Statistical Theory of (n, γ) and (p, γ) Excitation Functions*

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A method for the calculation of nucleon-capture cross sections and excitation functions is presented. The method is based on the statistical compound-nucleus model. The method is applied to the calculation of the excitation function for the reactions $In^{127}(n, \gamma)In^{128}$ and $Bi^{209}(p, \gamma)Po^{210}$. The result of the analysis of this paper indicates that the compound-nucleus mechanism is responsible for a considerably larger part of the cross section than previously believed.

I. INTRODUCTION

IN this paper, a method for the evaluation of cross tions and excitation functions for (n,γ) and (p,γ) reactions is presented. The method is based on the compound-nucleus statistical model. According to this picture the cross section and the excitation function depend mainly on the cross section for the formation of the compound nucleus, the radiation or γ width, and the nucleon width. The cross section for the formation of the compound nucleus is calculated using the optical model. The nucleon and γ widths are determined using the reciprocity theorem for nuclear reactions. According to this theorem, each width depends on the cross section of the appropriate inverse process. In particular, in the present study, the information about photonuclear cross sections and the shape of the giant resonance play a major role in the investigation of the radiation width.

The compound-nucleus picture proved very valuable in the interpretations of cross sections other than those related to radiative capture. However, it was realized[1–4] that the statistical model underestimates the radiative capture cross section, especially for heavy nuclei and higher nucleon energies. The lack of agreement between the statistical model and experiment was attributed to large contributions from direct reactions. Lane[5] and Lane and Lynn[6] developed the formalism for the study of direct capture reactions. Here, the previous ideas[1–3] were put into a quantitative form. The work of Lane and Lynn[5,6] was extended by many authors, chiefly by Lane and co-workers[7–9] and Brown.[10] Some of these studies compare the consequences of different models, others dwell particularly on direct reactions, and still others consider only very high-energy γ rays and semi-direct processes.

On the other hand, the statistical theory was successful in the interpretation of γ-ray spectra to various degrees of accuracy,[11–18] especially for lower energies. Benzi and Bortolani[19] applied the statistical model to the calculation of the excitation function for radiative capture. They ignored the contribution to the inverse cross section from the giant resonance. Also, their calculation is based on a nonrealistic spin dependence of the density of levels. Yet their results agree fairly well with experiment.

The latter successes of the statistical model motivated the present study. The purpose of this paper is to reexamine the applicability of the statistical model to the analysis of radiative capture processes, in particular, to the interpretation of excitation functions. It is shown here that the inclusion of the giant resonance in the cross section for the inverse process, a use of improved versions for the density of levels, and a more rigorous and up-to-date treatment of the nucleon width yield much better agreement between experiment and the statistical model. One learns, therefore, that the statistical processes play a more important role in radiative capture processes than older studies seem to indicate. It is also shown that the validity of the statistical model can be extended to higher nucleon energies. However, for very high energies (corresponding to energies above the giant resonance) and for very heavy nuclei, even the present approach fails to account for the entire (n,γ) or (p,γ) cross section.

II. THEORY

The cross section $\sigma(n,\gamma; E)$ for an incoming nucleon with an energy E, in which n stands for a nucleon and

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not for a neutron only, is written as a sum of terms

$$\sigma(n,\gamma;E;J_0)=\sum_J \sigma(n,\gamma;E;J,J_0). \quad (1)$$

In Eq. (1), $\sigma(n,\gamma;E;J,J_0)$ is the partial cross section when the intermediate state or the state of the compound nucleus has a spin J. In the compound nucleus, states with various spins are excited. In Eq. (1), J_0 is the spin of the initial state. According to the compound-nucleus theory, the cross section $\sigma(n,\gamma;E,J)$ can be written

$$\sigma(n,\gamma;E;J,J_0)=\sigma_c(n;E;J,J_0) \times[\Gamma_\gamma(E+B,J)/\Gamma_t(E+B,J)]. \quad (2)$$

In Eq. (2), $\Gamma_\gamma(E,J)$ and $\Gamma_t(E,J)$ are the radiation width and the total width, respectively, of a state with an excitation E and spin J. When the incoming nucleon has an energy E, the compound nucleus is excited to an energy $E+B$, where B is the nucleon binding energy. Therefore, the radiation width and total width are calculated for an energy $E+B$. The total width is the sum of widths for all possible decays. For most calculations, the total width is the sum of the nucleon and γ widths. Therefore, the present theory dwells mainly on the evaluation of the nucleon and γ widths.

The cross section for the formation of the compound nucleus at an energy $E+B$ with a spin J, by bombarding a target of spin J_0 with a nucleon having an energy E, becomes[20]

$$\sigma_c(n;E,J;J_0)=\pi\lambda\!\!\!^{-2}\sum_{j=|J_0-J|}^{J_0+J}\sum_{l=j-1/2}^{l=j+1|2}(2j+1)T_{l,j}(E). \quad (3)$$

The transmission coefficient $T_{l,j}(E)$ is a function of the nucleon energy E, its orbital angular momentum l, and total angular momentum j. These coefficients are determined, using the optical model. In the present paper, the transmission coefficients for neutrons and protons are obtained from the work of Mani *et al.*[21,22]

Now the nucleon width $\Gamma_n(E,J)$ is evaluated. This nucleon width is written

$$\Gamma_n(E,J)=\hbar\sum_{J'}\int_B^E S^n(E,J;E',J')dE'. \quad (4)$$

In Eq. (4), $S^n(E,J;E',J')$ is the nucleon decay rate or the probability of nucleon emission per unit energy range per unit time. This nucleon decay rate is evaluated by breaking it into a sum of terms,[23] each term $S(E,J;E',J';l,j)$ corresponding to the emission of a nucleon with a specified angular momentum l and total angular J, so that

$$S^n(E,J;E',J')=\sum_{j=|J-J'|}^{J+J'}\sum_{j=l-1/2}^{j=l+1/2}S^n(E,J;E',J';l,j). \quad (5)$$

The expression for $S^n(E,J;E',J';l,j)$ is obtained by using the reciprocity theorem for nuclear reactions. In the present treatment, the reciprocity theorem is applied to each channel with a specified l and j, so that each $S^n(E,J;E',J';l,j)$ is treated individually instead of applying the theorem directly to $S^n(E,J;E',J')$. This treatment yields[23]

$$S^n(E,J;E',J';l,j)=\frac{\sigma(E-E',l,j;E',J';E,J)}{8\hbar^2R^2\pi^4} \times\left\{\left[1-\left(\frac{l-\frac{1}{2}}{l_0}\right)^2\right]^{1/2}-\left[1-\left(\frac{l+\frac{1}{2}}{l_0}\right)^2\right]^{1/2}\right\}\frac{\rho(E',J')}{\rho(E,J)}. \quad (6)$$

In Eq. (6), R is the nuclear radius and

$$l_0=L_0/\hbar. \quad (7)$$

The critical angular momentum L_0 is related to the nuclear radius and the incoming nuetron energy E by

$$L_0=R_0(2mE)^{1/2} \quad (8)$$

In Eq. (6), $\rho(E,J)$ gives the nuclear density of states as a function of energy and angular momentum and will be discussed later in this paper. Here, $\sigma(E-E',l,j;E',J';E,J)$ is the inverse cross section for exciting a nucleus at an energy E' and spin J' to a nucleus with an energy E and spin J by absorption of a nucleon of energy $E-E'$ and of orbital angular momentum l and total angular momentum j. This cross section for the inverse process is written

$$\sigma(E-E',l,j;E',J';E,J)=(2j+1)\pi\lambda\!\!\!^{-2}T_{lj}(E-E'). \quad (9)$$

Now an expression for the radiation width $\Gamma_\gamma(E,J)$ is derived.

$$\Gamma_\gamma(E,J)=\hbar\sum_{J'=J-1}^{J+1}\int_0^E S^\gamma(E,J;E',J')dE'. \quad (10)$$

In Eq. (10), $S^\gamma(E,J;E',J')$ is the γ decay rate. Since only dipole radiation is considered, the summation over J' is from $J-1$ to $J+1$. Using the reciprocity theorem for nuclear reactions, the decay rate for γ emission can be written

$$S^\gamma(E,J;E',J') =\frac{\sigma(E-E';E',J';E,J)(E-E')^2}{\hbar^3c^2}\frac{\rho(E',J')}{\rho(E,J)}. \quad (11)$$

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In Eq. (11), $\sigma(E-E', E', J'; E, J)$ is the cross section for the absorption of a photon with an energy $E-E'$ by a nucleus with an energy E' and spin J', the photon exciting it to a state with an energy E and spin J. The inverse cross section for the photon absorption $\sigma(E-E', E'J'; E, J)$ is written

$$\sigma(E-E'; E', J'; E, J) = \sigma(E-E'; 0, J; E-E', J)(\delta_{J',J-1}+\delta_{J',J}+\delta_{J',J+1}) = \sigma(E_\gamma)(\delta_{J',J-1}+\delta_{J',J}+\delta_{J',J+1}). \quad (12)$$

The δ functions appearing in Eq. (12) reflect the selection rules for dipole transitions.

Equation (12) is based on the assumption that the photonuclear cross section of a nucleus in an excited state depends only on the γ-ray energy and not on the excitation energy or the spin of the initial state.[12] Different expressions in different energy ranges are used for $\sigma(E_\gamma)$.[6,24,25]

$$\sigma(E_\gamma)=3.8\left(\frac{E_\gamma}{7}\right)^3\left(\frac{A}{100}\right)^{7/3}\left(\frac{\Gamma_\gamma}{5}\right) \quad \text{for} \quad E_\gamma<3 \text{ MeV}, \quad (13)$$

$$\sigma(E_\gamma)=\sigma_0\frac{E_\gamma\Gamma_\gamma}{(E_\gamma-E_R)^2+\frac{1}{4}\Gamma_\gamma^2}\exp[b(E_\gamma-E_R)] \quad \text{for} \quad 3 \text{ MeV}<E_\gamma<9 \text{ MeV}, \quad (14)$$

$$\sigma(E_\gamma)=\sigma_0\frac{E_\gamma^2\Gamma_\gamma^2}{(E_\gamma^2-E_R^2)^2+E_\gamma^2\Gamma_\gamma^2} \quad \text{for} \quad E_\gamma>9 \text{ MeV}. \quad (15)$$

In Eqs. (13)–(15), all energies are in MeV and the cross section is in b. In these equations, E_R is the energy of the peak of the giant resonance, and Γ_γ is the width of this resonance. In Eq. (14), $b=0.3$ MeV^{-1}.[6]

The density of levels $\rho(E, J)$ appearing in Eqs. (6) and (11) is written as a product of a spin-dependent term and an energy-dependent term[26–29] so that

$$\rho(E, J)=(2J+1)\rho(E)\exp-[(J+\tfrac{1}{2})^2/2\sigma^2]. \quad (16)$$

The square of the spin cutoff parameter σ is related to the nuclear moment of inertia $\mathcal{I}$ and to the nuclear temperature T by[26]

$$\sigma^2=\mathcal{I}T/\hbar^2=gT\langle m^2\rangle. \quad (17)$$

In Eq. (17), g is the number of proton and neutron single-particle levels per MeV, and $\langle m^2\rangle$ is the mean square of the magnetic quantum numbers of the excited particles. It has been suggested[30–32] that for every specified energy there is a corresponding spin J_M such that at this energy there are no states with spin values higher than J_M and the density of levels vanishes. This property of the density of levels is included in the present study. The maximum spin is related to the energy by

$$J_M=(2Eg/\hbar^2)^{1/2}. \quad (18)$$

Nuclear densities of levels have been discussed by many authors.[26–29,33–38] Shell and pairing effects have been considered. In the present paper, the form suggested by Lang and LeCouteur[33] has been used. Following Lang and LeCouteur, the energy-dependent term in the density of levels $\rho(E')$ is given by

$$\rho(E')=C(E'+T)^{-5/4}\exp(2aE')^{1/2}. \quad (19)$$

Here the nuclear temperature and the excitation are related by

$$E'=aT^2-T, \quad (20)$$

where

$$E'=E \text{ for odd-odd nuclei}, \quad (21a)$$

$$E'=E-\Delta \text{ for odd-}A\text{ nuclei}, \quad (21b)$$

$$E'=E-2\Delta \text{ for even-even nuclei}. \quad (21c)$$

In Eqs. (21b) and (21c), Δ is the gap parameter. The

TABLE I. Comparison of experimental and theoretical-excitation-function results for the reaction $In^{127}(n,\gamma)In^{128}$. The cross σ_{expt} is the experimentally measured cross section, $\sigma_{theoret\,1}$ is the cross section as calculated by Benzi and Bortolani, and $\sigma_{theoret\,2}$ is the cross section as calculated in the present paper. All cross sections are in mb.

E	σ_{expt}	$\sigma_{theoret\,1}$	$\sigma_{theoret\,2}$
2.05	51±3	53.6	50.5
2.65	29±1	30.5	27.0
3.33	16.5±1	16.7	16.5
4.00	12.8±1	9.3	11.0
4.80	9.9±1	4.7	8.9
5.50	9.0±0.7	2.6	7.0
6.2	5.2±0.5	1.5	5.0
14.0	2.5±0.5	0.0	1.4

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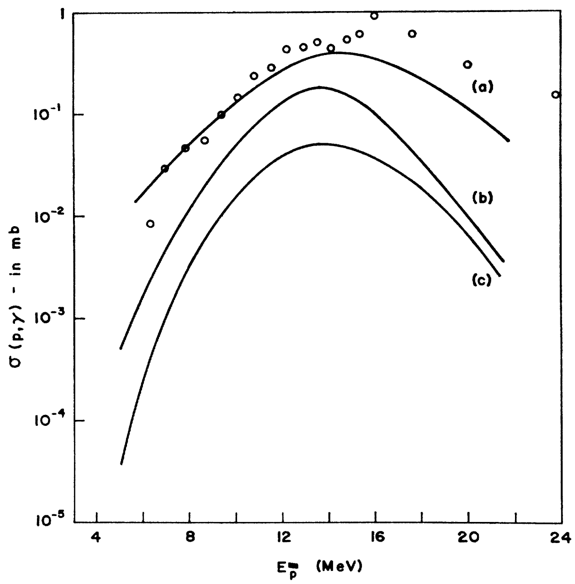

FIG. 1. The cross section for the reaction $Bi^{209}(p, \gamma)Po^{210}$ as a function of proton energy. The dots represent the observed cross sections. Curve (a) represents the results of the present calculations; curves (b) and (c) represent calculations by Lane and Lynn, using different densities of levels.

theory developed in this section is used in Sec. III for the calculation of the excitation function. The calculated excitation functions are then compared with experiment.

III. NUMERICAL RESULTS AND DISCUSSION

The power of the method described in this paper is demonstrated by performing sample calculations and comparing the results with experiment.

In the calculation performed in the present paper, the parameter "a", appearing in the form for the density of levels, is taken as $\frac{1}{10}A$ MeV^{-1} (here A is the number of nucleons). A rigid moment of inertia is assumed. The gap parameter is taken from the work of Nemirovsky and Adamchuk.[39]

Two reactions were chosen for comparison between theory and experiment, one for a (n,γ) reaction and one for a (p,γ) reaction. For a (n,γ) reaction, $In^{127}(n,\gamma)In^{128}$ was chosen. The capture cross section for this reaction was measured by Johnsrud *et al.*[40] The excitation function for the same reaction was analyzed by Benzi *et al.*[19] A comparison between the predictions of the present theory, the work of Benzi *et al.*,[19] and experiment is found in Table I. A comparison of the two theoretical results with experiment shows that the present theory is much more satisfactory, at least up to neutron energies of 14 MeV. For a (p,γ) reaction, $Bi^{209}(p,\gamma)Po^{210}$ is chosen. The excitation function for this reaction was measured by Andre *et al.*[41] and previously analyzed by Lane and Lynn.[6] A comparison between experiment, the work of Lane and Lynn,[6] and the present work is shown in Fig. 1. It can be seem from Fig. 1 that the present statistical analysis is more satisfactory than a similar analysis previously undertaken. In particular, the present theory gives excellent agree-between theory and experiment for proton energies less than 15 MeV. Above 15 MeV the discrepancy between theory and experiment, according to the present picture, is much smaller than the equivalent discrepancy arising from the previous compound-nucleus analysis.

In the present paper, the statistical model is used in a more rigorous way. The present values for the proton and neutron width is improved. This improvement stems from a more realistic evaluation of the cross section for the inverse reaction. The calculation of the inverse cross section is updated in two ways: (*i*) an optical potential replaces the simple square-well potential; and (ii) the angular momentum dependence of the inverse cross section is calculated more rigorously than in the older papers on capture processes. In the present analysis no parameters are adjusted, all parameters are obtained from the literature.

The present analysis suggests that the compound-nucleus mechanism accounts for a considerable fraction of the capture processes, certainly a larger fraction than believed in the past. This is true in particular when the energy of the nucleon is lower than the energy at which the giant resonance peaks. The compound-nucleus mechanism accounts for almost the entire cross section. On the other hand, the reduced, but existing, discrepancy between the prediction of the present theory for higher energies indicates clearly the existence of some mechanism other than a compound nucleus, one such as direct capture. However, the present analysis suggests a reduced contribution from such a mechanism.

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