PHYSICAL REVIEW VOLUME 184, NUMBER 5 25 AUGUST 1969

Regge-Pole Model for Invariant Functions. II. The Charge-Exchange Processes $pn \to np$ and $p\bar{p} \to n\bar{n}$

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(Received 20 January 1969)

The structure of the $pn \to np$ and $p\bar{p} \to n\bar{n}$ charge-exchange data is described by three Regge-type poles in two invariant functions.

I. INTRODUCTION

THE $pn \to np$ and $p\bar{p} \to n\bar{n}$ high-energy data show pronounced structure near the forward direction $(t=0)$[1,2]:

(i) The differential cross sections

$$d\sigma/dt(pn \to np) \sim d\sigma/dt(p\bar{p} \to n\bar{n}) \sim s^{2(\alpha-1)} \quad (1)$$

have a strong s dependence for $0 > \alpha > -\frac{1}{2}$.

(ii) The sum

$$\Sigma(t) = 2\pi s(s-4M^2)[d\sigma/dt(p\bar{p} \to n\bar{n}) + d\sigma/dt(pn \to np)] \quad (2)$$

falls by $\sim$20–40% in the small interval $0 \geqslant t \geqslant -0.02$ $(\text{GeV}/c)^2$.

(iii) The difference

$$\Delta(t) = 2\pi s(s-4M^2)[d\sigma/dt(p\bar{p} \to n\bar{n}) - d\sigma/dt(pn \to np)] \quad (3)$$

increases by $\sim$150–300% in the interval $0 \geqslant t \geqslant -0.02$ $(\text{GeV}/c)^2$.

Similar structure can be observed in the photoproduction processes[3]

$$\gamma p \to n\pi^+ \quad \text{and} \quad \gamma n \to p\pi^-, \quad (4)$$

where the same Regge trajectories occur. The two processes are related by crossing. Thus the $I^G = 1^+$ Regge trajectories provide the same contributions to both processes. The $I^G = 1^-$ contributions have opposite sign.

In Sec. II we derive bounds from the $pn \to np$ and $p\bar{p} \to n\bar{n}$ data on the residue function $(t-\mu^2)\phi_1(s,t,u)$ of the π pole, which appears only in the t-channel singlet amplitude $\phi_1(s,t,u)$. At $t=\mu^2$, the residue function $(t-\mu^2)\phi_1(s,t,u)$ is related to the πN coupling constant. The bound and the connection to the πN coupling constant require at least a 100% increase of the residue function $(t-\mu^2)\phi_1(s,t,u)$ in the small interval $0 \leqslant t \leqslant \mu^2$. Within the Toller-pole model[4,5] this behavior must be reproduced by the π residue function, if the π trajectory is assumed to be a class-III or a class-II conspirator. For example, the residue function of the the class-III conspirator in the Arbab–Dash fit I (see Ref. 6) increases about 200% in the interval $0 \leqslant t \leqslant \mu^2$. This remarkable feature can be understood if one starts from invariant functions[7] instead of helicity amplitudes.

$$\phi_1(s,t,u) = -(4M^2-t)zH_3 - tH_5 - 4M^2H_4. \quad (5)$$

As was pointed out in Paper I,[8] the π trajectory may be produced by Regge-type poles[9] $\alpha_3^{(-)}(t) = \alpha_\pi(t) - 1$ and $\alpha_5^{(+)}(t) = \alpha_\pi(t)$ in the invariant functions H_3 and H_5. The trajctory $\alpha_5^{(+)}(t)$ leads to an evasive solution and the trajectory $\alpha_3^{(-)}(t)$ to a conspiratorial solution of the kinematical constraint. However, the π pole at $t=\mu^2$ $(\alpha_\pi = 0)$ appears only in the invariant function H_5. The conspiring trajectory $\alpha_3^{(-)}(t) = \alpha_\pi(t)+1$ does not produce a $j^P = 0^+, 0^-$ parity doublet at $\alpha_\pi = 0$. Therefore, the πN coupling constant is related only to the residue of the evasive trajectory $\alpha_\pi(t) = \alpha_5(t)$. The increase of the π helicity residue function $(t-\mu^2)\phi_1(s,t,u)$ can be explained by a strong compensation between the conspiratorial part (H_3) and the evasive part (H_5) of the π trajectory [see Eq. (5)]. The $pn \to np$ and $p\bar{p} \to n\bar{n}$ data and the πN coupling constant yield a bound, which requires "only" a 20–30% increase of the residue function $(t-\mu^2)H_5(s,t,u)$ in $0 \leqslant t \leqslant \mu^2$.

This was probably the main motivation for the Arbab–Dash fit II, where the π trajectory is assumed to be evasive. The same compensation as described above is achieved by the introduction of an additional $I^G = 1^-$ class-III conspiring parity doublet $\alpha_d(t)$, $\alpha_{d'}(t)$. Within the Toller-pole model one has to expect a

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[9] See, for example, Eq. (4) of Paper I. The identification $\alpha_\pi(t) = \alpha_4^{(+)}(t)$ has been shown to be inadequate.

$j^p=0^+$ particle at $\alpha_d=0$ and a $j^p=0^-$ particle at $\alpha_{d'}=0$, unknown until now.

In Sec. III, we discuss a three-pole model for the processes $pn\to np$ and $p\bar{p}\to n\bar{n}$. The π is assumed to be evasive:

$$\alpha_\pi(t)=\alpha_5^{(+)}(t)=0.1(t-\mu^2). \quad (6)$$

The B is conspiratorial:

$$\alpha_B(t)=\alpha_3^{(+)}(t)+1=-0.14+t. \quad (7)$$

In addition, we introduce an $I^G=1^-$ conspirator C, which is parametrized, as in Pignotti, by

$$\alpha_C(t)=\alpha_3^{(-)}(t)+1=-1+[\alpha_C(0)+1]^2/[\alpha_C(0)+1-\alpha_C'(0)t], \quad (8)$$

where $\alpha_C(0)=-0.25$ and $\alpha_C'(0)=1.9$. The unconventional sequence of the trajectories (6)–(8),

$$\alpha_\pi(t)>\alpha_B(t)>\alpha_C(t), \quad (9)$$

has an important effect. *The contributions of the conspiring C [Eq. (8)] and the evasive π [Eq. (6)] provide simultaneously the decrease of the sum (2) and the increase of the difference (3).*

Exactly the same mechanism works in the photoproduction processes (4).[10] As mentioned above, no $j^p=0^+$, 0^- parity doublet occurs at $\alpha_C=0$, i.e., $\alpha_3^{(-)}=-1$. However, at[11] $\alpha_3^{(-)}=1$, i.e., $t\approx1.19$ $(\mathrm{GeV}/c)^2$, a quintet is predicted with $j^p=2^+$, 2^-, 1^+, 0^+, 0^-. Four $I^G=1^-$ resonances can be found in this region:

$$A_2(1300),\quad A_1(1070),\quad \pi_N(1016),\quad \text{and}\quad A_{1,5}(1170),\quad \delta(980), \quad (10)$$

with $j^p=2^+$, 1^+, 0^+. The j^p assignment of the latter resonances has not yet been determined. We have shown in Paper I that a trajectory $\alpha_3(t)$ has no coupling to any two-scalar or two-pseudoscalar system (e.g., $\pi\pi$, $K\bar{K}$). This is in good agreement with the dominant decay rates of $A_2\to\rho\pi$ and $A_{1.5}\to\rho\pi$, 3π, but in disagreement with the large $K\bar{K}$, $\pi\eta$ decay mode of the π_N. The recent listing of particle properties by Rosenfeld *et al.*[12] gives two A_2 resonances:

$A_{2H}(1315)$: $I^G=1^-$, $j^p=2^+$, dominant decay mode $\rho\pi$;

$A_{2L}(1270)$: $I^G=1^-$, $j^p=(-1)^j$, dominant decay mode $\rho\pi$.

The latter resonance may provide the 0^+ and 2^+ in our quintet. Furthermore, according to Rosenfeld *et al.*,[12] the $A_1(1070)$ may have two parts, with $j^p=1^+$ and $j^p=2^-$. A $j^p=0^-$ part in $A_{1.5}$ or δ would complete our quintet. At the second recurrence $\alpha_C=4$, i.e., $t=2.3$ $(\mathrm{GeV}/c)^2$, we expect a quintet $j^p=4^+$, 4^-, 3^+, 2^+, 2^-. The unnatural-parity members of this quintet could be identified with the resonance $\pi_A(1640)$. The B trajectory (7) produces a $j^p=1^+$, 1^- doublet. The 1^- partner of the B meson has not yet been observed. At $\alpha_B=3$, i.e., $t\approx3.2$ $(\mathrm{GeV}/c)^2$, a quintet $j^p=3^+$, 3^-, 2^-, 1^+, 1^- is predicted. Candidates for the unnatural-parity members 3^+, 2^-, 1^+ are provided by the resonance $\rho(1700)$. The ρ and R trajectories are well determined by an analysis of the charge-exchange processes $\pi^-p\to\pi^0n$ and $\pi^-p\to\eta n$:

$$\alpha_\rho(t)=+0.57+0.8t,$$
$$\alpha_R(t)=+0.4+0.7t.$$

Obviously, they cannot reproduce the strong s dependence[13] (1) or any other of the characteristic features of Eqs. (1)–(3). The same holds for the ρ and R contributions in the photoproduction processes (4). We therefore conclude that they cannot play a dominant role in the description of these processes, at least for small $|t|$ values.

II. BOUNDS FOR THE π RESIDUE FUNCTION

We calculate the $NN\to NN$ differential cross section in terms of the invariant[7] functions $H_i(s,t,u)$, $i=1,\ldots,5$:

$$\begin{aligned}2\pi s(s-4M^2)d\sigma/dt &=2|(t-4M^2)zH_3-4M^2H_4-tH_5|^2\\ &+2|(t-4M^2)zH_3-4M^2H_4-tz(H_2+H_3)+tH_4|^2\\ &+2|tH_2+4M^2H_3-z(t-4M^2)H_4|^2\\ &+2|tzH_3-(t-4M^2)H_1+4M^2zH_2|^2\\ &-16tM^2(z^2-1)|H_2+H_3|^2, \quad (11)\end{aligned}$$

where

$$z=-1+2s/(4M^2-t). \quad (12)$$

The π pole at $t=\mu^2$ can occur only in the s-u crossing-symmetric part $\phi_1^{(+)}(s,t,u)$ of the t-channel singlet amplitude (5).

$$\phi_1^{(\pm)}(s,t,u)=\tfrac{1}{2}[\phi_1(s,t,u)\pm\phi_1(u,t,s)]. \quad (13)$$

The residue of this pole,

$$\tilde{\phi}_1^{(+)}(s,t,u)=(t-\mu^2)\phi_1(s,t,u), \quad (14)$$

is related to the πN coupling constant

$$\lim_{t\to\mu^2}\tilde{\phi}_1^{(+)}(s,t,u)=\tfrac{1}{2}\mu^2g^2. \quad (15)$$

Only objects with unnatural parity and $I^G=1^+$ (e.g., the B meson) can be exchanged in the antisymmetric part $\phi_1^{(-)}(s,t,u)$.

From Eqs. (5) and (11), two simple inequalities can

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[11] This point is reached by a linear extrapolation $\alpha_C(t)=-0.25+1.9t$ of (8), for $t>0$.

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be seen:

$$2\pi s(s-4M^2)d\sigma/dt(pn\to np)\geqslant 2|\phi_1^{(+)}+\phi_1^{(-)}|^2, \quad (16)$$

$$2\pi s(s-4M^2)d\sigma/dt(p\bar{p}\to n\bar{n})\geqslant 2|\phi_1^{(+)}-\phi_1^{(-)}|^2. \quad (17)$$

Inserting the $pn\to np$ and $p\bar{p}\to n\bar{n}$ data, a restriction shown in Fig. 1 (dotted curve) is obtained for the π residue function (14):

$$\tfrac{1}{4}\Sigma(t)\geqslant|\tilde{\phi}_1^{(+)}(s,t,u)|^2/|t-\mu^2|^2. \quad (18)$$

Obviously, the connection with the πN coupling constant (15) requires an extremely strong increase of the π residue function $\tilde{\phi}_1^{(+)}(s,t,u)$. Within the Toller-pole model, this behavior must be reproduced by the π residue function (if the π is assumed to be a class-III or class-II conspirator). Starting from invariant functions, we have the following situation. The π pole appears only in the s-u crossing-symmetric part $H_5^{(+)}(s,t,u)$ of the invariant function $H_5(s,t,u)$. The π trajectory may also occur in the s-u crossing-antisymmetric part $H_3^{(-)}(s,t,u)$ of the invariant function $H_3(s,t,u)$. However, there is no pole at $t=\mu^2$ in $H_3^{(-)}(s,t,u)$. The πN coupling constant is related only to the residue

$$\lim_{t\to\mu^2}\tilde{H}_5^{+}(s,t,u)=\lim_{t\to\mu^2}(t-\mu^2)H_5^{(+)}(s,t,u)=-\tfrac{1}{2}g^2. \quad (19)$$

Two inequalities can be derived for $\tilde{H}_5^{(+)}(s,t,u)$ in the approximation $|t/4M^2|\ll 1$, independently of all other contributions to the differential cross section (11):

$$2\pi s(s-4M^2)d\sigma/dt(pn\to np)$$
$$\geqslant[t/(t-\mu^2)]^2|\tilde{H}_5^{(+)}+H_5^{(-)}(t-\mu^2)|^2, \quad (20)$$

$$2\pi s(s-4M^2)d\sigma/dt(p\bar{p}\to n\bar{n})$$
$$\geqslant[t/(t-\mu^2)]^2|\tilde{H}_5^{(+)}-H_5^{(-)}(t-\mu^2)|^2, \quad (21)$$

which give the sum

$$\tfrac{1}{2}\Sigma(t)\geqslant[t/(t-\mu^2)]^2[|\tilde{H}_5^{(+)}(s,t,u)|^2$$
$$+|H_5^{(-)}(s,t,u)(t-\mu^2)|^2]. \quad (22)$$

Figure 1 (dashed curve) shows the bound (22) for $H_5^{(-)}=0$. However, this bound cannot be reached, since the inequality (20) yields a lower bound in this case [cf. Fig. 1 (solid curve)].

It should be emphasized that, up to this point, our considerations have been model-independent. In any case, the bounds (20) and (22) and the connection (19) to the πN coupling constant require a strongly decreasing pionic form factor of the nucleon.

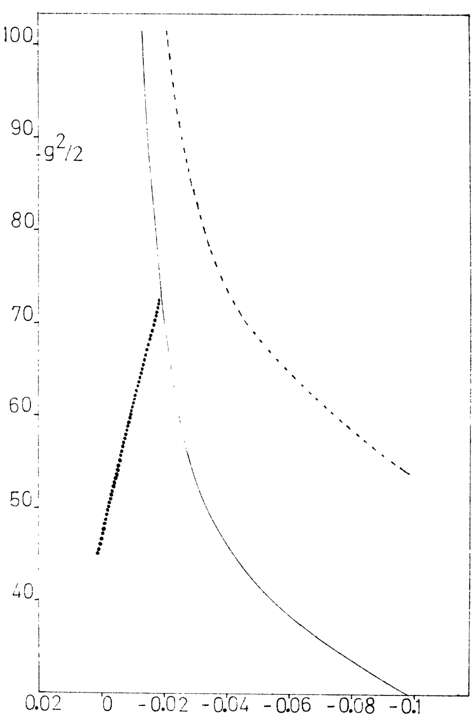

FIG. 1. Bounds for the π residue functions, calculated from the $pn\to np$ and $p\bar{p}\to n\bar{n}$ data for $E_{\rm lab}=8$ GeV. Dotted curve: $|\phi_1^{(+)}(s,t,u)|/\mu^2=\frac{1}{2}[\Sigma(t)]^{1/2}(\mu^2-t)/\mu^2$ [cf. Eq. (18)]; dashed curve: $|\tilde{H}_5^{(+)}(s,t,u)|=(1/\sqrt{2})[\Sigma(t)]^{1/2}(t-\mu^2)/t$ [cf. Eq. (22)]; solid curve: $|H_5^{(+)}(s,t,u)|=[2\pi s(s-4M^2)d\sigma/dt(pn\to np)]^{1/2}\times(t-\mu^2)/t$ [cf. Eq. (20)]. For $t=\mu^2$, the residue functions are related to the πN coupling constant [cf. Eqs. (15) and (19)].

III. THREE-POLE MODEL FOR CHARGE-EXCHANGE PROCESSES $pn\to np$ AND $p\bar{p}\to n\bar{n}$

We are restricting ourselves to the contributions of the π, B, and C trajectories. The last is an $I^G=1^-$ conspirator.

$$\alpha_\pi(t)=\alpha_5^{(+)}(t), \quad (23)$$

$$\alpha_B(t)=\alpha_3^{(+)}(t)+1, \quad (24)$$

$$\alpha_C(t)=\alpha_3^{(-)}(t)+1. \quad (25)$$

The invariant functions $H_i^{(\pm)}(s,t,u)$, $i=3, 5$, are Reggeized like the amplitudes in the spinless case:

$$H_i^{(\pm)}(s,t,u)=H_i^{(\pm)}(t)\Gamma(-\alpha_i^{(\pm)}(t))$$
$$\times\{1\pm\exp[-i\pi\alpha_i^{(\pm)}(t)]\}(s'|s_0)^{\alpha_i^{(\pm)}(t)}, \quad (26)$$

where

$$s'=s+\tfrac{1}{2}t-2M^2. \quad (27)$$

Then the sum (2) and the difference (3) can be written as follows:

$$\Sigma(t)=4\{[X_C(1+\cos\pi\alpha_C)+tX_\pi(1+\cos\pi\alpha_\pi)/\alpha_\pi]^2$$
$$+[X_C\sin\pi\alpha_C+tX_\pi\sin\pi\alpha_\pi/\alpha_\pi]^2$$
$$+4X_B^2(1-\cos\pi\alpha_B)+2X_C^2(1+\cos\pi\alpha_C)\}, \quad (28)$$

$$\Delta(t)=8X_B\{2X_C[1+\cos\pi\alpha_C-\cos\pi(\alpha_B-\alpha_C)-\cos\pi\alpha_B]$$
$$+tX_\pi[1+\cos\pi\alpha_\pi-\cos\pi(\alpha_B-\alpha_\pi)$$
$$-\cos\pi\alpha_B]/\alpha_\pi\}, \quad (29)$$

where

$$X_C = 2s_0 H_3^{(-)}(t)\Gamma(-\alpha_C(t)+1)(s'/s_0)^{\alpha_C(t)}, \quad (30)$$

$$X_B = 2s_0 H_3^{(+)}(t)\Gamma(-\alpha_B(t)+1)(s'/s_0)^{\alpha_B(t)}, \quad (31)$$

$$X_\pi = H_5(t)\Gamma(-\alpha_\pi(t)+1)(s'/s_0)^{\alpha_\pi(t)}. \quad (32)$$

The bound (21) (Fig. 1, solid curve) leads us, for the π residue function, to the parametrization

$$H_5^{(+)}(t)/\alpha_\pi' = -\tfrac{1}{4}[(t_0-\mu^2)/(t_0-t)]^2 g^2, \quad (33)$$

where $t_0 = 13.5\mu^2$.

As mentioned in the Introduction, the strong decrease of the sum can be understood by a compensation between the conspiratorial and evasive parts:

$$X_C(1+\cos\pi\alpha_C)+tX_\pi(1+\cos\pi\alpha_\pi)/\alpha_\pi. \quad (34)$$

On the other hand, the two contributions yield an increasing difference $\Delta(t)$, given opposite signs for the signature factors

$$1+\cos\pi\alpha_C-\cos\pi(\alpha_B-\alpha_C)-\cos\pi\alpha_B \quad (35)$$

and

$$1+\cos\pi\alpha_\pi-\cos\pi(\alpha_B-\alpha_\pi)-\cos\pi\alpha_B. \quad (36)$$

This follows from the assumed sequence (9) of the trajectories. The small value $\Delta(0)$ can be reproduced by a small signature factor (35) at $t=0$, i.e., at a small difference $\alpha_B(0)-\alpha_C(0)$. The choice of the intercepts $\alpha_B(0)$ and $\alpha_C(0)$ in (7) and (8) is suggested by the s dependence of the photoproduction processes (4).

The large differences in the slopes α_π', α_B', and α_C' [Eqs. (6)–(8)] lead to increasing signature factors (35) and (36) and in this way support the increase of the difference (3).

We have tried to fit the data with a linear C trajectory

$$\alpha_C(t) = -0.25+1.9t$$

rather than the Pignotti form (8). The sum (28) remains unchanged. However, the difference (29) is $\sim$10–30% smaller for larger values of t ($-0.2 \geqslant t \geqslant -0.5$). This results from a change in the sign of the signature factor (35) at $\alpha_C=-1$. One could argue that this deficit in the difference $\Delta(t)$ may be compensated for by ρ and R contributions. However, all interference terms which enter into $\Delta(t)$ are very small: ρR because of the smallness of the difference $\alpha_\rho - \alpha_R$; ρC because $\alpha C \approx -1$ for $t \approx -0.4$; and $\rho\pi$, which is exactly zero.

The only relevant interference term may be the RB one. Thus a considerable increase of the difference can be achieved only by large ρ and R residues, which enter quadratically in the sum. The critical quantity, however, is the ratio $\Sigma(t)/\Delta(t)$. For example, the data for $E_{\rm lab}=8$ GeV give

$$\Sigma(t)/\Delta(t)=1.6 \quad \text{for} \quad -0.2 \geqslant t \geqslant -0.5.$$

This value cannot be reproduced with large ρ and R contributions.

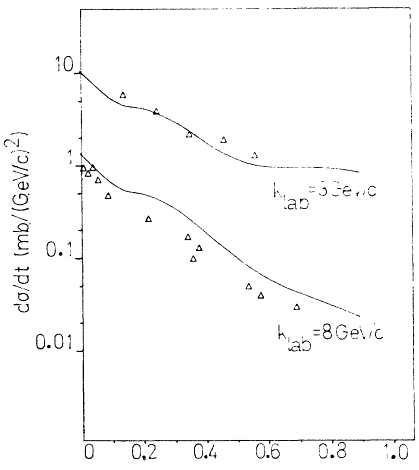

FIG. 2. Fit for the $pn \to np$ differential cross section at 3 and 8 GeV/c.

Figures 2 and 3 show our fits for the $pn \to np$ and $p\bar{p} \to n\bar{n}$ data. The scaling factor s_0 is assumed to be M^2 for all trajectories; for the πN coupling constant in Eq. (33) we took the correct value $g^2/4\pi = 14$. The masses of the resonances π, B, and (A_2, A_1) fix one

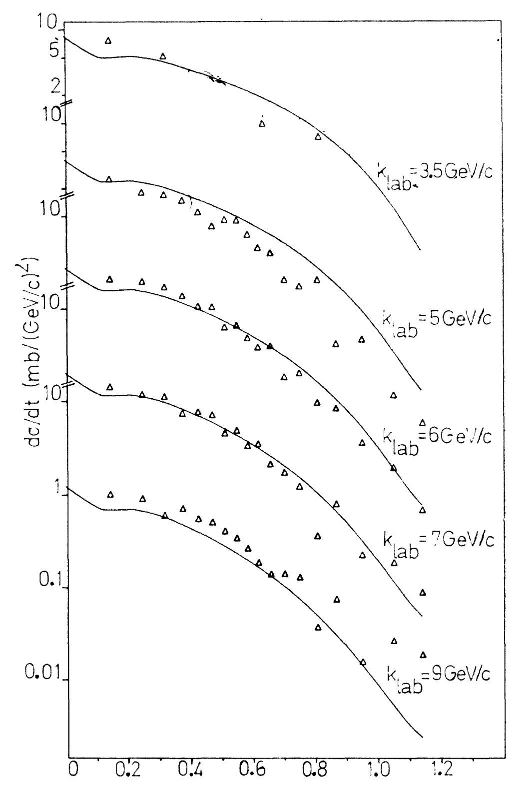

FIG. 3. Fit for the $p\bar{p} \to \bar{n}n$ differential cross section at 3.5, 5, 7, and 9 GeV/c.

parameter of the trajectories:

$$\alpha_\pi(\mu^2)=0\,,\quad \alpha_B(m_B{}^2)=1\,,\quad \alpha_C(m_{A_{2L},A_1}{}^2)=2\,.$$

In addition we used the six parameters

$$\alpha_\pi'(0)=0.1\,,\qquad t_0=13.5\mu^2\,,$$
$$\alpha_C'(0)=1.9\,,\quad H_3{}^{(-)}=24.49\ \text{mb}^{1/2}/(\text{GeV}/c)\,,$$
$$\alpha_B'(0)=1.0\,,\quad H_3{}^{(+)}=-14.85\ \text{mb}^{1/2}/(\text{GeV}/c)\,.$$

The $pn\rightarrow np$ data seem to be more energy-dependent than the $p\bar{p}\rightarrow n\bar{n}$. Much like Arbab and Dash, we were not able to reproduce this behavior correctly.[14] However, it should be emphasized that there are still systematic errors in the data. According to Refs. 1 and 2, the systematic normalization error of the 8-GeV/c $pn\rightarrow np$ data is 30–45%, and that of the $p\bar{p}\rightarrow n\bar{n}$ data is 15%.

IV. CONCLUSION

We have succeeded in giving a simple description for the structure of the $pn\rightarrow np$ and $p\bar{p}\rightarrow n\bar{n}$ charge-exchange processes. One strongly varying contribution —i.e., the contribution of the π pole—is available to explain two pronounced features:

(i) The sharp forward peak of the $pn\rightarrow np$ differential cross section. This is reproduced by subtracting the strongly increasing contributions of the evasive π trajectory from the large contributions of the conspirator C.

(ii) The forward dip of the difference $d\sigma/dt(p\bar{n}\rightarrow n\bar{n})-d\sigma/dt(pn\rightarrow np)$. This results from the increasing π-B and conspirator-B interference terms.

The similar structure of the photoproduction processes $\gamma p\rightarrow n\pi^+$ and $\gamma n\rightarrow p\pi^-$ is explained by the same trajectories π, B, and C. The $I^G=1^-$ conspirator $\alpha_C(t)$ with intercept $\alpha_C(0)=-0.25$ and slope $\alpha_C'(0)=1.9$ predicts (i) no resonances for $\alpha_C(t)=0$, (ii) a quintet $j^p=2^+, 2^-, 1^+, 0^+, 0^-$ for $\alpha_C(t)=2$, i.e., $t=1.19$ (GeV)^2, and (iii) a quintet $j^p=4^+, 4^-, 3^+, 2^+, 2^-$ for $\alpha_C(t)=4$, i.e., $t=2.3$ GeV^2. Candidates for the first quintet are A_{2L}, $A_{1.5}$, A_1, π_N, and δ; for the second π_A.

The Regge-pole model for invariant functions provides a simple method to reproduce conspiracy effects. Only one trajectory and one residue function are needed. The number of the parameters is much smaller than in the Toller-pole model, where two trajectories and three residue functions enter into the description of a class-III conspiracy.

ACKNOWLEDGMENTS

We would like to thank Professor F. Penzlin, Professor W. R. Theis, Dr. H. Kleinert, Dr. H. Meldner, and Dr. E. Tränkle for useful discussions.

[14] Arbab and Dash have multiplied (i) their $pn\rightarrow np$ calculations for 3 and 8 GeV by 1.0 and 0.75, and (ii) their $p\bar{p}\rightarrow n\bar{n}$ calculations for 5 and 6, 7, and 9 GeV/c by 1.0 and 1.115, respectively. Our calculations are not normalized.

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Regge-Pole Model for Invariant Functions. III. Photoproduction for Unpolarized and Linearly Polarized Photons

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(Received 20 January 1969)

It was shown in Paper II that the three Regge-type poles π, B, and C reproduce the $pn\rightarrow np$ and $p\bar{p}\rightarrow n\bar{n}$ data. The same trajectories are used in this paper to give a description of the $\pi^\pm$ photoproduction near the forward direction ($t\approx 0$). For larger values of $|t|$, however, inclusion of the ρ and R trajectories become necessary.

I. INTRODUCTION

It is well known that the Reggeization procedure for helicity amplitudes is complicated by kinematical singularities and constraints. Therefore, we have proposed[1] in Paper I to Reggeize the invariant functions $F_i(s,t,u)$ in a decomposition of the scattering matrix in terms of standard covariants[2] Q_i:

$$T=\sum_i F_i(s,t,u)Q_i\,. \tag{1}$$

This expansion reproduces the kinematics exactly. Therefore, the invariant functions are free of kinematical singularities and constraints and satisfy the Mandelstam representation. The simple Reggeization procedure of the spinless case can be applied. The physical interpretation of a Regge-type pole $j=\alpha_i(t)$ in the partial-wave amplitudes

$$F_i(j,t)=\int_{-1}^{1} dz\, F_i(s,t,u)P_j(z)$$

[1] K. H. Mütter, Nucl. Phys. **B8**, 311 (1968).

[2] K. Hepp, Helv. Phys. Acta **36**, 355 (1963); K. Hardenberg, K. H. Mütter, and W. R. Theis, Nuovo Cimento **62**, 385 (1969); H. Kleinert and K. H. Mütter, *ibid.* (to be published).