Veneziano Representation for $N\bar{N}$ Scattering*

MATTHEW A. JACOBS
University of California, San Diego, La Jolla, California 92037
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Linear combinations of Veneziano terms, starting at $J=0,1$, are written for the invariant amplitudes in $\Lambda\bar{\Lambda}\rightarrow\Lambda\bar{\Lambda}$ and $N\bar{N}\rightarrow N\bar{N}$ scattering. Using only the π, ρ, and Pomeranchuk trajectories, the residues in helicity amplitudes at $J=0,1,2$ along these trajectories are required to correspond to the boson spectrum. Then, it is found that the requirements of reality and factorization of coupling constants for the parent particles are impossible to impose. In addition, the coupling of the pion and the η are found to be zero to the accuracy of the calculation.

I. INTRODUCTION

RECENTLY, Veneziano[1] has suggested a representation for the $\pi\pi\rightarrow\pi\omega$ scattering amplitude which simply incorporates the properties of resonance poles in the direct-channel energy and related asymptotic behavior in the crossed-channel energy for linearly rising trajectories and which explicitly satisfies simple crossing relations. Although the Veneziano representation does not satisfy unitarity, it has been hoped that a small number of the modified Euler beta functions would be a good approximation to real two-particle scattering amplitudes. A number of people have applied this philosophy to $PP\rightarrow PP$,[2–10] $PP\rightarrow PV$,[1,11] and $PN\rightarrow PN$ [12–17] with reasonable measures of success. We have applied the simplistic philosophy of a small number of "low-order" modified beta functions (MBF's) to the equal-mass, $N\bar{N}\rightarrow N\bar{N}$ and $\Lambda\bar{\Lambda}\rightarrow\Lambda\bar{\Lambda}$, scattering processes. In Sec. II we discuss the basic restrictions imposed by requiring the poles of the scattering amplitude for fixed direct-channel energy to correspond to particles of a given spin J and possibly daughters of spin $J-1$, etc. Since the Sommerfeld-Watson transformation can be applied to MBF's, it is obvious that this will determine the correct asymptotic behavior in the crossed-channel energy. In Sec. III we describe the reasons for choosing the set of MBF's in our model and discuss the types of particles that we allow. We then discuss the results of the computer calculations on the model.

We should mention that Harari[18] has pointed out in an approximate and intuitive quark model of baryons and mesons that the dual description of scattering as resonances or Regge contributions should be expected in the other reactions investigated with the Veneziano representation, but not for $N\bar{N}\rightarrow N\bar{N}$ reactions because of the lack of simple "duality" diagrams describing the $N\bar{N}$ process. Since the result of this work is, in fact, that the simplest sets of MBF's do not provide acceptable solutions, one might be tempted to conjecture that this is in verification of Harari's prediction. However, since all MBF's exhibit "duality," this implication is valid only if it could be shown that no set of MBF's have acceptable solutions, which we have not been able to show.

II. RESIDUES OF POLES

The suggestion of Veneziano was that the requirement of poles in one Mandelstam variable and a power law asymptotic behavior in the crossed-channel variable is explicitly satisfied by modified beta functions of the form

$$\Gamma(m-\alpha_i(t))\Gamma(n-\alpha_j(u))/\Gamma(p-\alpha_i(t)-\alpha_j(u)) \quad (1)$$

for linear trajectories, where m, n, and p are integers (for meson trajectories). Poles occur where $\alpha_i(t)=J\geq m$ with residues polynomial in $\alpha_j(u)$ [and hence in $Z_t=(u-s)/(t-4m^2)$] of the order $J+n-p$. Since this is a finite polynomial, this can be related to the residue of a parent particle and a finite sequence of daughters.

The analytic structure of MBF's and their simple crossing properties suggest that linear combinations of MBF's are appropriately written for the invariant amplitudes of the process. For NN or $N\bar{N}$ scattering, there are five independent, invariant amplitudes. A convenient set $g_1, \cdots, g_5$, which we choose to be the

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$s(NN)$-channel invariant amplitudes, is

$$\begin{aligned} g_1&=P\,,\\ g_2&=V+T\,,\\ g_3&=S+A\,,\\ g_4&=-V+T\,,\\ g_5&=-S+A\,, \end{aligned} \tag{2}$$

where S, P, V, A, and T are the usual invariant amplitudes of Goldberger, Grisaru, MacDowell, and Wong (GGMW).[19] Fermi statistics [or equivalence of the t and u ($N\bar{N}$) channels] require $g_j(t,u)=-(-)^j g_j(u,t)$ for isospin-0 nucleons or $g_j{}^I(t,u)=-(-)^{I+j+1}g_j{}^I(u,t)$ for isospin-$\frac{1}{2}$ nucleons in an $I=0$, 1 combination of their s-channel isospin indices. Because of this symmetry, we will write linear combinations of MBF's for the s-channel invariant amplitudes [Eq. (2)].

The independent, reduced helicity amplitudes (half-angle factor removed) are

$$\begin{aligned} f_{00\pm}&=f_{++++}-f_{++--}\,,\\ f_{00\mp}&=f_{+++++}+f_{++--}\,,\\ f_{11\pm}&=(1/1+Z)f_{+-+-}-(1/1-Z)f_{+--+}\,,\\ f_{11\mp}&=(1/1+Z)f_{+-+-}+(1/1-Z)f_{+--+}\,,\\ f_{10\mp}&=(1-Z^2)^{-1/2}f_{+++-}\,, \end{aligned} \tag{3}$$

where $f_{\lambda_c\lambda_d,\lambda_a\lambda_b}$ are the Jacob-Wick[20] helicity amplitudes; the upper sign of $f_{\lambda\mu\epsilon}$ is chosen in the NN channel and the lower sign in the $N\bar{N}$ channels; $\lambda=\lambda_a-\lambda_b$, $\mu=\lambda_c-\lambda_d$. The partial-wave series for these amplitudes are[21]

$$f_{\lambda\mu\epsilon}=\textstyle\sum_J\,(2J+1)[F_h{}^{J\epsilon}e_{\lambda u}{}^{J+}(Z)+F_h{}^{J-\epsilon}e_{\lambda\mu}{}^{J-}(Z)]. \tag{4}$$

The relevant e's for $N\bar{N}$ scattering (from Ref. 21) are

$$\begin{aligned} &e_{00}{}^{J+}=P_J\,, && e_{00}{}^{J-}=0;\\ &e_{11}{}^{J+}=(P_J'+ZP_J'')/J(J+1)\,, && e_{11}{}^{J-}=-P_J''/J(J+1);\\ &e_{10}{}^{J+}=-P_J'/[J(J+1)]^{1/2}, && e_{01}{}^{J-}=0. \end{aligned}$$

Note that

$$e_{11}{}^{0\pm}=e_{10}{}^{0\pm}=0.$$

The normality of particles contributing to $F_h{}^{J\epsilon}$ is ϵ. When a Regge trajectory of normality ϵ passes through a non-negative integer, there is a pole in $F_h{}^{J\epsilon}$ with a residue proportional to the product of appropriate coupling constants. In terms of the s-channel invariants, the t-channel amplitudes are

$$\begin{aligned} f_{00-}{}^t=&\,(4m^2-t-\zeta)g_1+(4m^2+t-\zeta)g_2\\ &+(2m^2+\tfrac{3}{2}t-\tfrac{1}{2}\zeta)g_3+(-4m^2+5t-\zeta)g_4\\ &+(6m^2+\tfrac{5}{2}t+\tfrac{1}{2}\zeta)g_5\,,\\ f_{00+}{}^t=&\,(-4m^2+t+\zeta)g_1+(20m^2-5t\\ &+[(t+4m^2)/(t-4m^2)]\zeta)g_2+(-10m^2+\tfrac{5}{2}t\\ &+[(\tfrac{1}{2}t+6m^2)/(t-4m^2)]\zeta)g_3+(4m^2-t+\zeta)g_4\\ &+(-6m^2+\tfrac{3}{2}t-\tfrac{1}{2}\zeta)g_5\,,\\ f_{11-}{}^t=&\,(4m^2-t)(g_1+g_2+\tfrac{1}{2}g_3-g_4+\tfrac{3}{2}g_5)\,,\\ f_{11+}{}^t=&\,(4m^2-t)g_1+(4m^2+t)g_2+(2m^2+\tfrac{3}{2}t)g_3\\ &+(4m^2-t)g_4+(-2m^2+\tfrac{1}{2}t)g_5\,,\\ f_{01+}{}^t=&\,-8m^2(g_2+g_3), \end{aligned} \tag{5}$$

where $\zeta=u-s=$ numerator of Z_t. In order to be able to expand the residues of poles in $f_{\lambda\mu\epsilon}$ as finite polynomials in Z_t for an arbitrary number of sequential poles (due to a finite set of MBF's), we must require $p\leq m+n$ in Eq. (1). The residues of $f_{\lambda\mu\epsilon}$ due to particles of spin J or less are polynomial in Z_t as follows:

$$\begin{aligned} &f_{00\pm}\ \text{are polynomial order}\ J\,,\\ &f_{11\pm},\,f_{01+}\ \text{are polynomial order}\ J-1\,. \end{aligned}$$

In order to bound the order of polynomials occurring when $\alpha_i(t)=J$ to these values (for an arbitrary number of values of J), we must require $p\geq\max(m,n)$. In addition, the residues of the g's must have certain relations. For instance, at $\alpha_\pi(t_0)=0$,

$$\begin{aligned} &\text{residue of } g_1 \quad\text{at}\quad t_0=-g_\pi{}^2/16t_0\,,\\ &\text{residue of } g_2 \quad\text{at}\quad t_0=0\,,\\ &\text{residue of } g_3 \quad\text{at}\quad t_0=0\,,\\ &\text{residue of } g_4 \quad\text{at}\quad t_0=g_\pi{}^2/8t_0\,,\\ &\text{residue of } g_5 \quad\text{at}\quad t_0=g_\pi{}^2/8t_0\,. \end{aligned}$$

More complicated relations exist for the residues, polynomial in Z_t for higher spin particles, and are easily derived from Eqs. (5) and (4).

III. MODEL

If a trajectory $\alpha_i(t)$ evaluated at t_J is such that $\alpha_i(t_J)=J$, it produces a pole in Eq. (1) with residue

$$\begin{aligned} &[(-)^{J-m}/(J-m)!][n-1-\alpha_j(u)]\\ &\quad\times[n-2-\alpha_j(u)]\cdots[p-J-\alpha_j(u)]. \end{aligned} \tag{6}$$

The terms which contribute to $f_{\lambda\mu\epsilon}$ with highest Z_t power come from that set of MBF's with largest $n-p$ $(=0)$. The coefficient of the $Z_t{}^J$ term is

$$\mathfrak{M}_j{}^J\frac{(-)^m}{(J-m)!}\,,$$

where $\mathfrak{M}_j$ is the slope of the jth trajectory. Therefore, if there are v different values of m in the set of MBF's

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and all trajectories have the same slope, it is possible to eliminate all particles with spin greater than $\alpha_i(t)$ by eliminating them at v different integral values of $\alpha_i(t)$. From the form of Eq. (5), the residues of $f_{00\pm}$ proportional to Z^J and of $f_{11\pm}$ and f_{10+} proportional to Z^{J-1} have contributions from terms (1) with $n-p=-1$, 0, and hence, as a result of the expansion (6), have residue contributions dependent on $\alpha_j(0)$, n, and p. Many more than v values of J are required, therefore, to set the parity of the parents on a trajectory. There are further restrictions to make the daughters come out right. Since there is some evidence that the pion has a natural-parity conspirator (at $t=0$), we do not require a definite parity for parents along the entire trajectory, only at the low-spin sense points where the parity of the particles are known. However, we require the contributions from terms with spin$=\alpha_i(t)+1$ to vanish everywhere.

The particles we assume to exist in the model are determined as follows: Since there are only weak forces in the $B=2$ channel, we do not include any s-channel trajectories and try to obtain consistency with meson forces alone. In the meson channel we assume isospin degeneracy as an approximation. It is known that the ρ and ω are almost degenerate in mass,[22] as are the $A2$ and f. The $I=0$ axial-vector meson which we have assumed degenerate with the $A1$ is not yet established [it could be the $D(1285)$], but this is not too surprising, since the neutral member of the $A1$ is not established either.[23] The π and η are admittedly not degenerate, but we take them to be so to keep the calculation manageable and because it is not in disagreement with other results from the Veneziano model.[5] In addition, we do not include the φ, X^0, and their trajectories. In the nonet model, the physical φ is assumed not to couple to baryons and, so, is consistently left out of this calculation. In Wong's calculation of the boson spectrum from pseudoscalar-pseudoscalar scattering in the Veneziano model,[5] it was not necessary to include the X^0, and we leave it out for simplicity. We have the following three trajectories: (π,η), (ρ,ω), and Pomeranchuk.

We assume that the slopes of all trajectories are the same. This is consistent for the (π,η) and (ρ,ω) trajectories if the $A1$ lies on the (π,η) and if the $A2$ and f lie on the (ρ,ω). The slope of the Pomeranchuk trajectory has been the subject of much discussion. Most fits to data in the physical region would have its slope near zero. However, Miller[24] claims to have found a 2^+ particle at 1 BeV which is just where the first correct signature point of the Pomeranchuk trajectory should be if it has the normal slope of $\sim 1/M_n^2$. In the pseudoscalar-pseudoscalar problem, Wong[10] has shown that it is possible to allow the Pomeranchuk trajectory to have a canonical slope within the Veneziano model.

Experimentally, there are two $A2$ mesons of about the same mass. Since we have assumed the π and η degenerate, it is consistent to assume that the two $A2$ particles are degenerate, since it has been possible, in Wong's boson spectrum,[5] to show that the π-η mass splitting leads to $A2_L$, $A2_H$ separation from the soft-pseudoscalar consistency conditions. The other low-mass particles are the $\delta(962)$, $\eta(1070)$, and $B(1220)$. These particles can be considered as daughters of the f_P, f_P, and f whose masses have been shifted because of unitarity.

The trajectories and particles allowed are shown in Fig. 1.

The set of MBF's we choose to satisfy the implicit residue restrictions are determined as follows: For the pion trajectory m (or n) is ≥ 0; for the ρ and Pomeranchuk trajectories m (or n) is ≥ 1; p is restricted by $\max(m,n)\leq p\leq m+n$. The only other decision is to determine the largest acceptable m (or n). We choose this to be one. With this set of acceptable MBF's, there are 66 parameters in the isospin-0 nucleon scattering and 125 parameters in the isospin-$\frac{1}{2}$ case.

The results of the computer analysis for $\Lambda\bar{\Lambda}$ scattering is that the restrictions at $J=0$, 1, and 2 on the three trajectories provide 35 [33 if $\alpha_{\rm Pom}(0)=1$] linearly independent restrictions. The residues of the ω, f, and $f_{\rm Pom}$ in the amplitudes f_{00+}, f_{11+}, f_{10+} depend on only 2 (3) of the remaining 31 (33) independent parameters. However, there are further restrictions on these residues, since they are (essentially) g_E^2, g_M^2, and $g_E g_M$ of the ω, f, and $f_{\rm Pom}$, respectively, and hence must satisfy a positivity condition for the g_E^2, g_M^2 and a factorization condition for $g_E g_M$. The dependence of these numbers on the 2 (3) parameters is such that neither positivity nor factorization can occur for any of the three parent particles. In addition, the residue

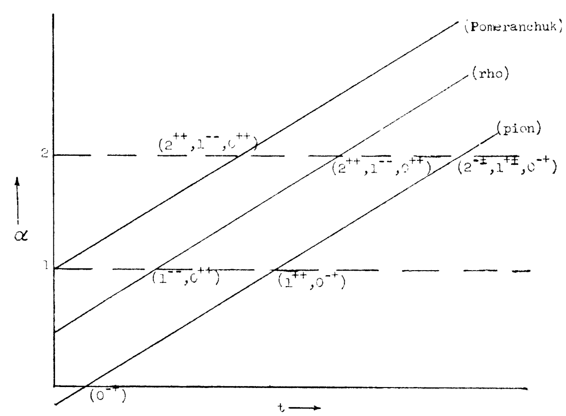

FIG. 1. Plot of the trajectories used and the particles on them. The notation (J^{PC}) is for a particle of spin J, parity P, and G-parity$=C(-1)^I$.

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of the η trajectory at the η pole is zero to the accuracy of the calculation.

The results of the computer analysis for $N\bar{N}$ scattering is that the restrictions at $J=0$, 1, and 2 on the three trajectories provide 71 [67 if $\alpha_{\text{Pom}}(0)=1$] linearly independent restrictions. The residues of the natural-parity parent particles in the amplitudes f_{00+}, f_{11+}, f_{10+} depend on only 3 (5) of the remaining 54 (58) independent parameters. Again it is impossible to satisfy either mutual positivity or factorization for any of the coupling-constant residues. It is also true here that the residue of the pion trajectory at the π-η pole is contrained to be zero to the accuracy of the calculation.

In neither the $\Lambda\bar{\Lambda}$ nor the $N\bar{N}$ calculation do we believe it is an important point to discuss the residue of daughter particles, since these are expected to be changed considerably by inclusion of MBF's with m, $n>1$ and by imposing, in some way, unitarity.[22] However, that the residues of the ρ, etc., are so well constrained to violate the reality and factorizability of coupling constants we believe to be important. Adding higher MBF's may change these results, but it is precisely the simplicity of the Veneziano model in incorporating duality which is most appealing. In the $\pi\pi\to\pi\pi$ scattering amplitude, Oehme[25] found an analogous problem. He was able to show that all but the first MBF term have an infinite number of negative g^2's and thus, that it would probably require an infinite sequence of MBF's to satisfy positivity.

The results of our calculation are with the input parameters $\alpha_\pi(0)\sim -0.02$, $\alpha_\rho(0)\sim 0.5$, $\alpha_{\text{Pom}}(0)\sim 1.0$, and slope ~ 1.0, although the results do not change critically with small variations of the parameters about these values.

To summarize, it is not possible, with 66 ($I_N=0$) or 125 ($I_N=\frac{1}{2}$) parameters to set up a Veneziano model which allows the correct parent structure up to $J=2$ on the pion, ρ, and Pomeranchuk trajectories.

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Equal-Time Commutators and Form Factors in Quantum Electrodynamics with Internal Symmetry

J. Katz* and J. Langerholc

II. Institut für Theoretische Physik der Universität Hamburg, Hamburg, Germany and Deutsches Elektronen-Synchrotron DESY

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Isospin currents are constructed in second-order perturbation theory by iterating the Yang-Feldman equations and renormalizing. The conventional method of subtracting divergences is replaced by a deductive approach in which Lorentz covariance and locality of the currents determine, up to a polynomial, the form factors which remain after renormalization. It is observed that any such polynomials occurring in addition to the usual expression for the form factors will cause not only an undesirable increase in the currents themselves at high energies, but also to a series of extra terms in the commutators of the electric currents. The commutators of the currents so defined are calculated by smearing the commutator in the time difference with a test function of class $\mathcal{S}$ which is allowed to approach a δ function in the appropriate topology (δ sequence). Attention is restricted here to those parts of the commutator connected with the vacuum-polarization correction to the electric form factors. It is found that the equal-time commutator does not exist in those cases where the result expected on the basis of a calculation with canonical commutation relations does not vanish. In this case, the renormalized current reappears along with several divergent integrals which are damped by the test functions of the δ sequence in p space. This causes the limit to be strongly dependent on the choice of the sequence, enabling either finite or infinite results to be obtained. The reason for this is that the commutator as a function of the time difference is not a continuous function at the origin, and thus the "equal-time value" has no meaning. The nature of the singularities occurring is studied, and they are displayed explicitly in x space. The question of the existence of time-ordered and retarded commutators is investigated.

I. INTRODUCTION

A. General Discussion

THE proposal that the equal-time limit of commutators of current operators could be abstracted from simple models and postulated as exact has had many experimental successes. It is therefore worthwhile to explore the foundations of this physical idea and hope that in the future it will be possible to establish a rigorous mathematical foundation for calculations in particle physics. Several problems have appeared during the development of local field theory because of the highly pathological behavior of expressions involving

* Present address: Purdue University, Lafayette, Ind., 47907.