Simple Model for Corrections to the Isobaric Multiplet Mass Equation*

E. M. HENLEY AND C. E. LACY
University of Washington, Seattle, Washington 98105
(Received 14 March 1969)

A simple nonperturbative model is used to estimate corrections to the first-order perturbation-theory formula for the isobaric multiplet masses. The leading-order correction to the equation, which is proportional to T_z^3, is found to be much smaller than expected from second-order perturbation arguments and from limits set by experiments.

THE completion of measurements of the masses of higher-isospin multiplets, especially all four members of several quartets,[1] allows one to test the isobaric multiplet mass equation (IMME)

$$M(A, T, T_z) = a(A, T) + b(A, T)T_z + c(A, T)T_z^2. \quad (1)$$

This relation is obtained from the first-order perturbation theory on the assumption that the charge-dependent perturbation arises from pair interactions only. The measurements[1] show the possibility of deviations from the quadratic form given by Eq. (1), and indicate the presence of a term $d(A, T)T_z^3$ of order $Z\alpha c$ (α is the fine structure constant). This is the order of magnitude anticipated from second-order perturbation theory.[2]

In this commentary, we give the results of calculations, based on a simple model, in which the Schrödinger equation is solved directly, with the charge-dependent potential included in the model Hamiltonian. The model consists of a closed, inert core plus three bound "valence" nucleons. Although this applies primarily to closed-shell-plus-three-nucleon systems, we apply it to other cases to study the Z dependence. It is assumed that each valence nucleon interacts with an average central potential produced by the core plus the other two valence nucleons. This potential includes a nuclear potential of Woods-Saxon shape, the Lane symmetry potential (proportional to $\mathbf{t}\cdot\mathbf{T}_1$, where $\mathbf{t}$ is the isospin of the "last" valence nucleon and $\mathbf{T}_1$ is the total isospin of the "first two" valence nucleons), and a Coulomb potential produced by a uniformly charged sphere of the same radius as that of the Woods-Saxon potential. The parameters for the nuclear and symmetry potentials are given in Table I.

TABLE I. Description of the single-particle potential.

$V_N = -V_0\{1+\exp[-(r-R)/D]\}^{-1}$
$V_S = (100/A)\{1+\exp[-(r-R)/D]\}^{-1}$ MeV
V_0: Adjusted as described in text.
$R = r_0A^{1/3}$
$r_0 = 1.4$ fm
$D = 0.5$ fm

In our model, only the principal parent from the parentage spectrum of the "quasicore" consisting of the inert core plus the "first two" valence nucleons is considered. We assume that this principal parent is coupled to $T_0=1$ and $J_0=0$. This implies that for each state of total isospin T, only one level of total angular momentum $J=j$ need be included.[3] We assume that Coulomb corrections from other states are negligible, since the states of the same angular momentum and parity tend to be far removed in energy.

Because the primary interest is to study the parameter d and its dependence on A, the depth of the nuclear potential was adjusted for each quartet so that the protons in the $T_z=-\frac{3}{2}$ member were bound by 0.2 MeV in the appropriate shell-model state. For the "quasicore", the Coulomb-energy differences were assumed to be derivable from Eq. (1) and were estimated from the semiempirical mass formula given by Myers and Swiatecki.[4]

For the members with $T_z=\pm\frac{3}{2}$, the nucleus is in a pure isospin state $T=\frac{3}{2}$ and the "last" nucleon satisfies a simple Schrödinger equation. For the members with $T_z=\pm\frac{1}{2}$, the nuclear state function is a mixture of $T=\frac{3}{2}$ and $T=\frac{1}{2}$ states. One obtains in these cases sets of coupled equations for the neutron and proton wave functions similar to the well-known Lane-Robson equations[5] for isobaric analog resonances.

The results of solving the coupled equations are shown in Table II, where we give the values of the coefficients b, c, and d for several values of A. For comparison, we also give the results of first-order perturbation theory and the experimental values.[1] Since the coefficient a involves the mass defect of the inert core, it is excluded.

The results of these calculations exhibit several

* Work supported in part by the U.S. Atomic Energy Commission, Grant No. AT(45-1)-1388.

[1] R. H. Stokes and P. G. Young, Phys. Rev. Letters **18**, 611 (1967); R. L. McGrath, J. Cerny, and E. Norbeck, *ibid.* **19**, 1442 (1967); H. Brunnader, J. C. Hardy, and J. Cerny, Phys. Rev. **174**, 1247 (1968); D. C. Hensley, Phys. Letters **27B**, 644 (1968); for a summary, see J. Cerny, Ann. Rev. Nucl. Sci. **18**, 27 (1968).

[2] J. Jänecke, in *Isospin in Nuclear Physics* [North-Holland Publishing Company, Amsterdam (to be published)], Chap. 8.

[3] The importance of the parentage spectrum has been pointed out especially by D. H. Wilkinson, *Isobaric Spin in Nuclear Physics* (Academic Press Inc., New York, 1966), p. 30.

[4] W. D. Myers and W. J. Swiatecki, Nucl. Phys. **81**, 1 (1966).

[5] D. Robson, Phys. Rev. **137**, 535 (1965).

TABLE II. Parameters of the IMME. The subscript p indicates the first-order perturbation, the subscript m stands for the model, and expt for the experimental results (see Ref. 1). Errors are given in keV in parentheses.

A	b_p (MeV)	b_m (MeV)	b_{expt} (MeV)	c_p (keV)	c_m (keV)	c_{expt} (keV)	d_m (keV)	d_{expt} (keV)	$Z\alpha c_{expt}$ (keV)
7	−0.69605	−0.68224	−0.588(50)	234.967	230.040	255(45)	0.65546	−11(30)	7
9	−1.14252	−1.12451	−1.332(7)	230.800	226.164	277(11)	0.06465	7.6(5)	9
13	−1.96550	−1.94171	−2.180(5)	221.059	217.234	255(4)	−0.43990	−1(12)	12
17	−2.70278	−2.68704	−2.882(7)	210.342	208.100	243(10)	−0.20362	8(42)	15
21	−3.39127	−3.37293	−3.660(9)	201.682	199.170	244(8)	−0.18168	52(21)	19
25	−4.04039	−4.01996	...	194.364	191.121	...	−0.04242	...	...
29	−4.81896	−4.77305	...	198.814	195.128	...	−0.86168	...	...
33	−5 24891	−5.22553	...	182.574	176.515	...	0.54547	...	...
37	−5.81765	−5.79338	−6.189(30)	177.711	169.604	182(30)	1.01287	5(17)	25

noteworthy features. First, the comparison with experiment seems remarkably good, considering the simplicity of the model. This indicates that Lane's symmetry potential and long-range Coulomb effects account very well for the principal features of the IMME and corrections due to refinements of the model, such as those discussed below, should be small. Of particular importance is the fact that the ratio b/c, plotted as a function of A in Fig. 1, lies close to the experimental values. This gives us confidence that the calculated ratio d/c has meaning. The ratio b/c is proportional to A, as predicted by perturbation theory.

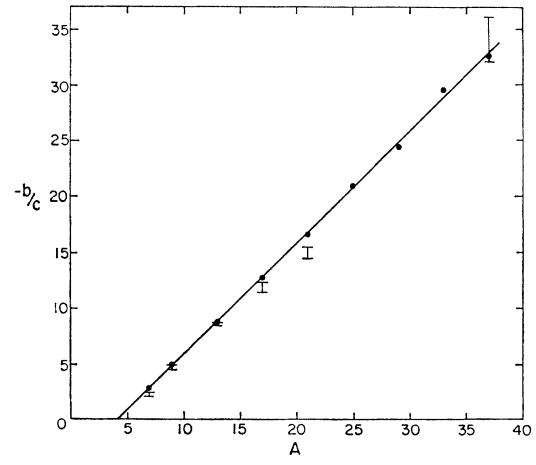

FIG. 1. Plot of the ratio $-b/c$ as a function of A.

Second, we see that the effect of the higher-order correction to the IMME is absorbed mostly in the coefficients b and c (in agreement with second-order perturbation-theory calculations[2]), since the change in these coefficients is large compared to d. Second-order perturbation theory predicts that d is only of order A^{-1} of the change of c.

Third, the calculated values of d are much smaller than $Z\alpha c$. Experimentally, the values of d are not well established (Table II), but appear to be of the order of or smaller than $Z\alpha c$.

The result of our work suggests that the coefficient d is generally much smaller than anticipated from higher-order perturbation theory; exceptions may occur for certain nuclei, when structure effects not considered in our model are taken into account.

In order to put these conclusions on firmer grounds, we are improving the model in several ways. This includes (a) the use of more realistic two-particle potentials for the residual interaction between the valence nucleons, (b) antisymmetrization of the wave functions, and therefore inclusion of more (or all) members of the "quasicore" parentage spectrum, and (c) the inclusion of other small charge-dependent effects. As was pointed out earlier, we expect, on the basis of the present calculations, that corrections from these refinements should be small.

We thank Dr. J. Cerny for making his data available to us prior to publication and Dr. L. Wilets for useful discussions.