Scattering of π^- Mesons in the Momentum Range 0.643–2.14 GeV/c from a Polarized Proton Target

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The asymmetry in the scattering of π^- mesons by polarized protons has been measured at 50 different momenta from 0.643 to 2.14 GeV/c. Results were obtained at values of $\cos\theta$ ranging from approximately $+0.9$ to -0.95 in the c.m. system at each incident pion momentum. The pion beam was incident on a 7.6-cm-long crystal assembly of lanthanum magnesium nitrate, in which the hydrogen in the water of crystallization was polarized by the "solid effect." The total momentum spread of the beam was 10% (full width at half-height) and data were collected simultaneously in 4 momentum channels, each with $2\frac{1}{2}\%$ full width at half-height. A gas Čherenkov counter was used to reject incoming electrons. Scattered particles were detected in scintillation counter arrays placed within the 10-cm gap of the polarized target magnet. Encoded information from each array was stored in the memory of a PDP-5 computer connected on-line to a fast electronic logic network. The computer was programmed to classify the events according to momentum and scattering angle and subdivide them into coplanar and noncoplanar categories. The latter provided a measure of the background. The results have been expressed in the form of an expansion in terms of first associated Legendre polynomial series and compared with the predictions of recent phase-shift solutions. It is concluded that although these analyses give satisfactory predictions of the general features of the results, no one solution gives complete agreement with the data above about 1.0 GeV/c.

I. INTRODUCTION

It has been known for many years that the pion-nucleon system exhibits several resonant states in the momentum range of the present experiment. The evidence for this came initially from measurements of total cross sections which defined the isotopic spin of each resonance and provided values of their masses and widths. The most recent set of total cross-section measurements has been made by Carter *et al.*[1] and is shown in Fig. 1. Although such measurements as these provide some indication of the main experimental features of the scattering process, further information is needed before the spins and parities of the resonant states can be firmly established and even before the list of resonances can in any way be regarded as complete. This further information has been provided by:

(a) measurements of differential cross sections for elastic[2–7] and charge exchange[8–10] scattering,

(b) measurements of the polarization of the recoil proton for elastic scattering,[11,12]

(c) asymmetry measurements, of the kind to be

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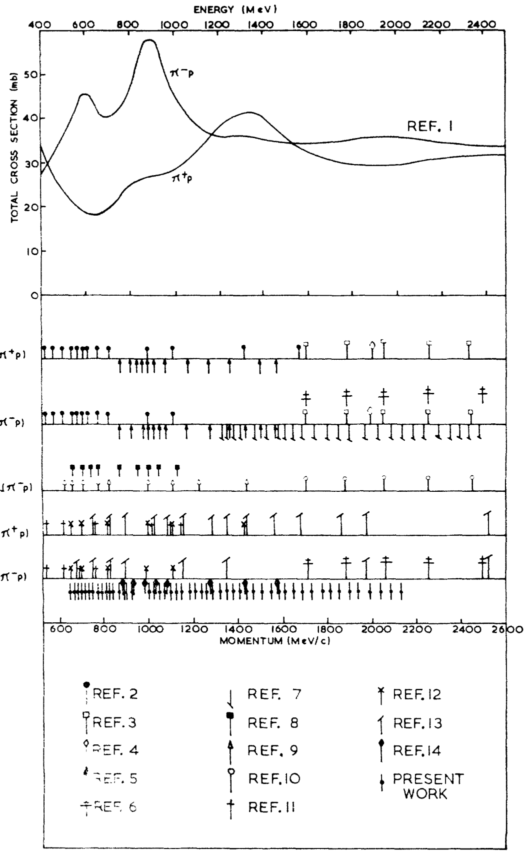

FIG. 1. Total π^-p and π^+p cross sections (see Ref. 1) showing momenta at which differential cross section and polarization or asymmetry measurements have been made. For details of these measurements, see Refs. 2–14.

described here, for elastic scattering from polarized protons.[6,13,14]

In fact, identical information is obtained from both (b) and (c) but, for technical reasons, those experiments listed under (c) have considerably greater statistical weight.

The experimental values of the quantities measured in (a) to (c) can be fitted with Legendre polynomial series to obtain a unique set of expansion coefficients whose behavior, as a function of momentum, can be related to that of the partial-wave amplitudes themselves. In the past, the study of these coefficients has revealed the spins and parities of some of the resonant states and has demonstrated the presence of other resonances whose existence could not have been deduced from the total cross-section measurements.[14] Unfortunately, it is only the highest-order coefficients which yield useful information. The values of these coefficients are determined by the highest angular momentum states involved, whereas the lower-order terms of the polynomial series contain contributions from many partial waves. It follows that a study of the expansion coefficients has a limited application and only a phase-shift analysis can hope to describe the behavior of all the angular momentum states. In order to resolve the Minami and, in some cases, the Yang ambiguities[15] in the phase-shift solutions, the experimental data for such an analysis must contain polarization or asymmetry measurements. In addition, these data have not, in general, been sufficiently precise to yield a unique solution at any one momentum. Some success has been achieved by invoking the requirement of continuity for each phase shift,[16,17] which requires that the measurements should be made at closely spaced values of the incident pion momentum.

The purpose of this experiment was to measure the asymmetry in π^-p elastic scattering on polarized protons as a function of momentum. Results are presented for 50 momenta between 0.643 and 2.14 GeV/c inclusive, with intervals between measurements varying from about 20 MeV/c at the lowest to about 50 MeV/c at the highest momentum in the range. At almost all the momenta, $\cos\theta$ ranged from about $+0.9$ to -0.95, θ being the scattering angle in the c.m. system. Figure 1 indicates the relationship between this experiment and earlier work. It can be seen that prior to these measurements, for π^-p scattering above about 1.2 GeV/c, asymmetry or polarization had only been studied at quite widely spaced momenta, except from 0.875 to 1.03 GeV/c,[14] where the spacing between measurements was about 50 MeV/c. The latter data were first included in a phase-shift analysis by Bareyre *et al.*,[17] which clarified the behavior of the partial-wave amplitudes in that region and revealed the existence of several new resonances. This analysis, which demonstrated the importance of careful measurements at closely spaced momenta, provided the initial stimulus for the work presented here.

II. THEORY OF PION-NUCLEON SCATTERING

The formulas given in this section have all been presented elsewhere (see, for example, Ref. 14). They are given here for completeness and for clarity in the subsequent discussion.

The hypothesis of charge independence permits the expression of any pion-nucleon scattering amplitude M

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[15] See, e.g., H. A. Bethe and F. de Hoffman, *Mesons and Fields* (Row, Peterson and Company, New York, 1955), Vol. II, pp. 72ff.

[16] P. Bareyre, C. Bricman, and G. Villet, Phys. Letters **18**, 342, (1965); P. Bareyre, C. Bricman, and G. Villet, Phys. Rev. **165**, 1730 (1968). The latter publication gives references to much earlier work.

[17] A. Donnachie, R. G. Kirsopp, and C. Lovelace, Phys. Letters **26B**, 161 (1968).

as a linear combination of two amplitudes M_1 and M_3 for states with total isotopic spin $I=\frac{1}{2}$ and $I=\frac{3}{2}$, respectively. The particular combination, which yields the amplitude M for $\pi^- p$ elastic scattering, is

$$M=\tfrac{1}{3}M_3+\tfrac{2}{3}M_1. \tag{1}$$

In order to take account of the two spin states of the nucleon, the amplitude M may itself be written in terms of two components f and g, which are functions of θ, such that,

$$M=f+ig\boldsymbol{\sigma}\cdot\hat{n}. \tag{2}$$

In (2), the $\boldsymbol{\sigma}$ are the Pauli spin matrices and $\hat{n}$ is a unit vector normal to the scattering plane defined by

$$\hat{n}=(\mathbf{k}_i\times\mathbf{k}_f)/(|\mathbf{k}_i\times\mathbf{k}_f|), \tag{3}$$

where $\mathbf{k}_i$ and $\mathbf{k}_f$ are the initial and final momenta of the pion (or the nucleon) in the c.m. system. The f and g are related to f_I and g_I for pure isotopic spin states by a linear relationship with the same coefficients as Eq. 1.

The f_I and g_I are related to the partial-wave amplitudes $A_{I,l+}$ and $A_{I,l-}$ for orbital angular momentum l and total angular momentum $J=l\pm\frac{1}{2}$, by

$$f_I(\theta)=\sum_{l=0}^{l_{\max}}[(l+1)A_{I,l+}+lA_{I,l-}]P_l^0(\cos\theta), \tag{4}$$

$$g_I(\theta)=\sum_{l=1}^{l_{\max}}[A_{I,l+}-A_{I,l-}]P_l^1(\cos\theta), \tag{5}$$

where $P_l^0(\cos\theta)$ and $P_l^1(\cos\theta)$ are the Legendre and first associated Legendre polynomials. The $A_{I,l\pm}$ are, in general, complex quantities which may be expressed in the form

$$A_{I,l\pm}=(\lambda\!\!\!^{-}/2i)[\eta_{I,l\pm}\exp(2i\delta_{I,l\pm})-1]. \tag{6}$$

The parameters $\eta_{I,l\pm}$ and $\delta_{I,l\pm}$ are both real. The former expresses the absorption of the partial wave in the scattering process, and is restricted by unitarity to

$$0\leqslant\eta_{I,l\pm}\leqslant 1. \tag{7}$$

The second parameter $\delta_{I,l\pm}$ is the phase shift, and $\lambda\!\!\!^{-}$ is given by

$$\lambda\!\!\!^{-}=\hbar/|\mathbf{k}_i|. \tag{8}$$

The f_I and g_I are not only related to the partial-wave amplitudes through Eqs. (4) and (5) but also to the experimentally measured quantities. In particular, the differential cross section $(d\sigma/d\Omega)_P$ for the scattering of pions on an assemblage of protons with polarization vector $\mathbf{P}_T$ is given by

$$(d\sigma/d\Omega)_P=|f|^2+|g|^2-(2\,\mathrm{Im}f^*g)\mathbf{P}_T\cdot\hat{n}. \tag{9}$$

In the case where $\mathbf{P}_T$ is zero, this reduces to

$$(d\sigma/d\Omega)=|f|^2+|g|^2 \tag{10}$$

for scattering from unpolarized protons.

The scattering asymmetry ϵ is defined by

$$\epsilon=[(d\sigma/d\Omega)_{P_+}-(d\sigma/d\Omega)_{P_-}]/[(d\sigma/d\Omega)_{P_+}+(d\sigma/d\Omega)_{P_-}], \tag{11}$$

in which the subscripts P_+ and P_- refer to states of target polarization $\pm|\mathbf{P}_T|$, the direction of $\mathbf{P}_T$ being parallel to $\hat{n}$. Equations (9)–(11) may then be combined to give

$$(\epsilon/|\mathbf{P}_T|)(d\sigma/d\Omega)=-(2\,\mathrm{Im}f^*g). \tag{12}$$

III. EXPERIMENTAL METHOD

The work described here used the same apparatus as the asymmetry measurements in K^-p elastic scattering by the present authors.[18] In Ref. 18, which will be denoted by I, full details are given of the beam transport equipment, polarized-proton target, scintillation counters, electronics, on-line computing, and data analysis. In this section of the present paper, the method of rejection of electrons from the incident beam will be described. The results themselves will be presented and discussed in Secs. IV and V.

It has been demonstrated by Chamberlain *et al.*[13] that electron-positron pairs, arising from electromagnetic processes induced by beam electrons in the target assembly, can easily simulate coplanar scattering events. In the present experiment, the measured electron flux was $(8\pm2)\%$ of the total incident beam at 1.05 GeV/c. This percentage decreased with increasing beam momentum but even at the highest momenta of the experiment the electron to pion ratio was appreciable. These electrons were rejected by means of a threshold Čherenkov counter, filled with Freon 13 (chemical formula: $CClF_3$), which was placed near the intermediate focus of the pion beam (see Fig. 1 of I). This counter, which was arranged to be sensitive only to electrons, was placed in anticoincidence with counter C to provide the strobe pulse K, as described in Sec. VI of I. The resolution of the DISC[19] counter, which was used in I to select kaons, was insufficient to distinguish pions from electrons. It was, therefore, not used in the experiment described here.

IV. RESULTS

The values of the quantity $\epsilon(\theta)$, defined in Eqs. (23) and (24) of I, are shown graphically in Fig. 2.[20] Only the statistical errors are shown in the figure and there is an additional scale error of $\pm10\%$ which is a combination of the following effects:

[18] C. R. Cox, P. J. Duke, K. S. Heard, R. E. Hill, W. R. Holley, D. P. Jones, F. C. Shoemaker, J. J. Thresher, J. B. Warren, and J. C. Sleeman, preceding paper, Phys. Rev. **184**, 1443 (1969).

[19] R. Meunier, J. P. Stroot, B. Leontic, A. Lundby, and P. Duteil, Nucl. Instr. Methods **17**, 20 (1962).

[20] Tables of numerical values of these results are available as Rutherford Laboratory Report No. RHEL/M/137 (unpublished). These tables are also available as NAPS Document 00500 from ASIS National Auxiliary Publications Service, c/o CCM Information Science Inc., 909 3rd Ave., New York, N. Y. 10022; by remitting $1 for microfiche or $3 for photocopies.

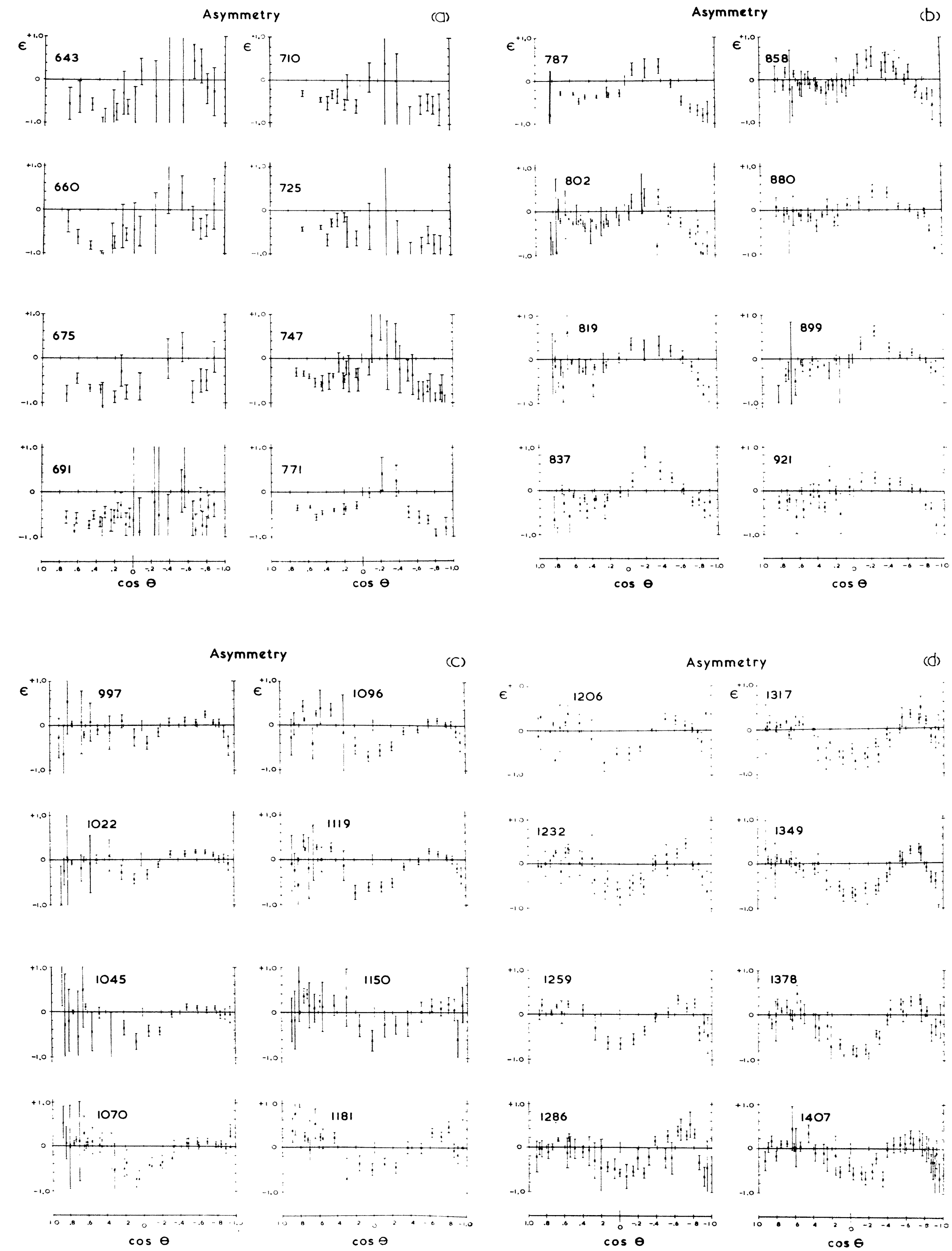

FIG. 2. Measured values of $\epsilon(\theta)$ for $\pi^- p$ scattering from 0.643 to 2.14 GeV/c: (a) 0.643–0.771, (b) 0.787–0.921, (c) 0.997–1.181, (d) 1.206–1.407, (e) 1.438–1.689, and (f) 1.733–2.14 GeV/c. Only statistical errors are shown.

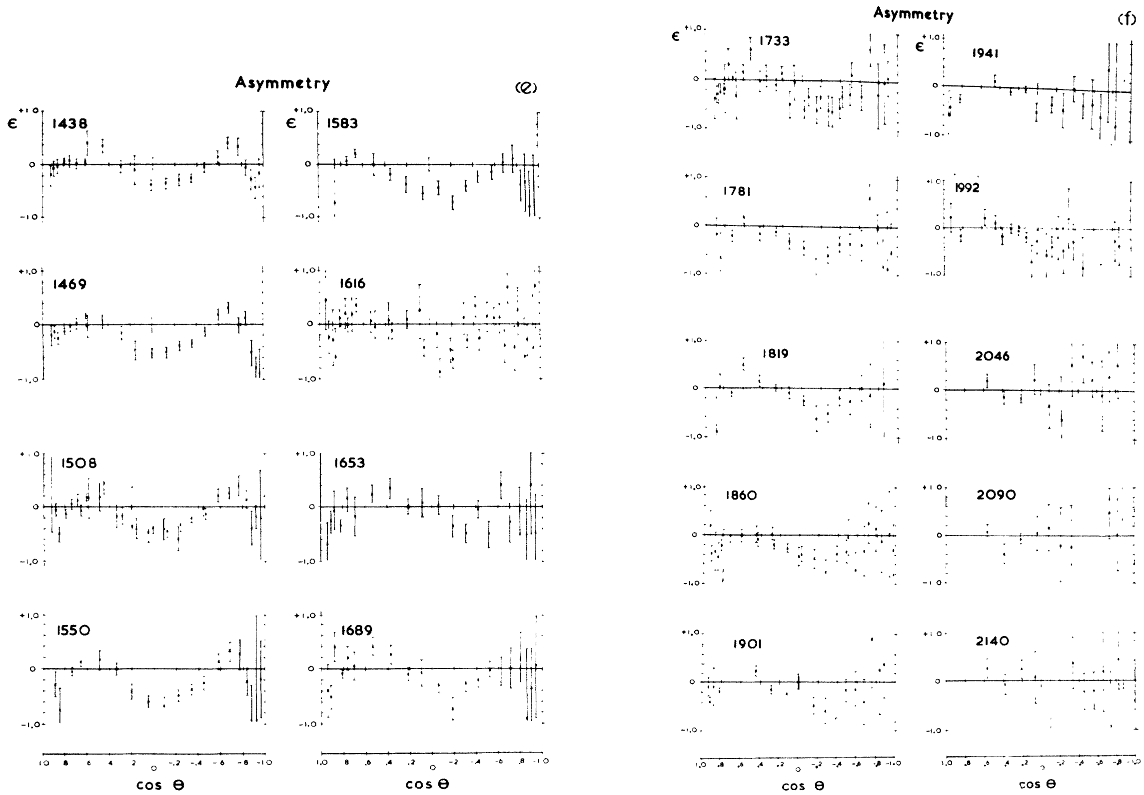

FIG. 2 (*continued*)

(a) The data used in the calibration of the target polarization[21] had a scale uncertainty of $\pm 7\%$.

(b) The measurements of proton-proton scattering (described in Sec. V of I) which were used to calibrate the polarization of the target introduced an additional error of $\pm 6\%$ in the target polarization.

(c) The error in the NMR monitoring system amounted to $\pm 2\%$.

(d) It was found, by a careful examination of the noncoplanar events, that the contamination of free hydrogen events in the background data amounted to $(5\pm 3)\%$ of the coplanar scattering rate from free protons. This was consistent, within the measuring errors, with multiple scattering of the outgoing particles in the target material and the walls of the cryostat. The same contamination was also detected in the proton-proton elastic scattering results. Because of the form of the defining equations for $\epsilon(\theta)$, the combination of these two corrections produced an effect on the asymmetry values which was much less than the statistical errors. However, the uncertainty in this correction increased the scale error by $\pm 4\%$.

The beam momenta were determined to $\pm\frac{1}{2}\%$ by the floating-wire technique. The widths of the momentum channels shown in Fig. 2 are a combination of the momentum dispersion at the intermediate focus and the statistical nature of the momentum losses in the beam counters and the target crystal.

The width of the intervals in $\cos\theta$ have been discussed in I. They were typically ± 0.06 for the central region and ± 0.04 for the outer edges of the angular distribution.

At most momenta, the asymmetries shown are a weighted mean of several sets of data taken at different times during the experiment. For a few of these there was a significant difference in the angle of incidence of the pion beam and in those cases the separate data sets have not been averaged.

V. DISCUSSION OF RESULTS

A. Legendre Polynomial Coefficients

A convenient way of summarizing the results of this experiment and of making a comparison with other work is to express them as an expansion in terms of first associated Legendre polynomials in the form

$$\epsilon\frac{d\sigma}{d\Omega}=\lambda\!\!\!^{-2}\sum_{n=1}^{n_{\max}} D_n P_n{}^1(\cos\theta)\,. \tag{13}$$

In (13), the D_n are real coefficients, which are to be

[21] P. G. McManigal, R. D. Eandi, S. N. Kaplan, and B. J. Moyer, Phys. Rev. **137**, B620 (1965); F. Betz, J. Arens, O. Chamberlain, H. Dost, P. Grannis, M. Hansroul, L. Holloway, C. Schultz, and G. Shapiro, *ibid.* **148**, 1289 (1966).

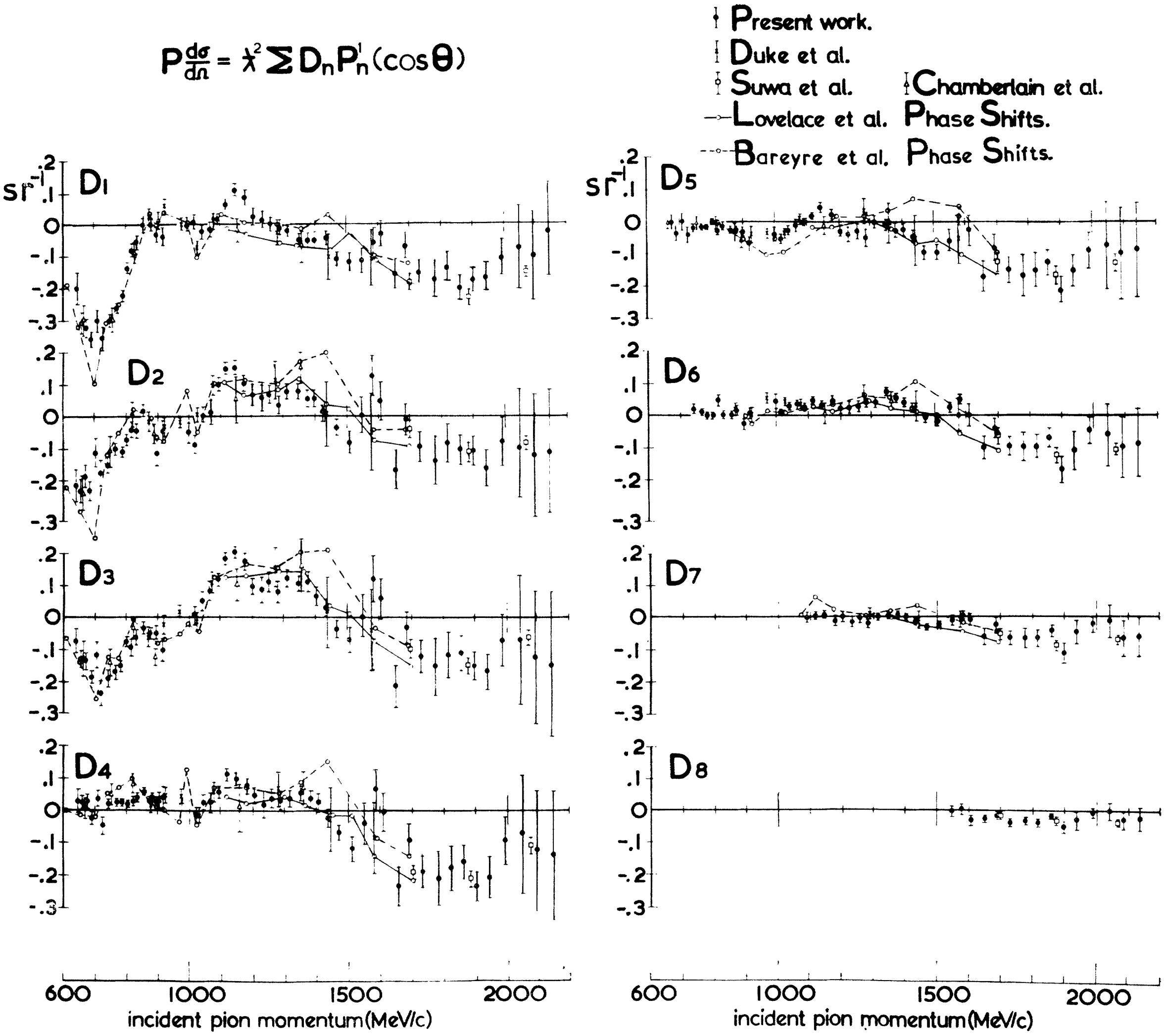

FIG. 3. The coefficients D_n of the first associated Legendre polynomials in the expansion

$$\epsilon(\theta)(d\sigma/d\Omega) = \lambda^2 \sum_{n=1}^{n_{\max}} D_n P_n^1(\cos\theta).$$

The figure also shows the experimental results of Refs. 6, 13, and 14, and values of D_n calculated from the phase-shift analyses of Refs. 16 and 17.

determined from a least-squares fit, of order $n_{\max}$, to the experimental values of $\epsilon\times(d\sigma/d\Omega)$. In this work, only ϵ was measured and it was necessary to obtain $d\sigma/d\Omega$ from previously published data, using a linear interpolation based on the coefficients of the polynomial fits given by Helland *et al.*,[2] Bellamy *et al.*,[3] Duke *et al.*,[5] and Suwa *et al.*[6] This procedure is open to two objections:

(a) The errors on the values of $d\sigma/d\Omega$ may not have been correctly determined because the full error matrix for the polynomial coefficients was not, in general, available. However, the uncertainties in the asymmetry data were sufficiently large that they completely dominated the final error on the quantity of interest.

(b) The method of interpolation may introduce an artificially smooth behavior into the values of $d\sigma/d\Omega$. In fact, as can be seen from Fig. 1, there exist measurements of π^-p differential cross sections at momentum intervals of 50 MeV/c or less up to 1.2 GeV/c and at intervals of about 100 MeV/c up to 1.6 GeV/c. In addition, Fig. 14 of Ref. 5 shows that the polynomial coefficients are in fact consistent with a smooth behavior up to that momentum. Above 1.6 GeV/c, the measurements of $d\sigma/d\Omega$ are more widely spaced so that this uncertainty can only be resolved by more detailed measurements.

The method of selecting the optimum order of fit has been described elsewhere (e.g., in Ref. 5) and will not be discussed again here. The values of D_n obtained by this procedure are given in Fig. 3. The figure also shows the results of Bareyre *et al.*,[11] Chamberlain *et al.*,[13] Duke *et al.*,[14] and Suwa *et al.*[6] There is, in general, good agreement between these earlier measurements and the present work.

From the coefficients presented in Fig. 3, the following observations may be made:

(a) An eighth-order fit is required from 1.6 GeV/c up to the highest momentum of this experiment. This fit gives values for the D_8 coefficient which are consistent with a contribution to the scattering from G and possibly H waves in this region.

(b) From 0.9 to 1.6 GeV/c, the variation of the coefficients with momentum is far from monotonic, which indicates the presence of important variations in the amplitude and phase of at least some of the partial waves in this region. It is interesting that seven of the nine new resonances proposed by Donnachie *et al.*[17] fall within this range as well as several of those more firmly established. In fact, over the total range of this experiment all partial waves up to $l=3$, and in both isotopic spin states, resonate at least once, if all the resonances listed by Donnachie *et al.*[17] are included. In view of this, it is also interesting that above 0.9 GeV/c all the coefficients, at least up to D_6, show a similar pattern of behavior, and there are no striking differences between the odd- and even-order coefficients such as would be expected if only a few well-separated resonant states were present in this region of momentum.

(c) Below 0.9 GeV/c, the known resonances are more widely separated and it is easier to relate the behavior of the coefficients to specific nucleon states. In particular, the dip in D_3, which reaches a minimum between 0.70 and 0.75 GeV/c, can be explained in terms of interference between the pair of resonances $D_{15}(1680)$ and $P_{11}(1470)$, together with interference between $D_{13}(1518)$ which resonates at 0.73 GeV/c and the real part of the $P_{33}(1236)$ amplitude which is still appreciable in this region. The minimum is even more apparent in D_1 and must be attributed to the $P_{33}D_{13}$ term as before, with an additional contribution from the D_{13} and the P_{11} resonances. Both D_1 and D_3 are formed from interference terms between angular momentum states of opposite parity, as given by Tripp.[22] By contrast D_2 is composed of products of amplitudes with the same parity and in the region under discussion the main contribution comes from the term containing $P_{11}(1470)$ and $P_{33}(1236)$, which is negative near the P_{11} resonance at 0.67 GeV/c and rises towards positive values above that momentum.

B. Partial-Wave Analysis

During the last few years, several phase-shift analyses of pion-nucleon scattering have been attempted. These analyses, which have been reviewed by Lovelace,[23] aim to provide values of $\eta_{I,l\pm}$ and $\delta_{I,l\pm}$ for each partial wave. These quantities are related to $\epsilon(\theta)$ by the equations given in Sec. II. A comparison has been made between the present data and three of the most recently published phase-shift solutions, namely, those of Bareyre *et al.*[16] and Donnachie *et al.*,[17] which will be referred to as SACL, CERN 1, and CERN 2, respectively. The solution CERN 2 is a dispersion relation iterative fit to the values of η and δ obtained in CERN 1. None of these solutions has used the data presented in this publication.

TABLE I. Comparison of the results of this experiment with predictions based on the phase-shift solutions of Bareyre *et al.* (Ref. 16), denoted by SACL, and Donnachie *et al.* (Ref. 17), denoted by CERN 1 and CERN 2. The last three columns of this table give the value of the goodness-of-fit parameter M defined in Eq. (14). For a good fit, the most probable value of M is numerically equal to the number of experimental points, denoted by N.

Momentum			M	
(MeV/c)	N	SACL	CERN 1	CERN 2
1120	23	260	61	···
1180	23	64	81	76
1290	34	64	110	130
1350	35	130	110	76
1440	23	61	30	78
1510	31	···	57	75
1580	18	66	43	30
1690	22	52	52	69

(a) Figure 3 shows values of the coefficients calculated using SACL and CERN 1. Below 1.05 GeV/c, only the SACL results are plotted since these are in good agreement with those of CERN 1. It can be seen that the phase-shift solutions agree well with the present experiment up to 1.05 GeV/c at which momentum the solutions diverge and the experimental results favor CERN 1. This continues to be the case up to about 1.6 GeV/c, above which momentum there is again good agreement between SACL, CERN 1, and the experimental data.

(b) A quantitative comparison of these solutions with the data of the present experiment between 1.12 and 1.69 GeV/c has been made by computing the goodness of fit of the solution to the experimental data. The goodness of fit parameter M is defined as

$$M=\sum_{i=1}^{N}\{[\epsilon(\theta)'_i-\epsilon(\theta)_i]/\Delta\epsilon(\theta)_i\}^2, \qquad (14)$$

in which N is the number of experimental points, $\epsilon(\theta)'_i$ is the value of the asymmetry calculated from the partial-wave solution and $\Delta\epsilon(\theta)_i$ is the error on the experimental point. The results of this comparison are shown in Table I. It can be seen that, in general, the predictions of CERN 2 give somewhat worse agreement

[22] R. D. Tripp, European Organization for Nuclear Research Report No. CERN 65-7 (revised) (unpublished).

[23] C. Lovelace, in *Proceedings of the Heidelberg International Conference on Elementary Particles, 1967* (North-Holland Publishing Company, Amsterdam, 1968), p. 79.

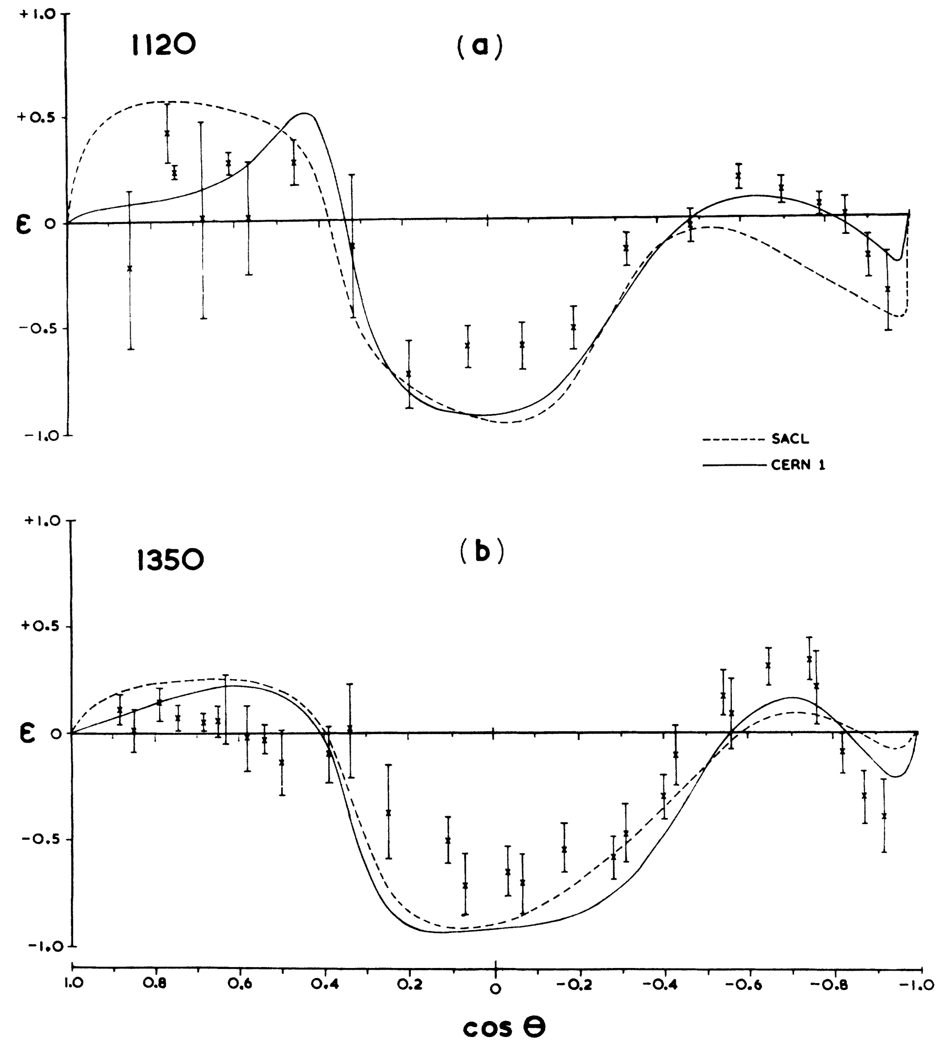

FIG. 4. Comparison between the experimental data and the predicted values for $\epsilon(\theta)$ based on the SACL and CERN 1 phase-shift solutions (Refs. 16 and 17) at (a) 1.12 and (b) 1.35 GeV/c.

with the experimental data than CERN 1. This is not too surprising because the former gives a smooth representation in which the fluctuations in η and δ, which are evident in CERN 1 (and also in SACL), are averaged out. However, the results of CERN 1 seem to fit the experimental data better than the SACL solutions as was also noted in the discussion of the Legendre coefficients. This difference in the value of M for SACL and CERN 1 is most marked at 1.12 GeV/c, but at 1.35 and at 1.69 GeV/c neither solution can be preferred. The solutions at 1.12 and at 1.35 GeV/c are compared with the experimental data in Fig. 4. Although in both cases the general shapes of the solutions are similar, they differ in their detailed predictions. At 1.12 GeV/c, the predictions are quite different for the behavior in both the forward and backward directions but they agree in the middle of the angular range. At 1.35 GeV/c, on the other hand, the two solutions themselves agree quite well in their predictions but both disagree with experiment in the region of $\cos\theta$ from +0.3 to 0.0. However, these disagreements are not large and the over-all behavior of the results is predicted quite well by the phase-shift solutions.

VI. CONCLUSION

Results for the asymmetry in $\pi^- p$ scattering from a polarized-proton target have been presented at 50 incident pion momenta from 0.64 to 2.14 GeV/c and a very brief comparison with existing phase-shift solutions has been made. Up to 1.1 GeV/c, the predictions of the analyses of Refs. 16 and 17 agree well with each other and with the experimental data. Above that momentum the results are not in complete agreement with any of the published solutions but they tend to favor the solutions of Donnachie *et al.*[17] However, no one set of solutions satisfies the entire data and a new solution, including these results, is required.

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Investigation of the Final States $K^-\pi^-\pi^+\pi^0p$ and $K^-\pi^-\pi^+\pi^+n$ Produced in K^-p Interactions at 4.6 and 5.0 BeV/c*

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We have studied the reactions $K^-p \to K^-\pi^-\pi^+\pi^0p$ and $K^-p \to K^-\pi^-\pi^+\pi^+n$ at 4.6- and 5.0-BeV/c incident K^- momenta. Production cross sections for the $K^-\pi^-\pi^-\pi^+\pi^0p$ state are 0.76±0.09 and 0.93±0.12 mb, and for the $K^-\pi^-\pi^+\pi^+n$ state 0.42±0.06 and 0.39±0.07 mb, at 4.6 and 5.0 BeV/c, respectively. The combined sample for the $K^-\pi^-\pi^+\pi^0p$ state contains 1305 events. Resonance contributions include: $\bar{K}^{*0}(890)$, (19±3)%; $K^{*-}(890)$, (7.2±1.7)%; $\Delta^{++}(1236)$, (14.5±2.5)%; $\Delta^+(1236)$, (8±3)%; $\Delta^0(1236)$, <2%; $\omega^0(784)$, (14.5±3)%; $\eta(548)$, (3.3±1.3)%; a 1060-MeV 3π effect identified as the $A_1^0(1080)$, (10.5±3)%; $A_2^0(1300)$, (3±1.5)%; $\rho^+(760)$, (9±2.5)%; $\rho^-(760)$, (5.5±2)%; and a ~960-MeV narrow 3π enhancement, (2.5±1)%. The A_1^0 is produced in this reaction in a way precluding contributions from the Deck effect. A spin-parity analysis for the A_1^0 provides good evidence that the spin-parity is in the sequence 0^-, 1^+, 2^-, ... Our results are consistent with the previously favored 1^+ assignment. We have 701 events in the combined sample for the $K^-\pi^-\pi^+\pi^+n$ final state. Contributions to this channel from accepted or suspected resonance states include: $\bar{K}^{*0}(890)$, (29±4)%; $\Delta^-(1236)$, (17±3)%; $\Delta^+(1236)$, (7±2)%; and a narrow 960-MeV $\pi^+\pi^+\pi^-$ effect, (6±2)%. Addition of $K^*\pi$ and $K\rho$ distributions from both final states corresponding to specific values of I_3 provides good evidence for production of $K_A(1320)$ and $K_N(1420)$. A similar treatment of $I_3=\frac{3}{2}$ $\Delta(1236)\pi$ and $N\rho(760)$ distributions indicates an effect near 1820 MeV.

I. INTRODUCTION

We here present results of an investigation of the following five-body final states observed in K^-p interactions at 4.6 and 5.0 BeV/c:

$$K^-\pi^-\pi^+\pi^0p\,, \quad (1)$$

$$K^-\pi^-\pi^+\pi^+n\,. \quad (2)$$

This paper deals only with the combined data from both momenta. No general investigations of these final states (at 4.6–5.0-BeV/c K^- momenta) have appeared in the literature. Certain aspects of reaction (1) at 3.8 BeV/c have been published previously.[1,2] Also, an analysis of these reactions at 4.1 BeV/c and, particularly, at 5.5 BeV/c is currently being undertaken by another group.[3]

In the present paper we provide a comprehensive review of our investigations of these reactions. In Sec. II we present pertinent details of our analysis procedures. Additional information on background estimates and on weighted phase space is given in Ap-

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