PHYSICAL REVIEW VOLUME 184, NUMBER 5 25 AUGUST 1969

Some Implications of a New Source of Cosmic-Ray Muons*

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(Received 17 Feburary 1969)

We study implications of the recent experiment by the University of Utah cosmic-ray group, which presents evidence for a new source of cosmic-ray muons. The data imply the existence of a new class of hadrons X, produced in pairs with a cross section $\gtrsim 0.3$ mb in primary cosmic-ray collisions of energy $>10^{12}$–10^{13} eV. The X, although stable under strong and electromagnetic interactions, decays with large branching ratio into states containing a muon and with lifetime $<10^{-7}$–10^{-8} sec. The total mass of the produced X pair is estimated to be less than 55 GeV. We examine the possible quantum numbers of X. We study possible new interactions of cosmic-ray muons and neutrinos underground. In particular, the muon electromagnetic field should photoproduce X; the cross section for this process is estimated to be $\sim 10^{-32}$ cm^2 and may lead to observation of pairs of muons under ground with small lateral separation or measurable angular divergence. The hypothesis $X=W=$ intermediate boson for the weak interactions is considered; the experimental limits on the production of W by neutrinos and muons underground, along the with absence of large ν-p elastic scattering at accelerator energies, place strong, and possibly fatal, constraints on this interpretation. The importance of the polarization of the muon beam underground in these considerations is pointed out. Finally, further experimental consequences of the existence of X are discussed.

I. INTRODUCTION

RECENTLY the Utah cosmic-ray group has presented evidence[1,2] for a new source of cosmic-ray muons of energy >1 TeV ($=10^{12}$ eV). Although this result has not yet been confirmed by an independent experiment, we shall here assume the experimental result to be correct and will study its implications, both theoretical and experimental. In this study we have been aided greatly by conversations with the Utah group, particularly with Keuffel, which we acknowledge with gratitude. Most of the ideas we present are already folklore (certainly within the Utah group),[3–6] and our purpose is to systematize and document in a semi-quantitative way, as best we can, theoretical options and possible further experimental consequences.

It is all too easy, in our opinion, for a theorist to quote the Utah experiment as possible evidence for the existence of some favorite hypothetical particle or interaction. However, there is a broad spectrum of such interpretations, and one experiment will not distinguish them. It is of the greatest importance to find other experimental consequences which are characteristic of all or some of these interpretations.

II. PHENOMENOLOGY

In brief, the Utah experiment examines the zenith-angle distribution, for a fixed depth, of cosmic-ray muons underground at slant depths of 2000–8000 hg cm^{-2} [1 hectogram (hg)$=100$ g]. This distribution should be[7] $\approx\sec\theta$ if the muons are decay products of π or K mesons, and should be constant if the muons are produced directly or as decay products of a short-lived parent. What is found[1] is a distribution less strong than $\sec\theta$, indicating a component of the latter type, which we call the X process.

In interpreting this result, we assume the extra X-process muons and the pions are predominantly produced in cosmic-ray proton-proton collisions high in the atmosphere.[8] At this stage, we suppose[9] these muons are produced either directly or as decay products of a short-lived parent X. To estimate the production cross section for X-muons, we

(1) assume $\sigma_{pp\to X\text{-muons}}\sim$const (or slowly varying with E);

(2) assume the distribution of the fraction of primary energy given to the muon (inelasticity distribution) is constant (or slowly varying) with energy;

(3) from assumptions 1 and 2, compute a sea-level flux of X-muons;

* Work supported in part by the U. S. Atomic Energy Commission.

[1] H. Bergesen, J. Keuffel, M. Larson, E. Martin, and G. Mason, Phys. Rev. Letters **19**, 1487 (1967).

[2] J. Keuffel, Proc. Utah Acad. Sci. **45**, 1 (1968).

[3] H. Davis and D. Davis, Bull. Am. Phys. Soc. **13**, 683 (1968).

[4] C. Callan and S. Glashow, Phys. Rev. Letters **20**, 779 (1968).

[5] P. V. Ramana Murthy, Phys. Letters **28B**, 38 (1968); see also C. H. Woo, Phys. Rev. Letters **21**, 1419 (1968).

[6] E. Lohrmann, DESY Report (unpublished).

[7] P. Barrett, L. Bollinger, G. Cocconi, Y. Eisenberg, and K. Greisen, Rev. Mod. Phys. **24**, 133 (1952).

[8] Callan and Glashow (Ref. 4) have suggested that what is detected underground is not muons, but a new heavy primary. Evidence against this has been presented by H. Kasha and R. Stefanski, Phys. Rev. Letters **20**, 1256 (1968); W. Kropp, F. Reines, and R. Woods, *ibid.* **30**, 1451 (1968); and Ramana Murty (Ref. 5); and F. Ashton, H. Edwards, G. Kelly, and A. Wolfendale, Phys. Rev. Letters **21**, 303 (1968).

[9] The hypothesis that the muons are predominantly the decay products of K mesons can be made (Ref. 6) to fit the data roughly, provided one allows a renormalization of the vertical depth-intensity measurements by a factor of $\lesssim 2$. This hypothesis, which we do not make, is clearly an experimental question.

(4) extrapolate the measured sea-level flux[10] (at energies <300 BeV) of muons from π and K decay to the Utah energies $E\sim3000$ BeV, using an energy dependence $\sim E^{-3.7}$; and

(5) from the magnitude of $\sec\theta$ effect,[2] estimate the ratio of X-muons to normal component at energies ~2 TeV, thereby obtaining, for a given assumption of inelasticity distribution, the cross section for $\sigma_{pp\to X\text{-muons}}$.

We find, in rough agreement with the more detailed calculations of Keuffel and Osborne,[11] a differential energy spectrum at sea level

$$n(E)=dn_\mu/dE=11E^{-3.7}(\sec\theta+E/3500)\ \text{cm}^{-2}\ \text{sec}^{-1}\ \text{sr}^{-1}\ \text{BeV}^{-1}, \quad (2.1)$$

with E in BeV. Because of the different energy dependence of the X-muon spectrum, there would not necessarily be a contradiction with experiment if the X-production threshold were low compared with 3 TeV. We take a conservative upper limit to the production threshold[12] as ~6 TeV.

From this sea-level muon spectrum, we estimate the cross section (per nucleon) for the X process to be $\gtrsim0.3$ mb. This estimate depends upon the mechanism of energy transfer from proton to muon (inelasticity distribution). We have assumed for this purpose that the muon is produced via a two-body decay of an object X, which in turn has a flat distribution of longitudinal momentum in the production reaction. We consider this an efficient mechanism of energy transfer. However, even if the muon *always* takes *all* of the primary proton energy in the X process, the estimate of production cross section is only reduced from the above $\frac{1}{3}$ mb by a factor of 7. We have also, of course, tacitly assumed one muon (on the average) produced per pp X-process collision. Some details of these considerations are in Appendix A.

III. INTREPRETATION OF X PROCESS

From the estimates given in the previous section we draw the following conclusions:

(1) It appears extremely difficult to explain this large a source of muons in terms of conventional electromagnetic[13] (μ pair) or weak production processes such as production of intermediate boson[14] W, or direct production of muons via the weak interactions.

(2) The muons are not produced together with stable particles such as quarks (via leptonic β decay of an unstable quark, for example). This follows from the rather stringent limits[15] placed on the production cross section for such stable particles, which is, for quarks of mass <10 BeV, less than $\sim10^{-32}$ cm^2 and, for stable heavy triplets of integer charge, $\sim10^{-30}$ cm^2 for masses in the range 3–10 BeV. Furthermore, such a β-decay mechanism is unlikely to be an efficient one for energy transfer from incident proton to final muon, a necessity by virtue of the large lower limit of 0.3 mb on the production cross section.

(3) It is rather unlikely that the muon is produced directly, as opposed to being the decay product of an intermediary. This is because the cross section of muons on protons at energies >1 TeV would also most likely be $\gtrsim0.3$ mb, which is at least a factor of ~20 larger than that tolerable from the observed attenuation of muons underground.[16]

These arguments, while far from being airtight, still strongly suggest a unique interpretation, which we hereafter adopt: In pp collisions in the TeV range, a new class of hadrons X_1, $\bar{X}_2$ is produced in pairs,[17] which are stable under strong and electromagnetic interactions, decay with high probability into a final state containing at least one muon, and have masses in the range $6<M_{X_1}+M_{\bar{X}_2}<55$ BeV and widths consistent with either weak or semiweak coupling.

That X is a hadron is implied, almost by definition, by the large production cross section which, with $M_X>3$ BeV as implied by accelerator experiments,[18] in any case is so large as to defy credulity.

That X is produced in pairs is implied by the stability of the pairs: If a combination of known hadrons couples strongly to a single X in production, and X is reasonably heavy,[19] it will also decay into combinations of known hadrons.

From the estimate of production threshold $E_0\lesssim6$ TeV, we conclude that strictly from kinematics $M_{X_1}+M_{\bar{X}_2}<110$ BeV, and when a more reasonable esti-

[10] S. Baber, W. Nash, and B. Rastin, Nucl. Phys. **B4**, 539 (1968), and references quoted therein. See also A. Aurela and A. Wolfendale, Ann. Acad. Sci. Fenn. **227**, 3 (1967).

[11] J. Keuffel and J. Osborne (private communication).

[12] Because of the steep fall of the primary energy spectrum, the energy of the primary proton is comparable to that of the detected muon. For example, for the "efficient" mechanism of energy transfer described in the next paragraph, the mean energy of a primary which yields a muon of energy E is $\sim2.6E$. If one decreases "efficiency" and raises the primary production threshold, one must also increase the magnitude of the cross section.

[13] For example, were the X muons produced from electromagnetic decay of ϕ mesons (Ref. 6), $\phi\to\mu^++\mu^-$, the muons from $\phi\to K^++K^-$ would overwhelm the direct muons by an order of magnitude.

[14] Early estimates give cross sections $\lesssim10^{-31}$ cm^2 for light W's; see Proceedings of the Argonne International Conference on Weak Interactions, 1965, p. 241 [Argonne National Laboratory Report No. ANL-7130, 1965 (unpublished)], for references.

[15] See the compilation of R. H. Dalitz, in *Proceedings of the Second Hawaii Topical Conference in Particle Physics, 1967* (University of Hawaii Press, Honolulu, 1968), p. 348.

[16] As discussed in Sec. VII, Ref. 42, there is a loophole in this argument. Suppose the X process involves negative-chirality (left-handed μ^-) muons only. Then muons from π and K decay, which are predominantly of positive chirality, would not suffer the absorption coming from the inverse processes.

[17] By distinguishing X_1 from X_2, we do not intend to imply that they are necessarily different.

[18] This is an overconservative limit based on the W-boson search of R. Burns *et al.*, Phys. Rev. Letters **15**, 830 (1965).

[19] This is done so that selection rules do not inhibit the decay into hadron channels.

mate[20] is made, $M_{X_1}+M_{\bar{X}_2}\lesssim 55$ BeV. Therefore, it is unlikely that the X is the $\sim$137-BeV particle conjectured by Lee.[21]

The lifetime of X can be as short as that characteristic of semiweak decays; this is discussed in terms of a specific model in Sec. V. (It may be somewhat shorter, although care must then be taken with respect to possible large muon absorption underground.) The lifetime of X can be as long as $\sim 10^{-7}$–10^{-8} sec (for a particle with $M_X\sim 10$ BeV) before again being limited by the secθ effect and atmospheric absorption of the X.

IV. QUANTUM NUMBERS OF X

Because X is a hadron, it must be assigned the quantum numbers appropriate to strong interactions: B, Q, Y, isotopic spin, possibly the $SU(3)$ representation, and even lepton number L. These quantum numbers themselves may be sufficient to guarantee the stability of X, or they may not. We may identify the following five (inclusive) options. In the first four, we assume strict conservation of additive quantum numbers B, Y, and L by strong interactions, octet (or triality zero) $SU(3)$-symmetry-breaking interaction, and in three of the cases, the possibility of assigning X to $SU(3)$ representations. The options for X are then as follows:

1. Heavy leptons.[22] If X has $B=\pm 1$ and X has integer spin, it follows that X has nonvanishing lepton number and is stable under strong interactions (e.g., a μ-p "resonance"). Likewise, if X possesses half-integer spin and $B=0$, it must have $L\neq 0$ and be stable.

2. Heavy triplets.[23] If the triality t of X is not zero [and if $SU(3)$-breaking forces have $t=0$], then X cannot decay strongly to known hadrons. No new additive quantum number is necessary in this case.[24,25]

3. Charm. Even if X has vanishing triality, X cannot decay strongly into known hadrons, provided $\langle Q\rangle_X \equiv$ [mean value of electric charge taken over the $SU(3)$ multiplet] $\neq 0$. In fact, $\langle Q\rangle_X$ is nonzero in all integer-charged triplet models, and also in the model of unitary singlet X identified with the hypothetical W boson of weak interactions[26,27] (which interacts strongly).

4. Ad hoc selection rule. If none of the above conditions is met (e.g., $B=L=\langle Q\rangle=t=0$, boson), there is no *a priori* reason (provided X is reasonably heavy) for expecting X to be stable. In this case, an *ad hoc* selection rule must be invoked.

5. Others. Other possibilities can be envisaged, provided either that lepton conservation is considered a multiplicative rather than an additive conservation law, or that $SU(3)$-symmetry violation includes a piece with nonvanishing triality, such as **3**, $\bar{\mathbf{3}}$, ... It is also possible that X is only coupled to hadrons via strong $SU(3)$-symmetry-violating interactions,[28] and that it is not possible to classify X in an $SU(3)$ representation. These possibilities, which we have not systematically studied are beyond the scope of this brief paper.

Of the possibilities above, the most familiar are the heavy integer-charged triplets,[23] which decay weakly, or the hypothesis $X=W$, with W in either **1** or **3**, and W decaying semiweakly.[25–27]

V. DECAY OF X INTO LEPTONS

If X has triality $t\neq 0$, or $\langle Q\rangle\neq 0$, or $L\neq 0$, and is not the intermediate boson W, a new interaction (such as an additional piece to the Cabibbo current) coupling X to leptons must be postulated. The detailed nature of any such new coupling is not easy to predict, and we shall not attempt it here. We shall limit our statements to the following, relevant to the question of the branching ratio for X decay into states containing a muon.

(1) If X is a heavy lepton, then it must always decay into states containing a lepton.

(2) If $X=W$, it is plausible, on the basis of current-algebra or especially field-algebra considerations,[29–31] to expect W to decay with large branching ratio into leptons.[32] Although this argument has assumed W not to be a hadron, and cannot be carried through in the same way if W is a hadron, it is unlikely that the situation changes drastically in this case.

VI. TWO DIFFICULTIES

The interpretation we have given has at least two difficult points. One is, in any model, to find a rationale

[20] If X_1, $\bar{X}_2$ are produced in the forward direction in the center-of-mass system, then the minimum momentum transfer to the target nucleon is $\Delta_{\min}\approx M_pE_0/E$, where E_0 is the threshold for the production of X. For $E\gtrsim 4E_0$, "diffraction dissociation" can begin to be a possible efficient production mechanism. See the discussion in Sec. VI.

[21] T. D. Lee, Phys. Rev. **171**, 1731 (1968).

[22] An example has been given by Y. Tanikawa and S. Watanabe, Phys. Rev. **113**, 1344 (1959). This case has apparently been ruled out by the experimental limits on elastic ν-p scattering. S. Ozaki [Progr. Theoret. Phys. (Kyoto) **34**, 868 (1965)] has argued against the existence of ν-N "resonant" reactions; however see Ref. 16 and Sec. VII, which might apply to the case of $X=$ heavy lepton as well as $X=W$.

[23] Triplet models have been discussed by many authors. See the summaries by T. D. Lee, Nuovo Cimento **35**, 933 (1965); F. Gürsey, T. D. Lee, and M. Nauenberg, Phys. Rev. **135**, 467 (1964).

[24] For example, if $X=W$, W is an $SU(3)$ triplet, and there is no new additive quantum number, triple β decay $3n\to 3p+3W\to 3p+3e^-+3\bar{\nu}$ occurs to order $G^{3/2}$ in amplitude.

[25] C. Ryan, S. Okubo, and R. Marshak, Nuovo Cimento **34**, 753 (1964); S. Pepper, C. Ryan, S. Okubo, and R. Marshak, Phys. Rev. **137**, B1259 (1965); see also T. Ericson and S. Glashow, *ibid.* **133**, B130 (1964).

[26] S. Pakvasa, S. F. Tuan, and T. T. Wu, Phys. Rev. Letters **20**, 1546 (1968).

[27] C. Callan, Phys. Rev. Letters **20**, 809 (1968).

[28] The symmetry-breaking effects at low energy could be suppressed by powers of M_p/M_x.

[29] J. Bjorken, Phys. Rev. **148**, 1467 (1966).

[30] V. Gribov, B. Ioffe, and I. Pomeranchuk, Phys. Letters **24B**, 554 (1967).

[31] J. Dooher, Phys. Rev. Letters **19**, 600 (1967).

[32] One may phrase this argument in the following way: Suppose W decays dominantly into hadrons. Then the process $\bar{\nu}_\mu+\mu^-\to$ hadrons is much bigger, at $E_{\mathrm{c.m.}}\approx M_W$, than $\bar{\nu}_\mu+\mu^-\to\bar{\nu}_e+e^-$. It follows (by conserved vector current and approximate chiral symmetry) that, at $E_{\mathrm{c.m.}}\approx M_W$, $e^++e^-\to$ hadrons is much larger than $e^++e^-\to\mu^++\mu^-$. However, this is generally considered unreasonable.

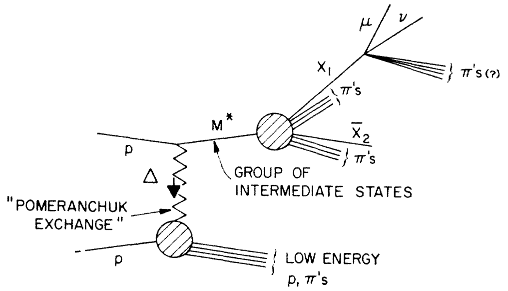

FIG. 1. Diffraction-dissociation model for X production.

for the large production cross section of $\sim\frac{1}{3}$ mb for $\sigma_{pp\to X}$ with a massive X. The other is associated with the depth-intensity relation of muons underground.

A. Production Mechanisms

The cross section for $\bar{p}$ production in p-p interactions rises to a value near 1 mb at laboratory proton energies near 25 BeV, approximately four times the threshold energy.[33] This cross section is approximately equal to πa^2, where $a=\hbar/M_pc$. Relative production of π, K, and $\bar{p}$ is also found to be roughly proportional to $M_\pi^{-2}:M_K^{-2}:M_p^{-2}$. If we take this simplest of arguments for production of X as well, then $\sigma_{pp\to X}\sim\pi(\hbar/M_Xc)^2<0.1$ mb for $M_X>3$ BeV, although admittedly, this is a long extrapolation in concept as well as energy.

There is the correlated problem of *efficient* production of energetic X (the question of inelasticity distribution). Because of the steep energy dependence of the primary spectrum, the muon flux is sensitive to the fraction of energy transferred from the proton. For considering production mechanisms, we have chosen to visualize the production at energies much greater than threshold. In this region, diffraction dissociation, Pomeranchuk-trajectory exchange, or something like it would seem to be the most reasonable hypothesis.[34] We have considered diagrams such as those in Fig. 1 (diffraction dissociation), which appear to yield high-energy muons. M^* is supposed to carry a major fraction of the incident energy and to represent a group of intermediate states, which decay into X_1 and $\bar{X}_2$ (and, very likely, some associated π's). Such a mechanism, when summed over all channels containing $X_1\bar{X}_2$ pairs, is expected to lead to a cross section roughly constant with energy. As the energy is decreased toward threshold, the minimum momentum transfer Δ^2 increases; at energies ~ 3 to 4 times the threshold, $\Delta^2>0.1$ BeV^2, so that suppression of the diffractive process can be expected. We cut off the cross section at this point.

Although the magnitude of the cross section for the diffractive processes is very uncertain, it is quite reasonable that X_1 and $\bar{X}_2$, which most likely carry some kind of quantum number akin to B or Y, should emerge with a sizeable finite fraction of the incident energy with good probability; this seems to be the case for baryon number (protons) and hypercharge (K and Y) at laboratory[35] and cosmic-ray energies.[36]

We have also estimated the production cross section of X-pairs via X exchange in the peripheral model[37] (Fig. 2). An adequate cross section can be obtained only if it is assumed that all $\Delta^2\lesssim M_X^2$ are effective, without significant damping by form factors at the vertices. Again, if one interprets this diagram in terms of groups of states being exchanged, this may not be totally unreasonable. However, the only certain statement that can be made is that optimism is required in order to obtain a large enough cross section.

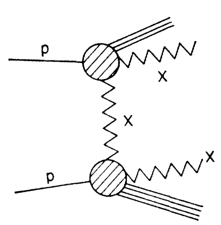

FIG. 2. Peripheral production of X via X exchange.

B. Depth-Intensity Problem

With the customary assumptions about how muons lose energy underground, the sea-level muon spectrum inferred from depth-intensity measurements is in satisfactory agreement with the π and K components alone.[38] A large additional absorption of muons seems to be necessary to maintain agreement with the depth-intensity curve in the presence of the new high-energy component of muons [cf. Eq. (2.1)] from the X process at sea level. Most of the energy loss, in the conventional picture, is accounted for in terms of presumably computable electromagnetic processes (ionization, pair production, and bremsstrahlung), with an estimated[39] 20% coming from the photonuclear process in Fig. 3. Keuffel and Osborne[11] estimate that ~ 5 times this is needed to restore agreement with the depth-intensity relation.

[33] R. K. Adair and N. Price, Phys. Rev. **142**, 844 (1966).

[34] R. K. Adair, Phys. Rev. **172**, 1370 (1968).

[35] See, for example, the data of L. G. Ratner *et al.*, Phys. Rev. **166**, 1353 (1968), where further references are given.

[36] See the review by Y. Fujimoto and S. Hayakawa, in *Encyclopedia of Physics*, edited by S. Flügge (Springer-Verlag, Berlin, 1967); also M. Koshiba, in Proceedings of the Tenth International Conference on Cosmic Rays, Calgary, 1967 (unpublished).

[37] F. Salzman and G. Salzman, Phys. Rev. **121**, 1541 (1961).

[38] See K. Kobayakawa, Nuovo Cimento **47**, 156 (1967), which contains references to earlier work.

[39] Kobayakawa (Ref. 38) finds $\sim 10\%$ of the muon loss contributed by the photonuclear process, with an assumed 70-μb photoabsorption cross section. In this estimate, we have chosen $\sigma_\gamma\sim 100\,\mu$b, constant with energy (consistent with the DESY measurements for $E\lesssim 6$ BeV), and have increased the contribution of virtual photons somewhat from Kobayakawa's estimate.

VII. NEW LEPTONIC PROCESSES

New leptonic interactions must be considered as a possible consequence of the existence of the X process. To illustrate, we choose the case $X=W$, which appears to be rich in additional implications. In this case, the reactions (Fig. 4)

$$\mu+p\to\nu+W+\text{hadrons}, \quad (7.1)$$

$$\nu+p\to\mu+W+\text{hadrons} \quad (7.2)$$

have a cross section possibly of the order of the photonuclear muon cross section ($\sim10^{-30}$ cm^2).[40] To compare process (7.1) with the photonuclear process illustrated in Fig. 3, we consider the corresponding cross sections $\sigma_{\mu\to\nu W}$ and $\sigma_{\mu\to\mu}$, differential only in q^2 and q_0, the square of four-momentum transfer and energy transfer, respectively, from leptons to hadrons:

$$\frac{d\sigma_{\mu\to\nu W}/dq^2dq_0}{d\sigma_{\mu\to\mu}/dq^2dq_0}\approx\frac{2\sqrt{2}GM_W^2}{4\pi\alpha}\,\frac{q^2/(q^2+M_W^2)^2}{q^{-2}}\,\frac{\sigma_{Wp}(q^2,q_0)}{\sigma_{\gamma p}(q^2,q_0)}. \quad (7.3)$$

Taking σ_{Wp} to be geometrical (in rock), taking $\sigma_{\gamma p}\sim10^{-28}$ cm^2/nucleon, and cutting off $\int dq^2/q^2$ at the nucleon mass, we find[41]

$$\frac{d\sigma_{\mu\to\nu W}/dq_0}{d\sigma_{\mu\to\mu}/dq_0}\approx9\times10^{-3}\left(\frac{M_W}{M_p}\right)^2\times\int\frac{q^2dq^2}{(q^2+M_W^2)^2}\,\frac{\sigma_{Wp}(q^2,q_0)}{\sigma_{Wp}(M_W^2,q_0)}\,\theta\left(q_0-\frac{2M_W^2}{M_p}\right). \quad (7.4)$$

If (and only if) momentum transfers q^2 comparable with the W mass are fully effective, the absorption of muons coming from process (7.1) will compete with the ordinary photonuclear losses of the muons (for $M_W\gtrsim10$ BeV). This would lead to an additional attenuation of high-energy muons of negative chirality (left-handed μ^-, right-handed μ^+) comparable to the photonuclear attenuation $\lambda_{\mu\to\mu}^{-1}=0.7\times10^{-6}$ g^{-1} cm^2. However, $\sim70\%$ of the muons from π decay and $\sim100\%$ of muons from K decay have positive chirality and will *not* be attenuated by the W-production process (7.1). On the other hand, all the muons from the X process have negative chirality and will be absorbed by the W-production process (7.1). It is therefore likely that W production by muons cannot by itself account for the extra absorption required by Keuffel and Osborne. This does not by itself rule out the possibility that the W production is present with a magnitude comparable to the photonuclear processes. Under these circumstances, high-energy neutrinos $E_\nu\gtrsim2M_W^2/M_p$ would be attenuated via process (7.2) as strongly ($\lambda_{\nu\to\mu W}\sim0.7\times10^6$ g cm^{-2}) as negative-chirality muons via (7.1).

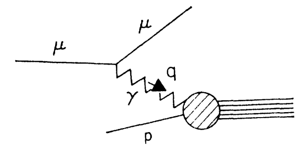

FIG. 3. Photonuclear interaction of muons.

With such large neutrino cross sections, one may question[5] whether the predicted flux of neutrino-induced muons underground is compatible with experiment. Using spectrum (2.1) and various assumed attenuation lengths, we have crudely estimated the flux of neutrino-induced muons underground. For the cases in which the incident neutrino flux is not appreciably attenuated, we find far too many neutrino-induced muons, even with an X-process threshold of 3 TeV, unless the absorption mean free path $\lambda_\nu\gtrsim5\times10^8$ g cm^{-2} ($\sigma_{\nu p}\lesssim3\times10^{-33}$ cm^2), in rough agreement with the arguments of Ramana Murthy.[5] It is possible, however, that the ν absorption underground is so strong that the neutrino beam is attenuated about as strongly as the muons.[42,43] Ramana Murthy[5] argues that this is not possible, because the muons would be attenuated more strongly than ob-

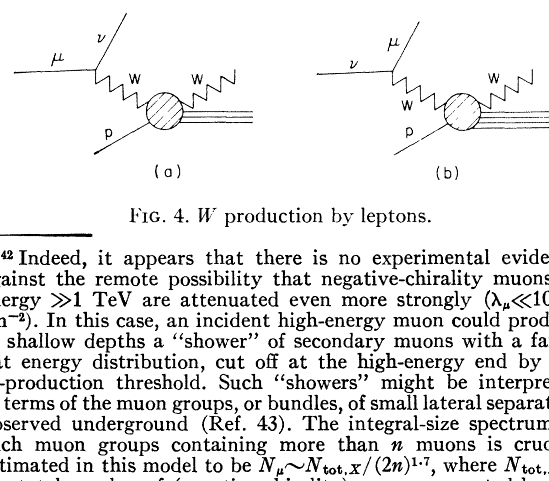

FIG. 4. W production by leptons.

[40] Here we refer to the cross section *without* the hypothesized increase of Keuffel and Osborne.

[41] Again we put the effective threshold as four times kinematical threshold; see Ref. 20 and Sec. VI.

[42] Indeed, it appears that there is no experimental evidence against the remote possibility that negative-chirality muons of energy $\gg1$ TeV are attenuated even more strongly ($\lambda_\mu\ll10^5$ g cm^{-2}). In this case, an incident high-energy muon could produce at shallow depths a "shower" of secondary muons with a fairly flat energy distribution, cut off at the high-energy end by the X-production threshold. Such "showers" might be interpreted in terms of the muon groups, or bundles, of small lateral separation observed underground (Ref. 43). The integral-size spectrum of such muon groups containing more than n muons is crudely estimated in this model to be $N_\mu\sim N_{\text{tot},X}/(2n)^{1.7}$, where $N_{\text{tot},X}$ is the total number of (negative-chirality) muons generated by the X process in proton collisions in the atmosphere. The magnitude and spectrum is in order-of-magnitude agreement with measurements of Bibliashvili *et al.* (Ref. 43) at 200 hg cm^{-2}. If this model were correct, the threshold for X production by muons must be >3 TeV in order that there be muons of energy ~3 TeV left to produce the secθ effect in the Utah experiment. This, in turn, would appear to require a rather high effective production threshold ($\gtrsim6$ TeV) in the pp collisions as well. In addition, the muon attenuation length <50 hg cm^2 would imply a μp cross section >0.3 mb. Thus, this case is well described by a direct-production mechanism involving negative-chirality muons only. From the arguments on elastic neutrino scattering given below in the text, only negative-chirality muons, and *not* neutrinos, would have to be coupled to hadrons in this case. Therefore, some additional parity violation in strong interactions might be anticipated. However, it is not hard to arrange this to be of order $\alpha M_X^{-2}\approx G$.

[43] Bibliashvili *et al.*, Can. J. Phys. **46**, S337 (1968), and references quoted therein.

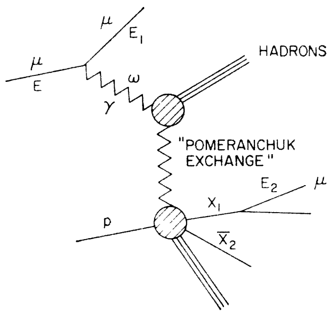

FIG. 5. Photonuclear production of "backward" X by muons.

served. However, he did not take into account the polarization of the beam and the chirality dependence of the absorption cross sections. But there is another argument which limits the magnitude of neutrino ab-absorption at high energy. The forward-scattering amplitude of neutrinos from nucleons, averaged over nucleon spins, is related to high-energy cross sections by a dispersion relation

$$\bar{T}_{\nu p}=\frac{1}{\pi}\int_0^\infty \frac{dW'\sigma_{\nu p}(W')}{W'-W}-\frac{1}{\pi}\int_0^\infty \frac{dW'\sigma_{\bar{\nu} p}(W')}{W'+W}$$

$$\cong\frac{1}{\pi}\int \frac{dW'}{W'}(\sigma_{\nu p}-\sigma_{\bar{\nu} p})+\frac{W}{\pi}\int \frac{dW'}{W'^2}(\sigma_{\nu p}+\sigma_{\bar{\nu} p}),$$

where W is the laboratory neutrino energy. Callan[28] has argued that the leading term could be approximately zero if W is an isotopic singlet. However, the next term is not zero. Experimentally, for $W\sim 1$–3 BeV,

$$T_{\bar{\nu} p}\lesssim 0.3(G\sqrt{2})=0.3T_{\text{"Fermi theory"}}.$$

Therefore, crudely,

$$(1/\pi)(W/W_{\text{threshold}})\sigma^{\nu p}\lesssim 0.3(G\sqrt{2})$$

or, for $W_{\text{threshold}}\lesssim 3$ TeV,

$$\sigma^{\nu p}< 1.6\times 10^{-29}\text{ cm}^2$$

or

$$\lambda_\nu \gtrsim 1.0\times 10^5\text{ g cm}^{-2}.$$

Considering the crudity of the calculations, we feel we can draw only the following conclusions. If $X=W$, then either

(a) the neutrino-production cross section of W is $\lesssim 3\times 10^{-33}$ cm^2 (this is difficult to reconcile with the copious production of W in p-p collisions, although, as always, one cannot estimate the rates well enough to draw a firm conclusion) or (b) the W-production cross section by neutrinos is $\sim 10^{-29}$ cm^2, with a threshold $\gtrsim 3$ TeV. In this case, elastic $\nu p\to\nu p$ scattering is of the order of the experimental upper limit, and the neutrino production of muons deep underground also of the order of the experimental limit. These conditions may well be mutually incompatible, but our calculations are too crude to establish this.

On this basis, we agree (but for different reasons) with the conclusions of Ramana Murthy[5] that the $X=W$ hypothesis involves difficulties.

Independently of any assumptions about W bosons, muons will produce X in the photonuclear process (Fig. 5) itself. We consider the X produced in the backward cone from the proton; the kinematics resembles strongly that in the p-p production process itself. We assume that (see also Appendix B):

(a) The X is produced with a flat, longitudinal momentum distribution in the center-of-mass frame, and decays isotropically into muon with two-body kinematics.

(b)

$$\frac{\sigma_{\gamma p\to X}}{\sigma_{\gamma p}{}^{\text{tot}}}\approx\frac{\sigma_{pp\to X}}{\sigma_{pp}{}^{\text{tot}}}\approx 8\times 10^{-3}, \qquad (7.5)$$

and we find the cross section for production of an X-muon of energy E_μ to be

$$\frac{d\sigma}{dE_\mu}\approx 10^{-32}\text{ cm}^2\left(\frac{\theta(E_0-E_\mu)}{2E_0}+\frac{E_0\theta(E_\mu-E_0)}{2E_\mu{}^2}\right), \qquad (7.6)$$

with $E_0=M_X{}^2/2M_p$.

The significance of this process is that it might produce muon pairs of small lateral separation and measurable angular divergence $\theta\sim M_p/M_X$, independent of incident muon energy, provided only the energy is well above threshold. The best depth for such observations appears to be $\sim$1000 hg/cm^2, where the high-energy muons required to initiate the process have not been too strongly attenuated. At this depth, we crudely estimate the fraction of these pairs to total number of muons to be between 3×10^{-4} (for $M_X\sim 3$ BeV) and 5×10^{-7} (for $M_X\sim 30$ BeV).

The X-muons produced in the forward cone in the hadron center of mass (Fig. 6) also lead to approximately parallel muon pairs underground, with small lateral spacing ($\lesssim 0.3M_X/M_p$ m at 1000 hg/cm^2). Using a total cross section of $\sim 10^{-32}$ cm^2 and inelasticity distribution as before, we estimate a ratio of narrow pairs to singles between $\sim 6\times 10^{-4}$ (for $M_X=6$ BeV) and 3×10^{-7} (for $M_X=30$ BeV) for depths $\gtrsim$1000 hg cm^{-2}. The mean lateral spacing is roughly independent of depth, because while the lateral spacing for a given pair increases with depth, the mean longitudinal momentum of the pairs at production increases with the depth at which they are observed.

VIII. CONCLUSIONS AND EXPERIMENTAL IMPLICATIONS

If we assume the validity of the Utah experiment, our study demands the existence of a new class of hadrons X, of mass in the range 4–30 BeV, stable under strong and electromagnetic interactions, and decaying with large branching ratio into states containing muons. The possible widths of X include those characteristic of weak and semiweak interactions, but not much beyond either. In order to be compatible with experiment, the production cross section of X must be $\gtrsim 0.3$ mb/nucleon, and the muon absorption must be significantly increased from that customarily assumed, with one remotely possible exception described below.

Some experimental consequences of these conclusions include the following:

(a) Probable existence of large transverse momenta[44] in the decay process $X \to \mu + ?$. This can be tested in extensive air-shower studies by observing the lateral distribution of muons (or electrons, if μ-e universality holds in the decay process) away from the shower core.[45] Such large transverse momentum might also be observed in the primary proton events themselves. Because the X is produced with high laboratory energy in the forward cone in the p-p collisions, they will also be produced with low laboratory energy in the backward cone, $E_X \gtrsim M_X^2/2M_p$. Given a transverse momentum $\sim \frac{1}{2} M_X$ (from a two-body decay), the laboratory angle is $\theta_\mu \lesssim M_p/M_X$. Measurement of such primary events at the $\frac{1}{3}$-mb level would appear to be within experimental possibility.

(b) The sea-level spectrum of muons, both in energy and angle, is modified from that normally expected [see Eq. (2.1)] at energies >1 TeV.

(c) The charge ratio of X-derived muons may differ from ~ 1, although we do not know how to predict it. A necessary condition for a charge ratio different from 1 is that in the reaction $p+p \to X_1 + \bar{X}_2 +$hadrons, there is no strong-interaction symmetry operation that takes X_1 into $\bar{X}_2$, while leaving the normal hadrons unaffected. This is guaranteed if X_1 and X_2 have different spins or masses. On the other hand, the model $X = W = SU(3)$ singlet does *not* satisfy this criterion, and in this case a charge ratio of unity is required.[46]

(d) If $X \to \mu + \bar{\nu}_\mu + \cdots$, an additional large component of sea-level neutrinos [equal to the X-muon component in Eq. (2.1)] exists.

(e) New underground lepton-induced phenomena may be anticipated. In increasing order of improbability, these are as follows:

(i) At the level of $\sim 1\%$ of the photonuclear muon absorption, the process $\mu + p \to \mu + X_1 + \bar{X}_2 +$hadrons

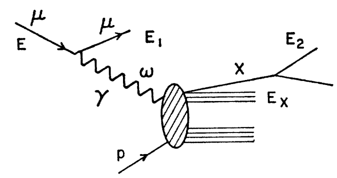

FIG. 6. Photonuclear production of "forward" X by muons.

(Fig. 6) occurs. It leads to a rate for observing underground μ pairs of between 6×10^{-4} and 3×10^{-7} per muon, detected at depths $\gtrsim 1000$ hg cm^{-2}. These pairs should have a spacing $\lesssim 0.3 M_X/M_p$ m. In addition, at a depth of ~ 1000 hg cm^{-2}, muons pairs having angular divergence $\theta_\mu \lesssim M_p/M_X$ rad should occur at a rate $\gtrsim 10\%$ of the narrow pair rate.

(ii) If $X = W$, the inverse reactions

$$\mu + N \to \nu + W + \text{hadrons},$$
$$\nu + N \to \mu + W + \text{hadrons}$$

might lead to attenuation of the normal-helicity muons produced from the X process, but not to the abnormal-helicity muons produced from π for K decay. However, a cross section above 10^{-33} cm^2 is already ruled out by the deep-mine neutrino experiments, unless it is $\sim 10^{-29}$ cm^2. A cross section larger than this is ruled out by the experimental upper limit for the process $\nu + p \to \nu + p$.

(iii) It is even conceivable that normal-helicity muons are strongly attenuated by the X process at the high energies above the X-production threshold, which in this case must be greater than 3 TeV. In this case, however, the neutrinos *cannot* be similarly attenuated, because of the experimental limit on elastic neutrino-proton scattering. Groups (bundles) of muons[42] with small lateral separation observed underground might be associated with such a showerlike absorption process. The predicted integral-size spectrum of these groups seems roughly to fit some of the observations. Further studies of these bundles would be very desirable.

ACKNOWLEDGMENTS

In addition to our acknowledgment to the Utah group, we wish to thank S. Flatté, C. J. Goebel, M. W. Peters, and W. Toner for very helpful discussions.

APPENDIX A: MODEL FOR COSMIC RAYS IN THE ATMOSPHERE

In this Appendix we describe the simple model of high-energy cosmic-ray propagation in the atmosphere which we have used.

We begin with the differential primary proton spectrum (at the top of the atmosphere) assumed to be[47]

$$dn_p = 2.2E^{-2.7}dE = n_p(0,E)dE, \tag{A1}$$

[44] If the dominant decay modes of X include several hadrons in the final state, as well as the leptons, there is no necessity for large transverse momentum. However, in this case the efficiency of conversion of primary energy into muon energy is low, and a very large production cross section for X is necessary.

[45] The Haverah Park cosmic-ray group has found evidence for such large transverse momenta. See C. McCusker, in Proceedings of the Tenth International Conference on Cosmic Rays, Calgary, 1967, Part A (unpublished).

[46] If $W = X$ is correct, then the process $\nu + p \to \nu + p$ is of first order in weak interactions, while the experimental limit is an order of magnitude smaller in cross section. Callan (Ref. 27) has given an argument for a suppression of this rate, which depends upon W^+p and W^-p strong interactions being identical. Therefore, the evidence on ν-p elastic scattering favors a charge ratio $\mu^+/\mu^- = 1$, if $X = W$.

[47] N. L. Grigorov *et al.*, in Proceedings of the Tenth International Conference on Cosmic Rays, Calgary, 1967, Part A, p. 512 (unpublished).

where $n_p(x,E)$ is the flux of protons per BeV sr sec at depth x (in g cm^{-2}) in the atmosphere. To determine $n_p(x,E)$, we take the differential probability dP of a proton of energy E to interact in thickness dx, producing a secondary proton of energy E' in dE' to be function[48] only of E'/E:

$$dP = f_{pp}\left(\frac{E'}{E}\right)\frac{dx}{\lambda_p}\frac{dE'}{E}, \quad f_{pp}\approx 1 \tag{A2}$$

where λ_p is the interaction mean free path in air. The solution of the appropriate diffusion equation is then

$$n_p(x,E) = n_p(0,E)e^{-x/\lambda_p'}, \tag{A3}$$

where the attenuation mean free path λ_p' is

$$\frac{1}{\lambda_p'} = \frac{1}{\lambda_p}\left(1 - \int_0^1 dt\, t^{1.7} f_{pp}(t)\right) \approx \frac{0.7}{\lambda_p}. \tag{A4}$$

We take $\lambda_p' \approx 120$ g cm^{-2}.

For the charged-pion spectrum, we again assume similar forms for $f_{\pi p}$ and $f_{\pi\pi}$, the probabilities of finding a π in a proton-air collision and π-air collision, as defined in Eq. (A2). Assuming that the attenuation mean free path of a pion λ_π', defined analogously to (A4), equals that of the proton, and neglecting loss from decay, we find the pion spectrum to be

$$n_\pi(x,E) \approx \frac{x}{\lambda_p} n_p(x,E) \int_0^1 dt\, t^{1.7} f_{\pi p}(t). \tag{A5}$$

Notice that

$$\int dt\, f_{pp}(t) \approx 1,$$

$\int dt\, f_{\pi p}(t) = \bar{n}_s =$ mean number of charged pions produced in a pp collision in this energy range,

$\int dt\, t f_{pp}(t) = \bar{E}'/E =$ mean energy retained by proton in a p-air collision,

and

$\int dt\, t f_{\pi p}(t) = \bar{E}_\pi/E =$ mean fraction of primary energy given to charged pions in a p-air collision.

We may proceed in a straightforward way to compute the muon flux from the decay pions.[7] The number of muons at sea level is found to be

$$n_\mu^{(\pi)}(E) = \frac{\lambda_p'}{\lambda_p} n_p(0,E) \int_0^1 ds\, s^{1.7} f_{\pi p}(s) \times \int_0^1 dt\, t^{1.7} f_{\mu,\pi}(t)\left(1 + \frac{E\cos\theta}{E_0^{(\pi)} t}\right)^{-1}, \tag{A6}$$

where $f_{\mu,\pi}(t)$ is defined as for the previous f's, and $E_0^{(\pi)} = mz_0/c\tau \approx 90$ BeV. z_0 is the scale height of the atmosphere taken to be ≈ 6 km (good for depths less than 250 g cm^{-2}).

Up to this point we have ignored muons from $K^\pm$ mesons; the contributions of these are of the same form as before, with π replaced by K. In this case, $E_0^{(k)} \approx 830$ BeV and $f_{\mu,k}(t) \approx 0.6$, the branching ratio of $K^\pm$ into the $K_{\mu 2}$ mode. For the energies in question, we may approximate, for pions,

$$\int dt\, t^{1.7} f_{\mu,\pi}(t)\left(1 + \frac{E\cos\theta}{E_0 t}\right)^{-1} \approx \frac{E_0}{E}\sec\theta \int dt\, t^{2.7} f_{\mu,\pi}(t) \approx 0.49\frac{E_0}{E}\sec\theta, \tag{A7}$$

while for kaons

$$\int dt\, t^{1.7} f_{\mu,k}(t)\left(1 + \frac{E\cos\theta}{E_0 t}\right)^{-1} \approx 0.22\left(1 + \frac{3.7E\cos\theta}{2.7E_0^{(k)}}\right)^{-1}. \tag{A8}$$

We may estimate $f_{\pi p}$ and f_{kp} by fitting the experimental sea-level muon spectrum[10] (at 100 and 500 BeV) by adjusting the values of the integrals involving these functions. The kaon contribution is not important, and we find a good fit with

$$\int dt\, t^{1.7} f_{\pi p}(t) \approx 0.08.$$

The neglect of the kaon contribution at higher energies, as emphasized by Lohrmann,[6] is not justifiable; however, even the assumption of 100% kaon parentage cannot explain the Utah data and vertical intensity measurements.[11] With the 20% K/π ratio favored by Osborne and Wolfendale,[49] addition of the kaon component does not greatly modify our estimates.

Turning to the X process, we assume the same kind of differential equations apply as for the π production. For example, the differential probability dP for making

[48] This roughly agrees with the facts [O. Czezewski, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), and Ref. 36] at accelerator as well as cosmic-ray energies.

[49] J. Osborne and A. Wolfendale, Proc. Phys. Soc. (London) **84**, 901 (1964).

X is taken to be

$$dP=f_{p,X}\left(\frac{E_X}{E_p}\right)\frac{dE_X}{E_p}\frac{dx}{\lambda_X}\int f_{pX}(t)\,dt=1\,,$$

where λ_X is the mean free path (in g cm^{-2}) in air for a proton to make an X (pion production of X is ignored):

$$\lambda_X^{-1}=(1/A^{1/3})\times 6\times 10^{23}\sigma(p+\text{air}\to X+\text{anything})\,,\qquad A\approx 15.$$

The sea-level muon spectrum is then found to be

$$n_\mu(E)=n_p(E)\left(\frac{5}{E}\sec\theta+\frac{\lambda_p'}{\lambda_X}\times\int_0^1 dt\,t^{1.7}f_{\mu,X}(t)\int_0^1 ds\,s^{1.7}f_{X,p}(s)\right),$$

where the X-production threshold has been assumed to be much lower than the energy at which the X-muons become important.

From the Utah data and the analysis of the angular distribution, the spectrum (2.1) can be deduced.

Upon assuming, as in Sec. II, that $f_{\mu,X}\approx f_{X,p}\approx 1$, we find

$$\frac{\sigma_{pp\to X}}{\sigma_{pp}^{\text{tot}}}\approx\frac{\sigma_{p,\,\text{air}\to X}}{\sigma_{p,\,\text{air}}^{\text{tot}}}=\frac{\lambda_p}{\lambda_X}=8\times 10^{-3}.$$

APPENDIX B: MUONS UNDERGROUND

In this Appendix, we discuss various possible muon- and neutrino-induced phenomena in the light of a supposed X process. We have not made detailed calculations of muon energy spectra underground, and these estimates must be considered, at best, of an order-of-magnitude nature. We consider (1) cross sections for the production of X muons by muons underground, (2) spectra of wide-angle muon pairs underground, (3) spectra of muons of narrow separation underground, and (4) neutrino-induced muon flux.

1. Production of Muons by Muons Underground

We consider the processes briefly discussed in Sec. VII for the production of muons by muons via the photonuclear X process. As in Fig. 5 and (7.5), we suppose that $\gamma p\to X$ proceeds by the diffraction-dissociation mechanism, with the ratio $\sigma_{\gamma p\to X}/\sigma_{\gamma p}^{\text{tot}}\approx 8\times 10^{-3}$, the same as in p-p collisions. To obtain a momentum spectrum of secondary muons, we assume that the longitudinal momentum distribution of X in the center-of-mass frame of photon and target proton is uniform, as was the assumed case for pp collisions. Therefore, the laboratory distribution of protons from the backward cone in the center of mass will be the same in the two cases and obtainable by Lorentz transformation from the center-of-mass frame. In that frame the momentum distribution p_X^* of the X is given by

$$dn_X/dp_X^*\cong 1/2E_{\text{c.m.}}\,, \tag{B1}$$

where $E_{\text{c.m.}}$ is the center-of-mass energy of photon or proton. Writing the laboratory energy E_X of X as

$$E_X=\frac{E_{\text{c.m.}}}{M_p}\left[E_X^*-\left(1-\frac{M_p^2}{E_{\text{c.m.}}^2}\right)^{1/2}P_X^*\right], \tag{B2}$$

we find

$$E_X\approx(E_{\text{c.m.}}/M_p)M_X^2/2E_x^*,\quad M_X\gg M_p. \tag{B3}$$

This leads to the distribution in the laboratory

$$dn_X/dE_X\cong E_0/E_X^2\,,\qquad\text{provided } E_X>E_0=M_X^2/2M_p. \tag{B4}$$

The distribution of muons implied by (B4) is obtained by folding an assumed two-body decay distribution $X\to\mu+??$ into the above spectrum. One finds, for the energy distribution of the muons,

$$\frac{dn_\mu}{dE}=\int_E^0\frac{dE_X}{E_X}\frac{dn_X}{dE_X}=\frac{\theta(E_0-E)}{2E_0}+\frac{E_0\theta(E-E_0)}{2E^2}. \tag{B5}$$

Notice that this spectrum is independent of $E_{\text{c.m.}}$, provided, of course, that $E_{\text{c.m.}}$ is high enough to produce the X. This feature is especially significant for direct studies of backward-production reactions in high-energy pp collisions. The extremely high incident energy need not be determined, only the energy and transverse momentum of the relatively slow secondary.

To obtain the cross section for production of backward X by photons, we use the estimate (7.5), and for production of backward X by muons (Fig. 5), we use the Weiszäcker-Williams expression,[50] and assume[39] $\sigma_{\gamma p}^{\text{tot}}\approx 100\ \mu$b:

$$\frac{d\sigma_{\mu p\to X}}{dE_1}\approx\frac{\alpha}{\pi}\ln\frac{q_{\max}^2}{m_\mu^2}\frac{\sigma_{\gamma p\to X}(\omega)}{\omega}\approx\frac{10^{-4}\sigma_{\gamma p}^{\text{tot}}}{\omega}\approx\frac{10^{-32}\text{ cm}^2}{\omega}, \tag{B6}$$

with ω the energy of the virtual photon: $\omega=E-E_1$, and we choose $q_{\max}\sim 1$ BeV. The cross section, differential in both muon energies, is, if we use (B5), roughly

$$\frac{d\sigma_{\mu p\to X}}{dE_1dE_2}\approx\frac{10^{-32}\text{ cm}^2}{E-E_1}\left(\frac{\theta(E_0-E_2)}{2E_0}+\frac{E_0\theta(E_2-E_0)}{2E_2^2}\right). \tag{B7}$$

For X muons produced in the forward cone (Fig. 6) by muons, we take (B6) and (B7), along with a flat longitudinal momentum distribution for the X, and

[50] R. H. Dalitz and D. R. Yennie, Phys. Rev. **105**, 1598 (1957).

obtain

$$\frac{d\sigma}{dE_1} \approx \frac{10^{-32}\ \mathrm{cm}^2}{\omega} \frac{dE_X}{\omega}; \tag{B8}$$

then, folding the assumed two-body decay distribution of X into $\mu+??$, we obtain

$$\frac{d\sigma}{dE_1 dE_2} = \frac{10^{-32}}{\omega^2} \int_{E_2}^{\omega} \frac{dE_X}{E_X} \approx \frac{10^{-32}}{(E-E_1)^2} \ln \frac{E-E_1}{E_2}. \tag{B9}$$

We take the minimum photon energy for producing X to be[41]

$$\omega_{\text{threshold}} \approx 2(m_{X_1}+m_{\bar{X}_2})^2/m_p \approx 8m_X^2/m_p, \tag{B10}$$

if X_1 and X_2 have comparable masses. For purposes of rough estimation, we shall often assume this, although there is admittedly little justification for doing so.

2. Spectra of Wide-Angle Muons Underground

Muon pairs produced underground via the backward-cone photonuclear process discussed in the previous section and with a measurable angular separation require a high-energy primary muon. It is therefore most favorable to search for such pairs fairly near the surface of the earth, the depth chosen to minimize background but to be less than an attenuation mean free path ($\sim$1000 hg cm^{-2}) for a high-energy muon. The "effective target thickness" for producing such pairs is determined by the range of the relatively slow secondary muon, which, according to (B4) and (B5), has momentum $\sim M_X^2/M_p$. Taking $M_X \lesssim 30$ BeV, this means a range less than that corresponding to a 900-BeV μ; the incident muon energies [$>8M_X^2/M_p$ according to (B10)] are considerably higher. To estimate crudely the spectrum of wide-angle pairs, we assume that (a) the depth of the detector underground is shallow enough so that attenuation of the high-energy sea-level muon flux of primary muons can be ignored and (b) the energy E_2 of the secondary X-derived muon is low enough so that only ionization loss need be considered. We also assume that $E_2 < E_1$, the energy of the other secondary muon.

The flux of pairs at depth x is then roughly

$$n_{\text{pairs}}(x) = (6\times10^{23}) \times 10^{-32} \int_0^x dx' \int_{\omega_t}^{\infty} dE\, n(E) \int_{kx'}^{E-\omega_t} \frac{dE_1}{E-E_1} \int_{kx'}^{\infty} dE_2 \times \left(\frac{\theta(E_0-E_2)}{2E_0} + \frac{E_0\theta(E_2-E_0)}{2E_2^2} \right) \mathrm{cm}^{-2}\,\mathrm{sr}^{-1}\,\mathrm{sec}^{-1}, \tag{B11}$$

where $n(E)$ is the sea-level differential vertical muon spectrum (2.1), $\omega_t \approx 8M_X^2/M_p$ according to (B10), $E_0 = M_X^2/2M_p$, k is the ionization loss ($\cong 2.4\times10^{-3}$ BeV g^{-1} cm^2), and other units as before.

Evaluating the integrals, assuming $E_0 < kx$, $kx' \ll \omega_t$, we find

$$n_{\text{pairs}}(x) = \frac{6\times10^{-9}E_0}{k} \times \int_{\omega_t}^{\infty} dE\, n(E) \left(\tfrac{3}{4} + \tfrac{1}{2} \ln \frac{kx}{E_0} \right) \ln \frac{E}{\omega_t}. \tag{B12}$$

The second logarithmic term corresponds to energetic secondary X-derived muons, which will have relatively small opening angles. We ignore this contribution for purposes of making a conservative rough estimate. We have, using the vertical spectrum (2.1) of sea-level muons,

$$n_{\text{pairs}} \approx 2\times10^{-6}E_0 \int_{\omega_t}^{\infty} dE\, n(E) \ln \frac{E}{\omega_t} \approx 8\times10^{-6}E_0\omega_t^{-2.7}\left(1+\frac{\omega_t}{2200}\right) \mathrm{cm}^{-2}\,\mathrm{sr}^{-1}\,\mathrm{sec}^{-1}, \tag{B13}$$

with ω_t and E_0 in BeV.

As an example, for $M_{X_1} \approx M_{X_2} \sim 10$ BeV, it follows that $E_0 \sim 50$ BeV, $\omega_t \sim 800$ BeV, and

$$n_{\text{pairs}} \approx 1\times10^{-11}\ \mathrm{cm}^{-2}\,\mathrm{sr}^{-1}\,\mathrm{BeV}^{-1}, \tag{B14}$$

and (at 1000 hg cm^{-2}),

$$n_{\text{singles}} \approx 1.5\times10^{-6}, \quad n_{\text{pairs}}/n_{\text{singles}} \approx 7\times10^{-6}. \tag{B15}$$

Notice that the result depends strongly upon M_X, allowing the ratio in (B15) to vary from $\sim 3\times10^{-4}$ to $\sim 5\times10^{-7}$ for $3 < M_X < 30$ BeV.

3. Muon Pairs of Narrow Separation

High-energy muon pairs associated with the process of Fig. 6 are also predominantly produced in the first muon attenuation length underground, i.e., the first 1000 hg cm^{-2}. In order to detect such pairs, it would seem advantageous to put the detector as close to the region of production as possible, namely, the first 1000 hg cm^{-2} or so.

To calculate the rate of such pairs, we proceed much as in the previous section. The main change will be that the survival probabilities of the secondary muons will be larger. We start with the cross-section estimate (B9), and compute the flux of pairs at depth x. As in the previous section, we ignore the attenuation of the incident beam of muons.

$$n_{\text{pairs}}(x) = (6\times10^{23})\times10^{-32} \int_0^x dx' \int_{\omega_t}^{\infty} dE\, n(E) \times \int_{kx'}^{E-\omega_t} \frac{dE_1}{E-E_1} \int_{kx'}^{E-E_1} \frac{dE_2}{E-E_1} \ln \frac{E-E_1}{E_2}. \tag{B16}$$

For $M_X > 6$ BeV and $x \sim 1000$ hg cm^{-2}, we have $kx \approx 240$ BeV$\lesssim \omega_t = 8M_X^2/M_p$. Under these circumstances we may take $kx' \ll \omega_t$, and the integrations simplify to

$$n_{\rm pairs}(x) \approx (6\times10^{-9})x\int_{\omega_t}^{\infty} dE\, n(E)\ln\frac{E}{\omega_t}\,,\qquad x \lesssim 1000\text{ hg cm}^2. \quad \text{(B17)}$$

What this means is simply that, essentially, all the secondary muons produced survive.

Again using (2.1), we find

$$n_{\rm pairs}(x) \approx (2.4\times10^{-8})x\omega_t^{-2.7}(1+\omega_t/2200). \quad \text{(B18)}$$

The ratio of narrow pairs to wide pairs, from (B12) and (B17), is

$$n_{\rm narrow}(x)/n_{\rm wide}(x) = 4kx/3E_0. \quad \text{(B19)}$$

For the previous parameters ($x\sim1000$ hg cm^{-2}, $M_X=10$ BeV, $E_0=50$ BeV) we find

$$n_{\rm narrow}/n_{\rm wide} \approx 6. \quad \text{(B20)}$$

For $M_X\sim6$ BeV, the ratio (B20) is ~20. The ratio of narrow pairs to singles varies from 6×10^{-4} to 3×10^{-7} as M_X varies from 6 to 30 BeV.

At depths greater than 1000 hg cm^{-2}, both the ionization loss and catastrophic losses (i.e., pair+bremsstrahlung+nuclear) must be considered in computing the muon spectra and resultant pair spectra.

4. Neutrino-Induced Processes

We here consider single W production by neutrinos (Fig. 4), according to the model described in Sec. VII. We assume that the neutrino beam is not appreciably attenuated underground, and we let the interaction probability dP in thickness dx be

$$dP = \frac{dx}{\lambda_\nu}\frac{dE_\mu}{E_\nu}\theta(E_\nu - E_0)\,, \quad \text{(B21)}$$

where E_0 is the threshold energy for producing W. The muon flux from the neutrinos at great depths will be

$$n_\mu \cong \int_{E_0}^{\infty} dE \int \frac{dx'}{\lambda_\nu}\int_{(k/b)(e^{bx'}-1)}^{E} \frac{dE_\mu}{E}(CE^{-2.7})\,, \quad \text{(B22)}$$

where $n(E)=CE^{-2.7}$ is the flux of negative-chirality neutrinos produced in the atmosphere by the X process itself. We neglect the contribution of π- and K-derived neutrinos, and determine C from the second term in (2.1). (The neutrino flux and μ flux from the W decays should be equal.) We also ignore range fluctuations of the secondary muons and take k as before, $b=7\times10^{-6}$ g^{-1} cm^2. Upon carrying out the integrations, we find

$$n_\mu = \int_{E_0}^{\infty} \frac{dE}{b\lambda_\nu} CE^{-3.7}\left[(E+E_c)\ln\left(1+\frac{E}{E_c}\right) - E\right]. \quad \text{(B23)}$$

With $E_c=k/b\approx340$ BeV,

$$n_\mu = \frac{CE_0^{-1.7}}{1.7b\lambda_\nu}\times[\ln(E_0/E_c)-1]\,,\quad \text{if } E_c \ll E_0$$
$$\times 1.2E_0/E_c\,,\quad \text{if } E_c \gg E_0. \quad \text{(B24)}$$

This flux is minimized by a large value of E_0, which is bounded above by 3 TeV. Taking $E_0\sim3$ TeV, $C=11/3500$ as in (2.1) and, conservatively, $n_\mu<10^{-12}$ cm^{-2} sec^{-1} sr^{-1} from experiment,[51] we get

$$\lambda_\nu > 5\times10^8\text{ g cm}^{-2},\quad \sigma_{\nu p} < 3\times10^{-33}\text{ cm}^2. \quad \text{(B25)}$$

This limit becomes more severe as the threshold energy E_0 decreases, for while the cross section for W production might be anticipated to decrease as $g^2\sim GM_W^{+2}$ as M_W decreases, the flux of muons, according to (B24), increases as $M_W^{-3.4}$.

[51] F. Reines *et al.*, Can. J. Phys. **46**, S350 (1968).