3}S_{31}e^{i\delta_1}-\tfrac{2}{3}(\sqrt{\tfrac{2}{5}})S_{33}e^{i\delta_3}+(1/\sqrt{15})S_{53}e^{i\delta_3},$$

$$P^+=-\tfrac{2}{3}P_{11}e^{i\delta_{11}}+\tfrac{1}{3}P_{13}e^{i\delta_{31}}+\tfrac{1}{3}P_{31}e^{i\delta_{11}}-\tfrac{2}{3}(\sqrt{\tfrac{2}{5}})P_{33}e^{i\delta_{31}}+(1/\sqrt{15})P_{53}e^{i\delta_{31}}.$$

The phase shifts here are for pion-nucleon scattering at center-of-mass momentum of about 190-MeV/*c* or 140-MeV incident pion kinetic energy. From Roper *et al.*,[6] $\delta_1=9.4°$, $\delta_3=-10.1°$, $\delta_{11}=-1.8°$, and $\delta_{31}=-3.5°$. These values are not as well known as for the Λ^0 case, and each has an uncertainty of $\sim\pm1.5°$. With more accurate forthcoming experimental data on Σ decay, it will soon be important to have better determinations of these phase shifts.

A. $\Delta I=\frac{1}{2}$ Rule

The traditional test of the $\Delta I=\frac{1}{2}$ rule for Σ decay is that $\sqrt{2}S^0+S^+-S^-=0$ and $\sqrt{2}P^0+P^+-P^-=0$. This is a necessary but not sufficient condition. These relations are still true when final-state interactions are included, but now the *S* and *P* amplitudes are complex. Hence, it is more desirable to test this rule by determining the isospin amplitudes from measurements of the decay parameters and decay rates. The problem is considerably overdetermined with the twelve (nine independent) measurables α, β, γ, and Γ for the three decay modes to be used to determine the four amplitudes S_{11}, S_{13}, P_{11}, and P_{13}. Such tests have been carried out by Franzini and Zanello,[15] Deans *et al.*,[16] and Jarlskog.[4] However, at the present time we do not have a good test of the $\Delta I=\frac{1}{2}$ rule in Σ decay because of the uncertainty in the value of α^0 from $\Sigma^+\to p\pi^0$. This parameter has been measured twice. Beall *et al.*[17] found $\alpha^0=-0.80\pm0.16$ from analyzing the polarization of the decay proton, and Bangerter *et al.*[18] found $\alpha^0=-0.986\pm0.07$ by applying the analysis of Tripp, Ferro-Luzzi, and Watson[19] to the observed polarization of Σ^+ hyperons from the reaction $K^-+p\to\Sigma^++\pi^-$. While these two determinations are not necessarily inconsistent, it is altogether likely that the true value lies closer to one or the other than to the weighted average. Good agreement with the $\Delta I=\frac{1}{2}$ rule requires $\alpha^0\approx-1.0$.

Although the experimental situation regarding α^0 at present is unresolved, it is interesting to estimate how large $\Delta I=\frac{3}{2}$ and $\frac{5}{2}$ amplitudes could be present in Σ decay consistent with the existing data. If the $\Delta I=\frac{1}{2}$ rule is not true for these decays, then

$$\sqrt{2}S^0+S^+-S^-=-3(\sqrt{\tfrac{2}{5}})S_{33}e^{i\delta_3}+(2/\sqrt{15})S_{53}e^{i\delta_3},$$

with a similar equation for the *P*-wave amplitudes. The values of the amplitudes determined by Berge[20] from present data (taking $\alpha^0=-0.960\pm0.067$) give $S_{33}/S_{13}\lesssim12\%$, where we have neglected the $\Delta I=\frac{5}{2}$ amplitude.

An interesting feature of including final-state interactions in the analysis is that $\alpha=0$ no longer need imply $\beta=0$ and $\gamma=\pm1$. Experimentally[7] it is known that α^+ and α^- are both consistent with zero[21] and that $\Sigma^+\to n\pi^+$ is primarily *P* wave ($\gamma^+\approx-1$) and $\Sigma^-\to n\pi^-$ is primarily *S* wave ($\gamma^-\approx+1$). Assuming the $\Delta I=\frac{1}{2}$ rule, then, if $\alpha^-=0$, we have the result that $P_{13}=0$ and $\beta^-=0$, $\gamma^-=+1$. On the other hand, $\alpha^+=0$ does not imply that $S^+=0$, and hence that $\beta^+=0$ and $\gamma^+=-1$. For example, if we take $\alpha^+=\alpha^-=0$, we find that $\beta^+=+0.23$ and $\varphi^+=\arctan(\beta^+/\gamma^+)=167°$. Experimentally,[7] $\varphi^+=160°\pm22°$.

B. Pseudo-$\Delta I\neq\frac{1}{2}$ Rules

If $\Delta I\neq\frac{1}{2}$,

$$\sqrt{2}S^0+S^+-S^-=-3(\sqrt{\tfrac{2}{5}})S_{33}e^{i\delta_3}+(2/\sqrt{15})S_{53}e^{i\delta_3},$$

regardless of the value of the $\Delta I=\frac{3}{2}$ amplitude S_{31}, which cancels out of the equation. There is a similar

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[20] J. P. Berge, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967), p. 46.

[21] However, it should be noted that recent determinations have been made with sufficient accuracy to give values which are significantly different from zero [reported by W. J. Willis, in *Proceedings of the Heidelberg International Conference on Elementary Particles, 1968* (North-Holland Publishing Co., Amsterdam, 1968), p. 281].

equation for the P-wave amplitudes. Clearly, there is a variety of special conditions under which $\sqrt{2}S^0+S^+-S^-$ can equal zero without imposing the $\Delta I=\frac{1}{2}$ rule, especially if we can change the relative sign of the amplitudes.[22] Determining the isospin amplitudes directly, as discussed above, provides a more stringent test of the $\Delta I=\frac{1}{2}$ rule and can distinguish among most of these pseudo cases.

Some of the pseudo-$\Delta I=\frac{1}{2}$ rule cases can be ruled out when the final-state interactions are included in the analysis. For example, consider the case $\sqrt{2}S^0-S^+=S^-$ and $\sqrt{2}P^0-P^+=P^-$, i.e., a pseudo-$\Delta I=\frac{1}{2}$ rule where the relative signs of the S^+ and P^+ amplitudes have been changed from that given above. If we perform the analysis including $\Delta I=\frac{3}{2}$ and $\frac{5}{2}$ amplitudes and force $\sqrt{2}S^0-S^+-S^-=0$, we must require that

$$e^{i\delta_1}(\tfrac{4}{3}S_{11}-\tfrac{2}{3}S_{31})+e^{i\delta_3}[-\tfrac{2}{5}S_{13}-(5/3)(\sqrt{\tfrac{2}{5}})S_{33}]=0.$$

Since $\delta_1\neq\delta_3$, this condition requires $S_{31}=2S_{11}$ and $S_{33}=-(\sqrt{\frac{2}{5}})S_{13}$. Making these substitutions into the original amplitudes yields

$$S^+=S^-=e^{i\delta_3}[\tfrac{3}{5}S_{13}+(1/\sqrt{15})S_{53}].$$

There will be similar equations for the P waves, also with the result that $P^+=P^-$. This relationship between the S and P amplitudes then predicts $\gamma^+=\gamma^-$. Since experimentally it is found that $\gamma^+\approx-\gamma^-$, we can rule out this particular pseudo-$\Delta I=\frac{1}{2}$ rule case.

Another way of ruling out the pseudo-$\Delta I=\frac{1}{2}$ rules in favor of $\Delta I=\frac{1}{2}$ rule is to test those predictions of $\Delta I=\frac{1}{2}$ rule which are not duplicated by the pseudo-$\Delta I=\frac{1}{2}$ rule. For example, for Σ decays the $\Delta I=\frac{1}{2}$ rule, in addition to the triangular sum rule for decays of Σ^+ and Σ^-, predicts two sum rules[23] involving the decays of Σ^0:

$$\langle\Sigma^0|p\pi^-\rangle=\langle\Sigma^+|p\pi^0\rangle,$$

$$2\langle\Sigma^0|n\pi^0\rangle-\langle\Sigma^+|n\pi^+\rangle-\langle\Sigma^-|n\pi^-\rangle=0.$$

In principle, a check of these sum rules would rule out pseudo-$\Delta I=\frac{1}{2}$ rules. Of course, it seems that observation and measurement of weak decays of Σ^0 is a long way off in the future.

C. If $\Delta I\neq\frac{1}{2}$

The analysis is complicated in Σ decay, since both $I=\frac{1}{2}$ and $\frac{3}{2}$ amplitudes can occur as well as $\Delta I=\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ transitions. There is a total of ten possible isospin amplitudes and only nine independent experimental parameters. If the $\Delta I=\frac{5}{2}$ amplitudes could be neglected, the remaining eight isospin amplitudes are determinable, but only if time-reversal invariance is valid in the decay.

D. Tests of Time-Reversal Invariance

Only the decay $\Sigma^+\to p+\pi^0$ can provide a sensitive test for time-reversal invariance. For this decay mode the S- and P-wave amplitudes are comparable in magnitude, in contrast to $\Sigma^+\to n+\pi^+$, which is almost totally P-wave, and $\Sigma^-\to n+\pi^-$ is predominantly S-wave, and the test of time-reversal invariance comes from observing the interference between the S- and P-wave amplitudes in a given decay mode. As in Λ^0 decay, the time-reversal test depends on knowledge of the validity of the $\Delta I=\frac{1}{2}$ rule in order to know which final-state-interaction phase shifts are to be included.

If the $\Delta I=\frac{1}{2}$ rule is valid for Σ decay, the test for time-reversal invariance in $\Sigma^+\to p+\pi^0$ is clearly defined. Good knowledge of the decay parameters allows one to determine β^0 in terms of the phase shifts. For example, if $\alpha^+=\alpha^-=0$, then

$$\beta^0/\alpha^0=\tfrac{1}{3}\tan(\delta_{11}-\delta_1)+\tfrac{2}{3}\tan(\delta_{11}-\delta_3)\approx0.03\pm0.05$$

if time-reversal invariance is valid, where the estimated uncertainty reflects the uncertainty in the knowledge of the phase shifts.

On the other hand, if the $\Delta I=\frac{1}{2}$ rule is not valid in Σ decay, we do not, in general, have an unambiguous test of time-reversal invariance. This is because in Σ decay there are more isospin amplitudes than experimental parameters, as discussed above, and even complete knowledge of the decay parameters and decay rates would not allow us to evaluate all the isospin amplitudes that occur in the expression for the time-reversal parameter β^0.

IV. Ξ DECAY

The amplitudes in Ξ decay are, for $\Xi^-\to\Lambda+\pi^-$,

$$S^-=S_{12}e^{i\delta_2}+\tfrac{1}{2}S_{32}e^{i\delta_2},$$

$$P^-=P_{12}e^{i\delta_{21}}+\tfrac{1}{2}P_{32}e^{i\delta_{21}},$$

and, for $\Xi^0\to\Lambda+\pi^0$,

$$S^0=(S_{12}e^{i\delta_2}-S_{32}e^{i\delta_2})/\sqrt{2},$$

$$P^0=(P_{12}e^{i\delta_{21}}-P_{32}e^{i\delta_{21}})/\sqrt{2}.$$

Here δ_2 and δ_{21} are Λ-π scattering phase shifts and are difficult to obtain experimentally. Attempts have been made to calculate them semitheoretically. Nath and Kumar[24] calculate $\delta_2=-18.7°$ and $\delta_{21}=-2.7°$. Martin[25] treats only P wave and finds $\delta_{21}=-1.2°$.

A. $\Delta I=\frac{1}{2}$ Rule

If the $\Delta I=\frac{1}{2}$ rule is valid, $S^-=\sqrt{2}S^0$ and $P^-=\sqrt{2}P^0$, with the consequences that (1) $\Gamma(\Xi^0)/\Gamma(\Xi^-)=\frac{1}{2}$, and (2) $\alpha^0=\alpha^-$, $\beta^0=\beta^-$, and $\gamma^0=\gamma^-$.

Including the $\Delta I=\frac{3}{2}$ amplitudes to first order, we obtain

$$\frac{\Gamma^0}{\Gamma^-}\cong\frac{1}{2}\left(1-\frac{3S_{12}S_{32}+3P_{12}P_{32}}{S_{12}^2+P_{12}^2}\right)$$

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[25] B. R. Martin, Phys. Rev. **138**, B1136 (1965).

and

$$\frac{\alpha^0}{\alpha^-}\cong\frac{1}{2}\frac{\Gamma^-}{\Gamma^0}\left[1-\frac{3}{2}\left(\frac{S_{32}}{S_{12}}+\frac{P_{32}}{P_{12}}\right)\right].$$

Experimentally,[26] $\Gamma^0/\Gamma^-=0.52\pm0.04$ and $\alpha^0/\alpha^-=0.94\pm0.30$, which implies $\Delta I=\frac{3}{2}$ amplitudes $\gtrsim 6\%$ (90% confidence level) in Ξ decay.

B. Pseudo-$\Delta I=\frac{1}{2}$ Rule

As in Λ^0 decay, the predictions of the $\Delta I=\frac{1}{2}$ rule result for a particular admixture of $\Delta I=\frac{3}{2}$ amplitudes, e.g., if $S_{32}=4S_{12}$ and $P_{32}=4P_{12}$. For this case, $\sqrt{2}S^0=-S^-$ and $\sqrt{2}P^0=-P^-$. Here, however, in contrast to Λ^0 decay, final-state interactions do not distinguish between the $\Delta I=\frac{1}{2}$ rule and the pseudo case. This is because the isotopic spin of the final state in Ξ decay is fixed at $I=1$, in contrast to the Λ^0 case, where two values, $I=\frac{1}{2}$ and $\frac{3}{2}$, are allowed.

C. If $\Delta I\neq\frac{1}{2}$

If $\alpha^0\neq\alpha^-$ and $\beta^0\neq\beta^-$, determination of α^-, β^-, α^0, and β^0 determines all amplitudes, but not their relative sign.

D. Tests of Time-Reversal Invariance

The test of time-reversal invariance in Ξ decay is that $\beta^-/\alpha^-=\beta^0/\alpha^0=\tan(\delta_{21}-\delta_2)$, independent of whether the $\Delta I=\frac{1}{2}$ rule is satisfied or not. Here, δ_{21} and δ_2 are Λ-π scattering phase shifts and are difficult to obtain experimentally, so this decay does not appear useful for this test. However, there is the interesting result that if $\beta^0/\alpha^0\neq\beta^-/\alpha^-$ for Ξ decay, *both time reversal and the $\Delta I=\frac{1}{2}$ rule are violated.* This test is possible. The converse, that if time reversal and the $\Delta I=\frac{1}{2}$ rule are both violated, then $\beta^0/\alpha^0\neq\beta^-/\alpha^-$ is only true if T violation gives unequal contribution to $\Delta I=\frac{1}{2}$ and $\frac{3}{2}$ amplitudes. Present data[26] are consistent with time-reversal invariance in Ξ decay and give $\delta_{21}-\delta_2=2°\pm16°$.

V. CONCLUSIONS

We have discussed how experiments sensitive to the final-state interactions can enhance our understanding of the nonleptonic hyperon decays. In effect, this means a good determination of the β parameters in these decays. An excellent place to search for time-reversal invariance is in the decays $\Lambda^0\to n\pi^0$ and $\Lambda^0\to p\pi^-$. The necessary interfering amplitudes in these decays are roughly comparable, and the test is relatively insensitive to the presence of small $\Delta I=\frac{3}{2}$ amplitudes. In Σ decay, only the mode $\Sigma^+\to p+\pi^0$ offers promise for a sensitive test, and only if the $\Delta I=\frac{1}{2}$ rule is shown to be valid in the decay. Although the β parameter for the decay $\Sigma^+\to n+\pi^+$ may be moderately large (i.e., ~0.25), this mode presumably would not provide a sensitive test, since it is characterized by only one amplitude, the P-wave. In Ξ decay we show how to provide a test for time-reversal invariance independent of knowledge of the Λ^0-π scattering phase shifts. A particular feature of this test is its sensitivity to T violation occurring in $\Delta I=\frac{3}{2}$ amplitudes.

In Λ^0 and Ξ decay, branching-ratio data indicate that $\Delta I=\frac{3}{2}$ amplitudes, if present at all, are $<5\%$ of the $\Delta I=\frac{1}{2}$ amplitudes. Independent verification of the $\Delta I=\frac{1}{2}$ rule in the decays from comparison of α^0/α^- awaits better experimental results on the neutral decay modes for both of these hyperons. A good test of the $\Delta I=\frac{1}{2}$ rule in Σ decay depends on redetermination of decay parameters for the decay $\Sigma^0\to p+\pi^0$. It should be noted that the $\Delta I=\frac{1}{2}$ rule could be valid in Λ^0 and Ξ decays but not in Σ decays, since Σ decays can have $\Delta I=\frac{5}{2}$ amplitudes, which Λ^0 and Ξ decays cannot.

If the $\Delta I=\frac{1}{2}$ rule is not valid in these decays, measurement of all the decay parameters leads to complete determination of all the isospin amplitudes in Λ^0 decay, and to within a relative sign for Ξ decay. For Σ decay, however, there will be an ambiguity, since there the number of amplitudes exceeds the number of available experimental parameters. If the $\Delta I=\frac{1}{2}$ rule is valid, in all cases the problem is overdetermined, and the isospin amplitudes may be determined directly from experiment. Indeed, the success in doing this for Σ decay constitutes the best test of the $\Delta I=\frac{1}{2}$ rule for this hyperon.

Finally, any substantial improvement in our knowledge of the decay parameters will require an improvement in the knowledge of the appropriate scattering phase shifts so that the effect of the final-state interactions may be properly included in the analysis. This is particularly true for Σ decay, where it would be desirable to know with greater accuracy the pion-nucleon scattering phase shifts for an incident pion kinetic energy of 140 MeV.

ACKNOWLEDGMENT

One of the authors (S.P.) would like to thank Professor S. P. Rosen for useful discussions and a useful communication.

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