It is a simple matter to verify that the second double sum involving $\tilde{d}^{\mu\nu;\lambda\kappa}$ does not contribute to the stress-energy tensor. The $l=m=0$ term of the first double sum is excluded, of course, by definition (2). The part of the first sum, with $m=0$, $l\neq 0$, gives the zero-temperature contribution to the stress-energy tensor which we have already considered; the part with $l=0$, $m\neq 0$ gives the blackbody contribution which we have just discussed. It is straightforward to show that the sum remaining with neither l nor m vanishing gives the finite-temperature, finite-plate separation correction quoted in Eqs. (17)–(22).

ACKNOWLEDGMENTS

We have enjoyed fruitful conversations on this topic with David Boulware and Robert Puff.

PHYSICAL REVIEW VOLUME 184, NUMBER 5 25 AUGUST 1969

Measurement of the Primary Cosmic-Ray Proton Spectrum between 40 and 400 GeV*

W. K. H. SCHMIDT, K. PINKAU, AND U. POLLVOGT

Max-Planck-Institut für Physik und Astrophysik, Institut für Extraterrestrische Physik, 8046 Garching bei München, Germany

AND

R. W. HUGGETT

Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana

(Received 18 November 1968; revised manuscript received 1 May 1969)

Equipment consisting of an ionization spectrometer and spark chambers has been exposed to primary cosmic rays in a balloon flight which allowed a data collection time of 14.3 h at an altitude of 5.7 g/cm^2. The purpose of this experiment was to study the flux, composition, possible time variations, and nuclear interaction properties of cosmic rays at energies between 40 and 400 GeV. The apparatus has also been exposed to 10-, 20.5-, and 28-GeV/c protons at the Brookhaven AGS in order to study the spectrometer response at three known energies and to be able to extrapolate this response to higher energies. The integral energy spectrum of primary cosmic-ray protons between 40 and 400 GeV was found to be $n(\geq E_0) = (0.91_{-0.2}^{+0.3})\, E_0^{-1.7\pm 0.1}$ (E in GeV). The corresponding intensity is a factor of 2 lower than that obtained from the Proton I and II satellite experiments.

EXPERIMENTAL PROCEDURE

THE apparatus, which has been described briefly in a previous publication,[1] is shown schematically in Fig. 1. This apparatus consists essentially of spark chambers for defining the beam, a target volume, and below these, an ionization spectrometer for determining the incident particle's energy. The spectrometer has a depth of about 3 interaction lengths.

In the flight reported here, the target consisted of Ilford G-5 nuclear emulsion.[2] In this flight, the trigger condition was set as follows. Let $Tn(m)=1$ indicate that counter Tn was required to produce a signal m times minimum ionization, and let $\bar{A}=1$ indicate that no signal has been received from counter A. If "+" is the logical "or" connection, and "o" is the logical "and" connection, the trigger requirement can be written as

$$\bar{A} \circ T7(1) \circ T6(2) \circ \{T1(13) \circ T2(13) + T3(13) \circ T4(13) + T5(13) \circ T6(13)\} = 1.$$

No pulse was required to occur from counter $T8$, since it was decided to have the possibility for the apparatus to be triggered by electromagnetic cascades from γ-rays.[3,4]

If the trigger requirement was met, the spark chambers were photographed. Alternate electrodes in this chamber were covered by 2-mm-thick glass. This made it possible to register very many simultaneous particles. The cameras did not only record sparks, but also the pulse heights of the three photomultipliers MI, MII, and MIII, each in 128 logarithmic channels. These multipliers combined the light output of each pair of adjacent scintillators, as indicated in Fig. 1. Furthermore, by photographing various discriminators and indicator lamps, the following additional data were recorded: $T8(4)$, $T8(9)$, $T8(16)$, $T7(1)$, $T7(2)$, $T1(2)$,

* Research supported jointly by the National Science Foundation under Grant No. GP/7169, and by Deutsche Forschungsgemeinschaft under Grant No. Pi 34/5.

[1] K. Pinkau, U. Pollvogt, W. Schmidt, and R. W. Huggett, in Proceedings of the International Conference on Cosmic Rays, London, 1965, p. 821 (unpublished).

[2] We have been successful in tracing many singly-charged and all multiply-charged particles from the spark chambers into the emulsion. Results on this will be published later.

[3] K. Pinkau, Phil. Mag. 2, 1389 (1957).

[4] R. Holynski, W. V. Jones, and K. Pinkau, Phys. Rev. 176, 1661 (1968).

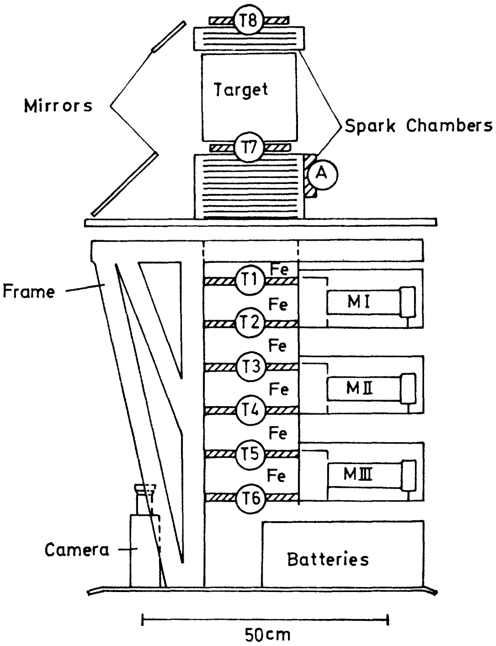

FIG. 1. Schematic drawing of the balloon instrument. Scintillation counter A is in anticoincidence, counters $T1, \ldots, T8$ record various trigger thresholds, and counters MI, MII, and MIII measure the cascade development in the ionization spectrometer.

$\overline{T1}(13)$, $T2(2)$, $T2(13)$, $T3(2)$, $\cdots$, $T5(2)$, $T5(13)$, $T6(2)$, $T6(13)$. In this way, information was obtained on the charge of the incident particle, the position of the first interaction, and additional information on the cascade development. Singly-charged particles were selected from the spark chambers' indication of the passage of a particle through counter $T8$ with the additional requirement that none of the higher levels of counter $T8$ had been triggered.

After finishing the balloon-flight program, the equipment was taken to Brookhaven National Laboratory for calibration measurements at the external No. 4 beam of the AGS. During calibration, the equipment was triggered externally by a telescope. This consisted of two scintillation counters $S1$ and $S2$ at distances 184 and 56 cm in front of counter $T8$. These counters $S1$ and $S2$ had dimensions of 12.5×12.5 cm^2. The equipment was triggered by a quadruple coincidence between counters $S1$, $S2$, $T8$, and $T7$. The spark chambers and all indicator lamps were photographed as in the balloon flights. In this way, the energy dependence of the recording efficiency of the instrument could be studied by extracting from the entire set subsamples which would have satisfied various trigger criteria.

DATA ANALYSIS AND RESULTS

The data are presented here only in so far as they pertain to the measurement of the primary proton energy spectrum. Furthermore, in the presentation of the Brookhaven results and the balloon data, we shall restrict ourselves to events in which (1) the primary particle interacts inside the spectrometer (rather than within the target, for example) for the first time, and (2) the axis passes through the entire spectrometer, with a minimum separation of 1 cm from the side of counter $T6$. It was these events which exhibited the smallest fluctuations. They can be identified clearly in both the Brookhaven and balloon data by noting the indicator lamps and the spark chambers.

We find that the sum of the three pulse heights N_1, N_2, and N_3 from the three multipliers MI, MII, and MIII is the parameter best suited to give a measure of the primary particle's energy. This sum is designated by $\sum N_i$, and is expressed in units of minimum-ionizing particles.

First, from the Brookhaven measurements, the distribution was established for the probability that a proton of energy E_0 produces a signal $\sum N_i$ in the ionization spectrometer. If we denote the average value of $\sum N_i$ by $\bar{N}$, we find that this distribution plotted as a function of $\sum N_i/\bar{N}$ changes slightly between 10 and 20.5 GeV/c, and changes even less between 20.5 and 28 GeV/c. These results are shown in Fig. 2(a).

We now assume that we have established in this way a fluctuation distribution that we may use over the entire energy range studied. In order to extrapolate to higher energies, we need only determine $\bar{N}$ as a function of energy.

This extrapolation is not straightforward, because theoretical calculations[5] that one might wish to take as a guide contain the contribution of noninteracting particles. These noninteracting particles make a large contribution at low energies. On the other hand, the balloon equipment was triggered during the flight only

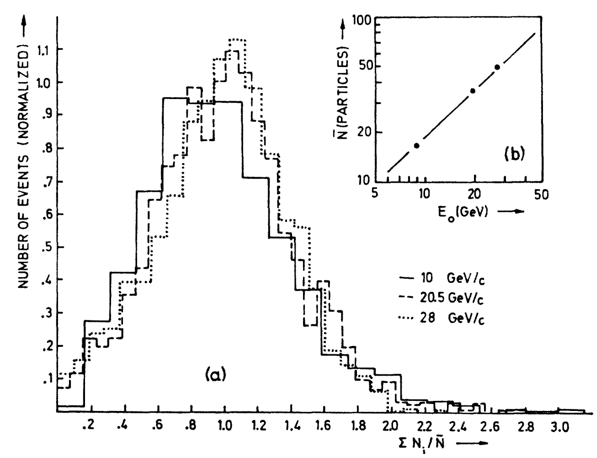

FIG. 2. Results of the Brookhaven measurements. (a) shows histograms of the number of events versus the energy deposit $\sum N_i$ in the spectrometer. $\sum N_i$ is expressed in units of its average, $\bar{N}$. The experimental results are for the three primary proton momenta indicated. (b) shows the dependence of $\bar{N}$ on the primary kinetic energy E_0.

[5] K. Pinkau and K. V. Thompson, Rev. Sci. Instr. **37**, **302** (1966).

by interacting particles. Thus, Fig. 2(a) contains also only the contribution from interacting particles.

The value of $\bar{N}$ including noninteracting particles has been plotted as a function of kinetic energy E_0 in Fig. 2(b). The straight line drawn through the points is an $E_0^{0.94}$ law which has been derived from the calculations by Pinkau and Thompson.[5] We therefore assumed this law to be valid up to the highest energies studied (about 400 GeV).

In order to obtain a correlation between $\bar{N}$ and E_0 for particles that interact inside the spectrometer and trigger the equipment, we next established a relationship between these two types of $\bar{N}$, and checked this at the three energies measured at Brookhaven. Again assuming that the contribution of noninteracting particles to $\bar{N}$ evaluated in this way was correct over the entire energy range (this essentially assumes that the total interaction cross section is constant), we could extrapolate using the $E_0^{0.94}$ law above.

Having obtained a correlation between $\bar{N}$ and E_0 for the balloon events, and having established the fluctuation distribution as a function of $\sum N_i/\bar{N}$, we now have available a distribution for the probability of recording a signal $\sum N_i$ for any primary energy E_0 in the range from 40 to about 400 GeV. We now fold this probability distribution with an assumed primary cosmic-ray spectrum of the form

$$n(\geq E_0) = AE_0^{-\gamma}$$

and determine the constants A and γ from a best fit of the resultant $\sum N_i$ spectrum to the experimental data. Figure 3(a) shows the experimental $\sum N_i$ spectrum and lines corresponding to various values of γ.

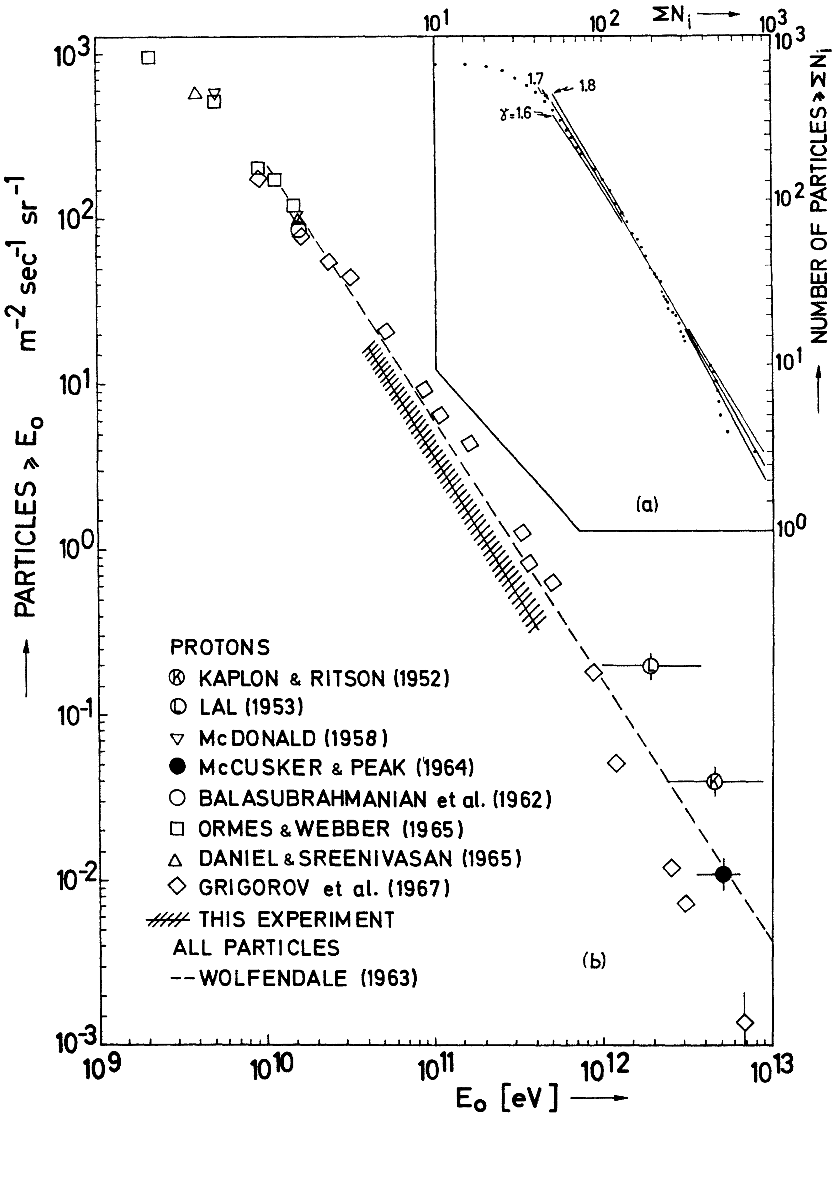

FIG. 3. Results of the balloon flight. (a) shows the experimental integral $\sum N_i$ spectrum for singly-charged particles that interact inside the spectrometer. Lines corresponding to various powers γ of an assumed primary proton power-law spectrum are shown. They are normalized to the point $n(\sum N_i \geq 200) = 37$. (b) shows the resulting primary proton integral energy spectrum together with the results of other authors (Refs. 6–14).

The fit of the line with $\gamma = 1.7$ is considered to be good.

The constant A can also be found from this fit if the geometrical factor of the instrument and the observation time are taken into account. Also, the following corrections have been considered:

(a) Change of efficiency with energy. We found that the "13-particle-requirement" was met automatically if $\sum N_i$ is sufficiently large. The deviation of the experimental points from the line $\gamma = 1.7$ below $\sum N_i = 60$ is due to the drop in efficiency. Allowance has also been made for the number of particles that do not interact in the spectrometer, and for the energy dependence and reduction in efficiency due to the $T6(2)$ requirement.

(b) Deadtime. A correction was made to account for the deadtime during which the equipment was inactivated during event analysis.

(c) Correction of overlying matter. Atmospheric absorption was taken into account using an attenuation length of 120 g/cm^{-2}. Since local interactions could be seen in the spark chambers, local matter (gondola, spark chambers, target material) was taken into account by using the inelastic interaction cross section.

The results for the primary spectrum of singly charged particles may be summarized by the expression

$$n(\geq E_0) = (0.91_{-0.2}^{+0.3}) E_0^{-1.7 \pm 0.1} \mathrm{cm}^{-2} \mathrm{sec}^{-1} \mathrm{st}^{-1},$$

with E_0 in Gev. This spectrum is shown in Fig. 3(b) together with results of other authors.[6–14] The errors indicated on our measurements include what we estimate could be systematic deviations.

DISCUSSION

Our results are consistent within the limits of error with those obtained at lower energies by McDonald[7] and Ormes and Webber.[8] Wolfendale[14] has predicted a primary energy spectrum of nucleons from the muon measurements at sea level and underground. This spectrum is consistent with our results within the limits of error, since so far we have measured only singly charged particles.

The only direct measurements which have been performed in the same energy range are those from the Proton I and Proton II satellite experiments of Grigorov *et al.*[6]

We disagree with the results of Grigorov *et al.*[6] by a factor of about 2. We feel that several arguments exist which seem to indicate that their spectrum may be erroneous. These arguments are:

(a) Over the energy range considered here, their spectrum of "all particles" and of "protons" coincides. However, a sizeable fraction of multiply-charged particles at these and higher energies have been observed in this and other[15] experiments.

(b) Their "event definition" and definition of the aperture of their equipment were based on scintillation counter evidence and not on any visual devices such as our spark chambers.

(c) They apparently did not study their equipment by exposing it to particles of known energies, as was done in our case.

ACKNOWLEDGMENTS

We thank Dr. W. V. Jones for helping with the analysis of the data. We also wish to express our appreciation to the Brookhaven National Laboratory, its 80-in. bubble chamber staff and its beam No. 4 staff, for their assistance with the calibration runs. The balloon flights could not have been successfully completed without the constant support and the great help of the NCAR balloon crew.

[6] N. L. Grigorov, V. E. Nesterov, I. D. Rapoport, I. A. Savenko, G. A. Skuridin, and F. A. Titenkov, Cosmic Research **5**, 342 (1967).

[7] F. B. McDonald, Phys. Rev. **109**, 1367 (1958).

[8] J. F. Ormes and W. R. Webber, in Proceedings of the International Conference on Cosmic Rays, London, 1965, p. 349 (unpublished).

[9] M. F. Kaplon and D. M. Ritson, Phys. Rev. **88**, 386 (1952).

[10] D. Lal, in *Proceedings of the Indian Academy of Science*, (Indian Academy of Sciences, Hebbal, Bangalore 6, India, 1953), Vol. 38, Sec. A, p. 93.

[11] C. B. A. McCusker and L. S. Peak, Nuovo Cimento **31**, 525 (1964).

[12] V. K. Balasubrahmanian, S. M. Ganguli, G. S. Gokhale, N. Kameswara Rao, P. K. Kunte, M. G. K. Menon, and M. S. Swami, in Proceedings of the International Conference on Cosmic Rays, Kyoto, 1962, Vol. III, p. 8 (unpublished).

[13] R. R. Daniel and N. Sreenivasan, Nuovo Cimento **35**, 391 (1965).

[14] A. W. Wolfendale, in Proceedings of the International Conference on Cosmic Rays, Jaipur, 1963, Vol. 6, p. 3 (unpublished).

[15] W. R. Webber, *Handbuch der Physik* (Springer-Verlag, Berlin, 1967), Vol. XLVI/2, p. 115.