Pion-Nucleon Scattering in the Veneziano Representation*

S. K. Bose and K. C. Gupta†
Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556
(Received 28 April 1969)

The leading Veneziano terms for low-energy π-N scattering are studied. It is found that they do not provide an acceptable model for low-energy scattering.

In this paper, we study a Veneziano representation[1] for pion-nucleon scattering at low energies. The leading Veneziano terms for this process, consisting of a degenerate ρ-f trajectory in the t channel, and N_α, Δ_δ, and N_γ trajectories in the s and u channels, have been written down and studied by Igi.[2] There are several reasons for reinvestigating this problem. First, Igi has not studied the low-energy behavior of this model in sufficient detail; in particular, the S-wave scattering lengths and the various resonance widths can be computed and compared with experiment. Second, a consistent treatment of this amplitude necessarily requires the baryon trajectories N_α, Δ_δ, and N_γ to be degenerate. This degeneracy must be taken seriously for low-energy calculations. Finally, in view of the degenerate baryon trajectories, we use a different parametrization of the π-N amplitude than that of Igi.[2]

The necessity of taking degenerate baryon trajectories in the amplitude written by Igi[2] is seen as follows: Consider the nucleon trajectory $\alpha_{N\alpha}(s)$. The Veneziano amplitude has simple poles when $\alpha_{N\alpha}(s)=\frac{1}{2}$ and $\alpha_{N\alpha}(s)=\frac{5}{2}$. Isospin symmetry requires the residues of these poles to vanish in the $T=\frac{3}{2}$ channels ($T=$isospin). There exist similar constraints for the other baryon trajectories; for instance, the pole resulting from $\alpha_{\Delta\delta}(s)=\frac{3}{2}$ should exist in the $T=\frac{3}{2}$, but not in the $T=\frac{1}{2}$ channel. All these requirements cannot be simultaneously satisfied unless the baryon trajectories are degenerate[3]: $\alpha_{N\alpha}(s)\alpha_{\Delta\delta}(s)=\alpha_{N_\gamma}(s)$. How good is a model with degenerate baryon trajectories? If we fix the linear trajectory by the nucleon mass and a universal slope of $0.86/(\mathrm{BeV})^2$, we get $\alpha(s)=-0.26+0.86s$. This trajectory then predicts the mass of the ($J^P=\frac{3}{2}^+$, $T=\frac{3}{2}$) and ($J^P=\frac{3}{2}^-$, $T=\frac{1}{2}$) isobars to be 1.43 BeV. Experimentally, these masses are 1.24 and 1.52 BeV, respectively.[4] Thus, the predicted mass values look acceptable.

The Veneziano form of the π-N scattering amplitude with a degenerate baryon trajectory is written below:

$$A^{(-)}(s,t)=\beta_1[C(1-\alpha(t),\tfrac{3}{2}-\alpha_B(s)) -C(1-\alpha(t),\tfrac{3}{2}-\alpha_B(u))], \quad (1)$$

$$B^{(-)}(s,t)=\beta_2[B(1-\alpha(t),\tfrac{1}{2}-\alpha_B(s)) +B(1-\alpha(t),\tfrac{1}{2}-\alpha_B(u))] +\beta_3B(\tfrac{1}{2}-\alpha_B(s),\tfrac{1}{2}-\alpha_B(u)), \quad (2)$$

$$A^{(+)}(s,t)=\beta_4[C(1-\alpha(t),\tfrac{3}{2}-\alpha_B(s)) +C(1-\alpha(t),\tfrac{3}{2}-\alpha_B(u))] +\beta_5C(\tfrac{3}{2}-\alpha_B(s),\tfrac{3}{2}-\alpha_B(u)), \quad (3)$$

$$B^{(+)}(s,t)=\beta_6[B(1-\alpha(t),\tfrac{1}{2}-\alpha_B(s)) -B(1-\alpha(t),\tfrac{1}{2}-\alpha_B(u))]. \quad (4)$$

In the above, $\alpha_B(s)$ is the degenerate baryon trajectory with mixed isospin and parity, $\alpha(t)$ is the degenerate ρ-f trajectory, and β_i ($i=1, 2, \ldots,6$) are constant parameters. $B(x,y)$ is the Euler function $\Gamma(x)\Gamma(y)/\Gamma(x+y)$, and $C(x,y)=\Gamma(x)\Gamma(y)/\Gamma(x+y-1)$. $A^{\pm}(s,t,u)$ and $B^{\pm}(s,t,u)$ are the usual invariant functions of π-N scattering. To determine the six unknown parameters above, we proceed as follows: First, the $T=\frac{3}{2}$ partner of the nucleon (the resonance with $J^P=\frac{1}{2}^+$ and isospin $\frac{3}{2}$, degenerate in mass with the nucleon) is eliminated, as there seems to be no experimental evidence for this particle. This gives[5]

$$\beta_6=\beta_2+\beta_3. \quad (5)$$

Similarly, the $T=\frac{1}{2}$ partner of the ($J^P=\frac{3}{2}^+$, $T=\frac{3}{2}$) resonance is decoupled from the amplitude. We get

$$2\beta_1+\beta_4-\beta_5=(m^*-m)(\beta_6+2\beta_2-2\beta_3). \quad (6)$$

In Eq. (6), m^* and m are the masses of the ($J^P=\frac{3}{2}^+$, $T=\frac{3}{2}$) resonance and the nucleon, respectively. Equality of the slopes of the baryon and meson trajectories has been used in deriving Eq. (6). Three more parameters can be fixed by extrapolation to the nucleon and ρ-meson poles. For the ρ-exchange Born term, we neglect the magnetic-moment-type coupling of ρ to the nucleons, and assume further that ρ is universally coupled to the isospin current.[6] The residue of the ρ pole is now given by a single constant g_1, which is related to the ρ width; experimentally, $g_1^2/4\pi=2$. Extrapolating amplitudes

* Research supported in part by the U. S. Atomic Energy Commission, under contract No. AT(11-1)-427.
† Research supported by the National Science Foundation under the University Science Development Program.

[1] G. Veneziano, Nuovo Cimento **57A**, 190 (1968).
[2] K. Igi, Phys. Letters **28B**, 330 (1968).
[3] This is reminiscent of the situation found in meson trajectories. See K. Kawarabayashi, S. Kitakado, and D. H. Yabuki, Phys. Letters **28B**, 432 (1969); C. Lovelace, *ibid.* **28B**, 264 (1968).
[4] N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, A. H. Rosenfeld, P. Söding, C. G. Wohl, M. Roos, and G. Conforto, Rev. Mod. Phys. **41**, 109 (1969).
[5] Note that the parity partner of the nucleon is absent from Eqs. (1)-(4).
[6] See, for instance, E. Abers and C. Zemach, Phys. Rev. **131**, 2305 (1963).

(1)–(4) to the ρ pole, we obtain

$$\beta_1=0, \quad \beta_2=-g_1{}^2b/16\pi. \tag{7}$$

Similarly, extrapolation to the nucleon pole yields

$$\beta_6=g^2b/4\pi. \tag{8}$$

In the above, b [$=0.86$ (BeV)$^{-2}$] is the universal slope of the Regge trajectories, and g is the usual pion-nucleon coupling constant, $g^2/4\pi\simeq 14.4$. Finally, we require the amplitude to satisfy the hypothesis of the partially conserved axial-vector current consistency condition of Adler[7]:

$$A^{(+)}(m^2,\mu^2,m^2)=g^2/4\pi m. \tag{9}$$

From Eqs. (3) and (9) we get[8]

$$2\beta_4+\beta_5=g^2/4\pi m. \tag{10}$$

Equations (5)–(8) and (10) completely fix the parameters β_i $(i=1, 2, \ldots, 6)$, and hence the π-N scattering amplitude. This amplitude now makes the following predictions:

(1) The decay width of the $(J^P=\frac{3}{2}^+,\ T=\frac{3}{2})$ resonance to the π-N channel is given by

$$\Gamma=(q^{*3}/m^{*2})(E^*+m)(m^*-m)b(g^2/4\pi+g_1{}^2/8\pi). \tag{11}$$

In the above, q^* and E^* are the c.m. momentum and the nucleon energy at the position of the $(J^P=\frac{3}{2}^+,\ T=\frac{3}{2})$ resonance. There now exists an inherent ambiguity in making numerical predictions, depending on whether one used for m^* the experimental value (1.24 BeV) or the value (1.43 BeV) given by the linear degenerate baryon trajectory, in calculating the kinematic factors occurring in Eq. (11). We obtain $\Gamma=360$ MeV (60 MeV) corresponding to the input $m^*=1.43$ (1.24) BeV. Experimentally, Γ is known to be 120 MeV.[4]

(2) The partial width of the $(J^P=\frac{3}{2}^-,\ T=\frac{1}{2})$ resonance to the π-N system turns out to be negative,

$$\Gamma=-(2q^{*3}/3m^*)(E^*-m)(g^2/4\pi+g_1{}^2/4\pi), \tag{12}$$

in serious conflict with the unitarity condition.[9]

(3) The π-N amplitude predicts the existence of five additional resonances at 1.43 BeV, with quantum numbers $(J^P=\frac{3}{2}^-,\ T=\frac{3}{2})$, and $J^P=\frac{1}{2}^\pm$ in both $T=\frac{1}{2}$ and $T=\frac{3}{2}$. Of these, the isobar with $(J^P=\frac{1}{2}^+,\ T=\frac{1}{2})$ turns out to have a partial width of 203 MeV and, hence, may be identified with the P_{11} (Roper) resonance at 1.46 BeV.[4] Similarly, the isobar with $(J^P=\frac{3}{2}^-,\ T=\frac{3}{2})$ has a calculated partial width of 40 MeV and may be identified with the D_{33} resonance at 1.67 BeV.[4] No S_{13} resonance $(J^P=\frac{1}{2}^+,\ T=\frac{3}{2})$ is observed at this energy. As for the rest, S_{11} and S_{13} resonances are experimentally observed at 1.53 and 1.63 BeV, but the calculated partia widths turn out to be absurdly large (≈ 2 BeV).

(4) Finally, the S-wave scattering lengths are calculated. Once again, these depend on the choice of m^* (see above). For $m^*=1.24$ BeV (as observed), we get, in units of the pion mass,

$$a_{1/2}=0.33\mu^{-1}, \quad a_{3/2}=-0.04\mu^{-1},$$

while for $m^*=1.43$ (as given by linear trajectory), we get

$$a_{1/2}=0.36\mu^{-1}, \quad a_{3/2}=-0.01\mu^{-1}.$$

Experimental values for these are[10]

$$a_{1/2}{}^{\mathrm{exp}}=0.178\mu^{-1}, \quad a_{3/2}{}^{\mathrm{exp}}=-0.087\mu^{-1}.$$

To summarize, only the S-wave scattering lengths and some of the isobar widths are of the right order. For the S_{11} and S_{13} resonances, the calculated widths are wrong by an order of magnitude, and for D_{33}, the width is negative, in blatant conflict with unitarity. Hence, we conclude that the leading Veneziano terms do not provide an acceptable model for low-energy π-N scattering.[11] Whether one can construct a more realistic amplitude by the inclusion of further (non- eading) terms remains to be seen.

We wish to thank Professor J. T. Cushing and Professor W. D. McGlinn for fruitful discussions.

[7] S. L. Adler, Phys. Rev. **137**, B1022 (1965).

[8] This conclusion is different from that of M. Ademollo, G. Veneziano, and S. Weinberg, Phys. Rev. Letters **22**, 83 (1969). The latter authors conclude from Adler's condition that the intercepts of the N_α and Δ_δ trajectories differ by $\frac{1}{2}$. Indeed, in a model in which N_α and Δ_δ trajectories are degenerate, such a conclusion is clearly impossible.

[9] See also, R. Oehme, University of Chicago report, 1969 (unpublished).

[10] J. Hamilton and W. S. Woolcock, Rev. Mod. Phys. **35**, 737 (1963).

[11] We have also attempted a different parametrization of the amplitude. Now, instead of Eq. (7) which was obtained by extrapolation to the ρ pole, we obtain two linear constraints, by eliminating the "undesired" resonances $(J^P=\frac{3}{2}^-,\ T=\frac{1}{2})$ and $(J^P=\frac{1}{2}^+,\ T=\frac{3}{2})$. However, this solution leads to a negative value for $g_1{}^2$.