$$I_{(0,3)}=\left[-\frac{(b+M^2)^3\omega^3}{32b^3}+\frac{M^2\omega^3(b+M^2)}{16b^2}+\frac{m^2\omega(b+M^2)^3}{16b^2}-\frac{m^2M^2\omega(b+M^2)}{4b}\right]I_{(0,0)},$$

$$I_{(-3,0)}=\left[\frac{(b-M^2)^3\omega^3}{32b^3}-\frac{m^2\omega(b-M^2)^3}{16b^2}\right]I_{(0,0)}, \qquad \text{(B4)}$$

$$I_{(1,1)}=-\tfrac{1}{2}\omega I_{(1,0)}+I_{(0,0)},$$
$$I_{(1,2)}=-\omega I_{(0,0)}+\tfrac{1}{4}\omega^2 I_{(1,0)}+I_{(-1,0)},$$
$$I_{(1,3)}=-\tfrac{1}{8}\omega^3 I_{(1,0)}+\tfrac{3}{4}\omega^2 I_{(0,0)}-\tfrac{3}{2}\omega I_{(-1,0)}+I_{(-2,0)},$$
$$I_{(2,1)}=-\tfrac{1}{2}\omega I_{(2,0)}+I_{(1,0)},$$
$$I_{(2,2)}=-\omega I_{(1,0)}+\tfrac{1}{4}\omega^2 I_{(2,0)}+I_{(0,0)},$$
$$I_{(-1,1)}=-\tfrac{1}{2}\omega I_{(-1,0)}+I_{(-2,0)},$$
$$I_{(-2,1)}=-\tfrac{1}{2}\omega I_{(-2,0)}+I_{(-3,0)},$$
$$I_{(-1,2)}=\tfrac{1}{4}\omega^2 I_{(-1,0)}-\omega I_{(-2,0)}+I_{(-3,0)}. \qquad \text{(B5)}$$

The recurrence relations may be checked by noting that $\delta((B-p)^2+a)$ fixes $P\cdot p$, i.e., using $p\cdot B=-\frac{1}{2}\omega$,

$$P\cdot p=-\tfrac{1}{2}\omega+p\cdot k. \qquad \text{(B6)}$$

We can then express $I_{(1,1)}$ as

$$I_{(1,1)}=\left(\frac{2}{\pi}\right)^2\int d^4k\,\delta(k^2)\delta((P-k)^2+b)\times\int d^4p\,\delta(p^2+m^2)\delta((B-p)^2+a)\left(\frac{-\frac{1}{2}\omega+p\cdot k}{p\cdot k}\right)$$
$$=-\tfrac{1}{2}\omega I_{(1,0)}+I_{(0,0)}. \qquad \text{(B7)}$$

The other recurrence relations may be derived analogously and in general permit the integrals $I_{(m,n)}$ to be expressed in terms of $I_{(m,0)}$ and $I_{(0,n)}$.

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Electric Dipole Moment of the Neutron: Expected Order of Magnitude

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Sideways dispersion relations yield a theoretically well-defensible "lower limit" on the expected order of magnitude of the neutron electric dipole moment. If T violation is due to the weak interactions, then 10^{-24} e cm appears as a reasonable expectation, and 10^{-23} e cm the most one can hope for. If T violation is due to the electromagnetic interactions of hadrons, we still cannot safely expect more than 10^{-23} e cm, although optimistic guesses can easily yield 6×10^{-22} e cm or more.

The electric dipole moment $e\beta$ of the neutron would vanish under reflection (P) invariance, or under time-reversal (T) invariance[1]; since neither is exact,[2] one should estimate the theoretically expected order of magnitude of β. The experimental upper limit, $\beta<5\times10^{-23}$ cm,[3] already falls below some predictions.[4] Some other models of T violation predict that β should effectively vanish; we call these null-β theories.[5] We discuss first the case where T violation is due to the weak interactions, and second, the case where it is due to the electromagnetic (EM) interactions of the hadrons[6] (which remain, however, P-invariant).

In comparison with our predecessors, we claim only (i) that our input assumptions bridge only those gaps in the experimental situation that cannot at present be sidestepped, (ii) that we are conservative rather than optimistic about the dynamics, and (iii) that we have isolated a less model-dependent and better calibrated expression for certain almost unavoidable contributions, which should set a theoretically well defensible order-of-magnitude "lower limit" on β, unless there are accidental cancellations, or conspiracies which effectively reduce the theory to the null-β type.

We begin with weak T violation. To motivate a fairly careful treatment, recall that a quasidimensional estimate would read thus: $\beta=$ (strength of T violation) $\times$ (strength of P violation) $\times$ (typical hadronic length). The first factor is generally agreed at around 10^{-3}; but

[1] L. D. Landau, Nucl. Phys. 3, 127 (1957).

[2] For references to T violation, see R. C. Casella, Phys. Rev. Letters 22, 554 (1969).

[3] J. K. Baird *et al.*, Phys. Rev. 179, 1285 (1969).

[4] Some recent predictions are P. Babu and M. Suzuki, Phys. Rev. 162, 1359 (1967): $\beta>2.2\times10^{-22}$ cm; K. Nishijima, Progr. Theoret. Phys. (Kyoto) 41, 739 (1969): 2×10^{-22} cm; P. McNamee and J. C. Pati, Phys. Rev. 178, 2273 (1968): (0.9 to 1.5)$\times10^{-22}$ or (5 to 8)$\times10^{-24}$ cm, in two alternative models.

[5] L. Wolfenstein, Phys. Rev. Letters 13, 562 (1964); R. J. Oakes, *ibid.* 20, 1539 (1968). For a difficulty in Oakes' theory: B. H. J. McKellar, *ibid.* 21, 1822 (1968).

[6] J. Bernstein, G. Feinberg, and T. D. Lee, Phys. Rev. 139, B1650 (1960).

the second has varied between 10^{-7} and 10^{-5}, and the third between the nucleon and pion Compton wavelengths, $2–14\times10^{-14}$ cm. The first main assumption is the self-coupled current model[7] of nonleptonic P violation. Second, we assume that the order of magnitude of joint P and T violation at low energies is obtainable quasiphenomenologically by inserting T-violating phases η into P-violating amplitudes, and taking, everywhere, $\eta\approx|\eta_{+-}|\simeq2\times10^{-3}$, where η_{+-} is the measured amplitude ratio[8] $(K_L{}^0\rightarrow\pi^+\pi^-)/(K_S{}^0\rightarrow\pi^+\pi^-)$. Thus, T-violating admixtures to $\Delta S=0$ and $|\Delta S|=1$ amplitudes are assumed to be comparable.

These assumptions cover a wide range of detailed models, e.g., those with various universal T-violating phases[9] and those of the Nishijima and Okubo type.[10]

We estimate β through the sideways dispersion relation[11] for the neutron form factor Γ_λ, defined by

$$\Gamma_\lambda(p',p)=i\left(\frac{p_0'}{m}\right)^{1/2}\times\int dx\, e^{ipx}\langle p'|[j_\lambda(0),\bar{\chi}(x)]\theta(-x_0)|0\rangle. \quad (1)$$

$p^2\equiv W^2$ is the dispersion variable, m the nucleon mass, j_λ the electromagnetic current density, χ the neutron source; and the photon momentum $k=p'-p$ is kept on shell: $k^2=0$. Then, for a neutral particle, the Dirac form factor vanishes, and for calculating β a sufficiently general form of Γ_λ is

$$\Gamma_\lambda=(2m)^{-1}\bar{u}(p')\{i\sigma_{\lambda\mu}k^\mu[(m+\not{p})E_2{}^+ +(m-\not{p})E_2{}^-]+i\sigma_{\lambda\mu}k^\mu\gamma_5[(m+\not{p})E_4{}^+ +(m-\not{p})E_4{}^-]\}. \quad (2)$$

The form factors $E_i{}^\pm$ depend on W^2; $E_2{}^+(m^2)=e\kappa/2m$, where κ is the Pauli moment, and $E_4{}^+(m^2)=-e\beta$. We assume an unsubtracted dispersion relation for $E_4{}^+$, whence

$$-e\beta=\pi^{-1}\int dW^2\frac{\mathrm{Im}E_4{}^+(W^2)}{W^2-m^2}. \quad (3)$$

The absorptive part A_λ of Γ_λ, and $\mathrm{Im}E_4{}^+$, turn out to be

$$A_\lambda=(2\pi)^3\sum_n(p_0'/m)^{1/2}\langle p'|j_\lambda|n\rangle\langle n|\bar{\chi}|0\rangle\delta(n-p), \quad (4)$$

$$\mathrm{Im}E_4{}^+=\tfrac{1}{2}\sum_{\mathrm{spin}}\pi A_\lambda(m+\not{p})m(W^2-m^2)^{-2}i\sigma^{\lambda\nu}k_\nu\gamma_5u(p'). \quad (5)$$

Equations (1)–(5) follow from a straightforward modification of standard theory.[11,12]

[7] See S. L. Adler and R. F. Dashen, *Current Algebras* (W. A. Benjamin, Inc., New York, 1968.)
[8] J. W. Cronin, in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna*, 1968 (CERN, Geneva, 1968), p. 281.
[9] B. G. Kenny, Ann. Phys. (N. Y.) 43, 25 (1967); also, P. McNamee and J. C. Pati (Ref. 4).
[10] K. Nishijima (Ref. 4); S. Okubo, Nuovo Cimento **54A**, 491 (1968).
[11] A. M. Bincer, Phys. Rev. **118**, 855 (1960).
[12] S. D. Drell and H. R. Pagels, Phys. Rev. **140**, B397 (1965).

An integral like (3) has been used successfully by Drell and Pagels[12,13] to calculate the Pauli moments. They retain only the lowest-mass intermediate states, baryon plus (charged) pseudoscalar, and they impose an upper limit $\Lambda(m+\mu)^2$, where μ is the boson mass and $\Lambda\approx2.8$; this cutoff secures good numerical results for both isovector and isoscalar moments. It also permits one to use, reasonably,[12] the Born approximation to both amplitudes in (4), relying[14] in part on the Kroll-Ruderman theorem. We follow their precedent, with their value of Λ, and for clarity retain at first only the $p\pi^-$ state. Thus, the input is the normal pion-nucleon coupling constant $g_\pi\approx13.5$, and the P- and T-violating constant g_π''. To find g_π'', consider first the P-violating but T-invariant constant g_π' defined by the effective coupling[15]

$$L_1=ig_\pi'(\bar{p}n\phi^*-\bar{n}p\phi). \quad (6)$$

By our second assumption, we insert a T-violating phase, obtaining $ig_\pi'(\bar{p}n\phi^*e^{-i\eta}-\bar{n}p\phi e^{i\eta})$; with $\sin\eta\approx\eta$, the P- and T-violating part is

$$L_2=g_\pi'\eta(\bar{p}n\phi^*+\bar{n}p\phi), \quad (7)$$

so that $g_\pi''=g_\pi'\eta$. The value of g_π' is obtained by following McKellar[16] and Fischbach,[17] who normalize through the nonleptonic S-wave hyperon decays, relying on the hypothesis of partially conserved axial-vector current and current algebra. Thus we find $|g_\pi'|\approx4\times10^{-8}$; g_π' is so small because it turns out to be due entirely[16] to the $\sin^2\theta$ part of H(weak), (where θ is the Cabibbo angle, $\tan\theta\approx0.2–0.25$), whereas the hyperon amplitudes are proportional to $\sin\theta\cos\theta$.

Near threshold, the Pauli moments and the dipole moment itself contribute negligibly to (4), and we find

$$|\beta_\pi|\approx\frac{\sqrt{2}|g_\pi g_\pi'\eta|m}{8\pi^2}\int_{(m+\mu)^2}^{\Lambda(m+\mu)^2}\frac{dW^2}{(W^2-m^2)^2}\ln\left(\frac{q_0+q}{q_0-q}\right), \quad (8)$$

$$|\beta_\pi|\approx0.5\times10^{-24}\text{ cm},$$

where $q_0(W^2)$ and $q(W^2)$ are the intermediate-state proton energy and momentum in the c.m. frame; the suffix on β_π identifies the $(p\pi^-)$ contribution. If *combined* P and T violation (g_π'') does not suffer the $\tan\theta$ suppression affecting g_π', then g_π'' and β_π would increase, roughly, by a factor of $\cot^2\theta$, giving $|\beta_\pi|\approx10^{-23}$ cm.

Though our method is inspired by the success of Drell and Pagels, the details are rather different. Thus, in the limit $\mu\rightarrow0$, the κ remain finite,[12,13] while β_π would diverge logarithmically, eventually yielding as its "leading term" $\beta_\pi\approx(\sqrt{2}g_\pi g_\pi'\eta/8\pi^2m)\ln(m/\mu)\approx0.8\times10^{-24}$ cm. Hence we suspect qualitative estimates of β which use κ as a direct "scaling factor."

[13] H. R. Pagels, Phys. Rev. **140**, B999 (1965).
[14] G. W. Gaffney, Phys. Rev. **161**, 1599 (1967).
[15] G. Barton, Nuovo Cimento **19**, 512 (1961).
[16] B. H. J. McKellar, Phys. Letters **26B**, 107 (1967).
[17] E. Fischbach (unpublished report).

Because of the $\tan\theta$ suppression of g_π', we have evaluated, with the same Λ, the $(\Sigma^- K^+)$ contribution β_K. Here g_K' is proportional to $\cos^2\theta$, and we find $g_K' \approx 1.8\times10^{-6}$; but β_K is suppressed by the higher threshold and by the relatively small value of the strong coupling constant $g_K \approx 3.5$.[18] Because the kaon is so massive, the Born approximation is more suspect; nevertheless, the K^+ photoproduction data[19] show that it is quite good in practice. We find

$$|\beta_K| \approx 1.1\times10^{-24}\ \text{cm}. \tag{9}$$

The photoproduction data can be used directly in a modified semiphenomenological calculation, with numerical results close to (8) and (9), respectively.

We also try to guess the contribution of the 1550-MeV S_{11} resonance, as typical of the resonance region. We use the simplest pole approximation to Γ_λ, whereby $n \to S_{11} \to (n+\gamma)$. In the residue, the electromagnetic factor is taken from the observed photoproduction of the S_{11} (actually of the charged, not the neutral component):

$$(E_S E_n/m_S m)^{1/2}\langle S_{11}|j_\lambda|n\rangle = e\nu\bar{u}(S_{11}) i\sigma_{\lambda\mu}k^\mu\gamma_5 u(n)\,, \tag{10}$$

where $\nu \approx 0.96/2m$,[20] and ν is real under T invariance. T and P are broken in the $n\to S_{11}$ factor of the residue. We obtain it by assuming that the form factor corresponding to the coupling L_2 obeys an unsubtracted dispersion relation in the nucleon mass, and that in the absorptive part one need retain only the S_{11} contribution. This, surely an overestimate, leads immediately to

$$|\beta_{S_{11}}| \approx |\nu g_\pi'\eta/g_{Sp\pi^-}| \approx 1.1\times10^{-24}\ \text{cm}\,, \tag{11}$$

where $g_{Sp\pi^-} \approx 0.7$ is the strong $Sp\pi^-$ coupling constant.

Finally we consider electromagnetic T violation. Here one still expects the above contributions, via g_π'' and g_K'', because radiative EM corrections to nonleptonic coupling constants[21] do seem to be of the order $\alpha/2\pi$. Intermediate states in (4) containing a photon could contribute comparably. However, the main effect should enter through the T-violating part of the current j_λ appearing explicitly in (4), and should not be a correction of order e^2 at all. Although P invariance, current conservation, and Hermiticity force the photon-nucleon vertex (i.e., Γ_λ) with on-shell nucleons to be automatically T-invariant,[6] this is not so, say, for the $Sn\gamma$ vertex (10). In such a model one would expect an imaginary part of the amplitude ν, and if T violation in general is to have the characteristic EM strength (unless forbidden accidentally, as in Γ_λ on-shell), then $|\text{Im}\nu|$ should have the same order of magnitude as $|\nu|$. A very crude "upper limit" on β can be obtained by adapting the pole model. T is broken by writing $\nu\approx i|\nu|$ in (10); and S_{11} is now assumed to dominate the form factor of the P-violating but T-invariant coupling L_1 of Eq. (6). Thus we find

$$|\beta_{S_{11}}| < |\nu g_\pi'/g_{Sp\pi^-}| \approx 6\times10^{-22}\ \text{cm}\,, \tag{12}$$

well above the experimental limit.

However, we should be on safer ground by allowing for such resonance couplings through Eqs. (1)–(5) with $p\pi^-$ intermediate states, as follows. P is broken by using L_1 in the second amplitude in (4), and T is broken in the first (photoproduction) amplitude through resonance-*exchange* contributions. We have calculated the contribution of the Roper P_{11} (1470-MeV) resonance, using a pure-imaginary transition moment $i|\nu_R|$ with[22] $|\nu_R| = (g_{Rn\pi}/g_\pi)(\kappa_P/2m)$, $g_{Rn\pi}^2/4\pi \simeq 2.5$, and for simplicity setting $m_R = m$, $\mu=0$. Eventually one finds

$$|\beta_{R,\text{ex}}| = \left(\frac{g_{Rn\pi}}{g_\pi}\right)^2 \frac{\sqrt{2}|g_\pi g_\pi'\kappa_P|m}{64\pi^2}\int_{m^2}^{\Lambda m^2}\frac{dW^2}{W^4(W^2-m^2)^2} \times\left(-(W^4-m^4)+2W^4\ln\frac{W^2}{m^2}\right), \tag{13}$$

$|\beta_{R,\text{ex}}| \approx 10^{-23}$ cm.

The reason that in EM (as compared to weak) T violation there is a much greater difference between the pole model and the threshold estimate is simply that near threshold the kinematics suppresses the effects of Pauli-type couplings. Further, if the Y^*NK and $N^*N\pi$ vertices and the $Y^*\Sigma\gamma$ and $N^*N\gamma$ vertices, respectively, are comparable, then the Σ^-K^+ intermediate state could appreciably increase both estimates (12) and (13), since in this contribution g_K' replaces g_π'.

Meanwhile, however, conclusions must rest on the view that the only really well-authenticated contributions are those leaning on the successes of Drell and Pagels,[12,13] i.e., (8), (9), and (13). We believe that these "lower-limit" numbers are unlikely to misrepresent the true near-threshold contributions by more than a factor of 2, granting our basic qualitative assumptions. If T violation is EM, then (13) suggests that in order to allow a reasonably conclusive confrontation with theory, the experimental limit on β needs to be lowered by at least a factor of 5. In this case, (12) could reflect merely the inadequacy of the pole approximation. If T violation is weak, then (8), (9), (11), and the remark just below (8), all suggest that the limit needs to be lowered by at least a factor of 10.

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[18] J. K. Kim, Phys. Rev. Letters **19**, 1079 (1967); N. Queen *et al.*, Nucl. Phys. **B11**, 115 (1969), and Phys. Letters **29B**, 311 (1969), have questioned some of Kim's values, but propose no drastic change in the value of $g_{\pi\Sigma^-K^+}$.

[19] B. D. McDaniel *et al.*, Phys. Rev. **115**, 1039 (1959); R. L. Anderson *et al.*, Phys. Rev. **123**, 1003 (1961).

[20] N. Dombey, Phys. Rev. **174**, 2127 (1968). G. Menessier quotes half this value for ν; Phys. Letters **29B**, 75 (1969).

[21] M. St. J. Stevens, Phys. Letters **19**, 499 (1965).

[22] For a summary of the relevant parameters, see T. Muta, Phys. Rev. **171**, 1661 (1968), especially Sec. III B. The vertex is like our Eq. (10) without the γ_5.