$N^{15}(He^3, d)O^{16}$ Reaction and Structure of Oxygen-16†

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The structure of O^{16} has been studied by means of the $N^{15}(He^3, d)O^{16}$ reaction. The experiments were performed with 16.00- and 24.90-MeV He^3 beams from the tandem Van de Graaff accelerator at Rochester. Reaction products were analyzed and recorded automatically by means of a high-resolution magnetic spectrometer equipped with a sonic-spark counter connected to an on-line computer system. In the region of excitation up to about 22 MeV, 14 lines have been identified with levels of O^{16}, all having odd parity except the ground state (which was strongly excited), the first excited state (which was weakly excited), and the third excited state (which was barely detectable). Many well-known states were not seen. The experimental data have been analyzed by use of distorted-wave Born-approximation (stripping theory) calculations, and the results have been compared with expectations from wave functions of the shell model and of the "mixed model" exemplified in the work of Brown and Green and of Kelson. The strengths of the transitions are mostly found to be in good agreement with predictions of the mixed model and, where applicable, the shell model, although the $J=2^-$, $T=0$ state at 12.53 MeV seems somewhat too strongly excited. However, a strongly excited state tentatively identified as a $J^\pi=3^-$, $T=0$ state of O^{16} lying at 13.12 MeV is not predicted by any of the theories used in the comparisons.

I. INTRODUCTION

FOR a long time the nucleus O^{16} has received special attention because it is the lightest doubly closed-shell nucleus sufficiently complex to be interesting and challenging spectroscopically. Because it has been the subject of extensive investigation, both theoretical and experimental, a large amount of information about its states exists. The energy levels of more than 50 states are known. The spins and parities of at least 40 are known, and in many cases so are the total widths known. Several $T=1$ states are also known,[1,2] as well as the low-lying $T=2$ states.[2] Several rotational bands have been identified,[3,4] and a considerable amount of information on γ-decay transition rates has been accumulated. An abbreviated summary of the known properties[1,5] of those states of special interest in the present paper is given in Table I.

Although in recent years great progress has been made toward a satisfactory theoretical description of the low-lying states, made possible by the development of the shell model and the collective model of nuclear structure, understanding is still far from complete. It has long been known that the ground state consists mainly of a doubly closed-shell configuration. Therefore, according to the simplest shell-model picture, if a single particle is raised from the p to the sd shell, then a negative-parity state is formed, which in many cases is found to have properties closely resembling those of a known state. In 1957 Elliott and Flowers[6] made the first one-particle–one-hole ($1p-1h$) independent-particle shell-model calculation of the structure of O^{16}, allowing holes in either of the p subshells and corresponding particles in any of the sd subshells. More modern and extensive investigations along these same lines have been carried out by several workers.[7,8] These calculations have been fairly successful in predicting the correct spins and approximately the correct energies for the negative-parity states in O^{16}, but they are incomplete, because the basis sets used are too simple; in particular, they do not give an account of the positive-parity states. Many other calculations have been made in which additional, more complicated configurations have been allowed, including two-particle–two-hole ($2p$-$2h$), three-particle–three-hole ($3p$–$3h$), and four-particle–four-hole ($4p$-$4h$) combinations. Besides being more generally realistic, these calculations provide the possibility of describing even-parity states without requiring the use of shell-model orbitals beyond the sd shell.

A description of the low-lying even-parity states has emerged chiefly from the work of Bassichis and Ripka,[9]

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[1] T. Lauritsen and F. Ajzenberg-Selove, in *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences–National Research Council, Washington 25, D.C.), Report No. NRC 61-5, 6-229, 1962; F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. **11**, 1 (1959).

[2] J. Cerny, R. H. Pehl, and G. T. Garvey, Phys. Letters **12**, 234 (1964).

[3] E. B. Carter, G. E. Mitchell, and R. H. Davis, Phys. Rev. **133**, B1421 (1964); G. E. Mitchell, E. B. Carter, and R. H. Davis, Phys. Rev. **133**, B1434 (1964).

[4] J. Borysowicz and R. K. Sheline, Phys. Letters **12**, 219 (1964).

[5] C. P. Browne and I. Michael, Phys. Rev. **134**, B133 (1964).

[6] J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **A242**, 57 (1957).

[7] A. Kallio and K. Kolltveit, Nucl. Phys. **53**, 87 (1964); A. M. Green, A. Kallio, and K. Kolltveit, Phys. Letters **14**, 142 (1965).

[8] V. Gillet and N. Vinh-Mau, Nucl. Phys. **54**, 321 (1964).

[9] W. H. Bassichis and G. Ripka, Phys. Letters **15**, 320 (1965).

Brown and Green,[10] and Kelson.[11] It will be referred to here as the "mixed model," because it combines aspects of the rotational model with the shell model. The clue for its development came from the observation that the 0^+ 6.05-MeV first excited state, the 2^+ state at 6.92 MeV, the 4^+ state at 10.35 MeV, and the 6^+ state at 16.21 MeV have approximately the spacings characteristic of a rotational band.[3,4] This implied a strongly deformed intrinsic state, which suggested substantial many-particle–many-hole components. Bassichis and Ripka[9] showed that the correct energies could be achieved by use of a judicious choice of the

TABLE I. Properties of some energy levels of O^{16}.[a]

Level	E^* (MeV)	$J\pi$	T	Γ (keV)
0	0.0	0^+	0	
1	6.05	0^+	0	
2	6.131	3^-	0	
3	6.916	2^+	0	
4	7.115	1^-	0	
5	8.870	2^-	0	
6	9.614	1^-	0	650
7	9.847	2^+	0	0.8
8	10.353	4^+	0	27
9	10.952	0^-	0	
10	11.080	3^+	0	
11	11.094	4^+	0	
12	11.260	0^+	0	2500
13	11.520	2^+	0	80
14	11.630	3^-	0	1200
15	12.050	0^+	0	
16	12.437	1^-	0	89
17	12.528	2^-	0	0.8
18	12.798	0^-	1	38
19	12.964	2^-	1	2
19a	13.010	2^+		150
20	13.080	1^-	1	113
20a	13.120	3^-[b]	0	128
21	13.250	3^-	1	26
45	17.120	1^-		41
46	17.209	1^-		84

[a] Most of the information in this table has been taken from the compilations in Refs. 1 and 5.
[b] Tentative assignments (see text).

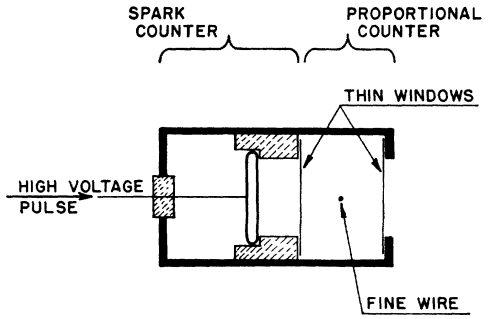

FIG. 1. Cross section of spark counter. Schematic, not to scale. The microphone is at one end of the spark gap.

amplitudes for the $0p$-$0h$, $2p$-$2h$, and $4p$-$4h$ components. It turned out that a very large $4p$-$4h$ component was required and that the calculated electromagnetic transition rates involving the members of the rotation band then could be made to agree with experimental values. A similar Hartree-Fock investigation has been reported by Kelson.[11] Brown and Green consider wave functions for both the odd-[12] and the even-[10] parity states which are obtained from shell-model states along with deformed states produced by exciting particles out of a deformed core.

Until recently the mixed model offered the most satisfactory description of the states of O^{16}, although a number of attempts were made to find an improved description within the framework of the pure shell model with an expanded basis.[13] In general, however, shell-model calculations have proved to be much better for odd-parity than for even-parity states. In particular, they have usually predicted that the first 0^+ excited state should lie far above its actual value of 6.05 MeV. In this respect Wong's shell-model results are an exception. Wong's calculations[14] allowed all possible two-phonon ($2\hbar\omega$) excitations, that is, all possible $2p$-$2h$ excitations of particles from the $1p$ shell to the sd shell and all possible $1p$-$1h$ excitations from the $1p$ to the $1f$ and $2p$ shells. However, his success in getting the correct energy for the 6.05-MeV state depended on a predicted strong excitation of the $2p$ shell, which is considered unrealistic because the $2p$ shell lies so high in energy.

More recently, the work of Zuker, Buck, and McGrory (ZBM)[15] has given strong encouragement for believing that exact pure shell-model calculations will eventually provide a satisfactory theory of O^{16}. Restricting their calculation to the $1p_{1/2}$, $2s_{1/2}$, and $1d_{5/2}$ subshells, ZBM allowed the excitation of all combinations up to $4p$-$4h$. Predicted energy levels and γ-ray

[10] G. E. Brown and A. M. Green, Nucl. Phys. **75**, 401 (1966).
[11] I. Kelson, Phys. Letters **16**, 143 (1965). However, the validity of all Hartree-Fock calculations with adjustable parameters has very recently been challenged by S. J. Krieger [Phys. Rev. Letters **22**, 97 (1969)].
[12] G. E. Brown and A. M. Green, Phys. Letters **15**, 168 (1965).
[13] N. Vinh-Mau and G. E. Brown, Phys. Letters **1**, 36 (1962); S. S. M. Wong, *ibid.* **20**, 188 (1966).
[14] S. S. M. Wong, Ph.D. thesis, University of Rochester, 1965 (unpublished).
[15] A. P. Zuker, B. Buck, and J. B. McGrory, Phys. Rev. Letters **21**, 39 (1968).

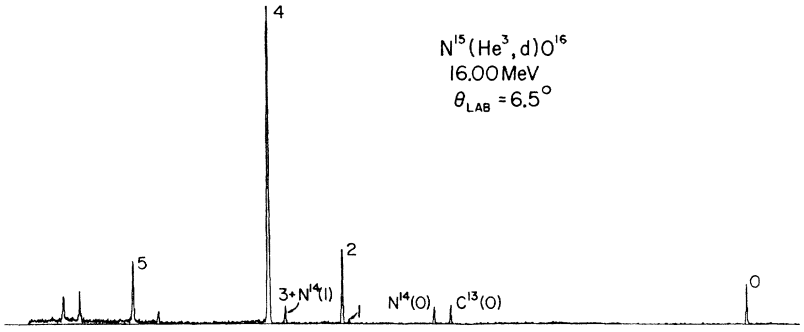

FIG. 2. Deuteron spectrum corresponding to excitation energies up to about 10 MeV in O^{16}. Target: TaN^{15} on thin self-supporting carbon backing. All groups identified with states of O^{16} are labeled in accordance with Table I. Impurity groups due to carbon and nitrogen are unusually strong here, because of the carbon backing and because of contamination of the N^{15}-gas sample used in sputtering. Here as in most spectra, all groups have been accounted for. (An exception is seen in Fig. 5.)

decay rates were found to be very satisfactory. Their calculation is obviously limited in scope. Also, it does not eliminate spurious-state effects, but it apparently gives the first successful pure shell-model explanation of the even-parity states.

The $N^{15}(He^3, d)O^{16}$ work described in the present paper was undertaken to obtain spectroscopic information for checking theoretical wave functions. We were particularly stimulated by the work of Wong.[14] In the beginning, we wished especially to study the character of the even-parity states. Surprisingly, little useful information was available when this experiment was started, because most of the existing results from nuclear reactions could not be analyzed to yield spectroscopic factors. The best cases available were from studies of the resonant scattering and capture of protons[16] and α particles,[3,17] but only the higher-lying unbound states are accessible through this means. We chose a single-nucleon transfer reaction in order to be able to make a standard distorted-wave Born-approximation (DWBA) analysis. The (He^3, d) reaction had already been established as a satisfactory spectroscopic tool. For a variety of other reasons the $N^{15}(He^3, d)O^{16}$ reaction seemed to be the best for our purposes.

II. EXPERIMENTAL METHOD

The experiments to be described were carried out with the tandem Van de Graaff accelerator at the Nuclear Structure Laboratory of the University of Rochester. Two beam energies are used, 16.00 and 24.90 MeV. Most of the work is done with the aid of a high-resolution double-focusing magnetic spectrograph of the Enge-type, although some data are taken by use of a counter telescope. The detector used with the spectrograph is a sonic spark counter[18] built six years ago, but never before used in an experiment. It is essentially one-dimensional, having a window 61.5 cm long and 1 cm high, with a proportional-counter section in front of the spark gap (see Fig. 1). For a given magnetic field strength and particle type, the equation $E_{max}/E_{min}=2$ is obtained. The proportional-counter pulses are used to trigger the logic circuits and the spark gap. They also provide a measure of the ionization density of the particles entering. Output pulses from the counter are analyzed by a computer system operated on-line. Particles are identified by type (p, d, H^3, He^3, He^4), separate spectra being accumulated automatically. To protect the counter against excessive counting rates a preset dead time is imposed electronically following each spark, and in addition, the beam is simultaneously switched off for the same length of time. Thus, the proportional counter is protected against pileup effects, and beam-current monitors can be used without live-time corrections. Dead times of 20–50 msec are used at different times, permitting maximum counting rates of 50 events per sec. The efficiency of the detector is essentially 100%. Under favorable conditions the over-all spatial resolution is better than 1 mm.

Spectra can be monitored continuously on a cathode-ray tube and can be traced out accurately by use of a Calcomp incremental plotter. Output data are stored on magnetic tape for later reference. Magnetic rigidities can be calculated quite accurately from line positions by use of the simple formula $B_\rho=B(\rho_0+\beta x)$, where ρ_0 and β are constants obtained empirically by least-squares fitting to known spectra, B is the magnetic induction in the magnet gap, and x is the coordinate of the line plotted by the incremental recorder. This method is accurate to about 0.05%. Each spectrum is stored in 2048 channels, of which about 1800 correspond to the open window of the counter. The data acquisition system will be the subject of a separate paper.[19]

The N^{15} targets were of several types, all derived from samples of nitrogen gas or ammonia, enriched to greater than 99% abundance in N^{15}. Targets most frequently used consisted of TaN sputtered onto a gold or carbon substrate. They proved stable under bombardment.

[16] D. H. Wilkinson, Phil. Mag. **1**, 379 (1956); 1031 (1956); F. B. Hagedorn, Phys. Rev. **108**, 735 (1957); D. F. Hebbard, Nucl. Phys. **15**, 289 (1960); D. S. Gemmell, B. W. Hooton, and G. A. Jones, Phys. Letters **1**, 269 (1962).

[17] R. W. Hill, Phys. Rev. **90**, 845 (1953); J. W. Bittner and R. D. Moffat, *ibid.* **96**, 374 (1954); A. J. Ferguson and G. J. McCallum, Bull. Am. Phys. Soc. **6**, 235 (1961).

[18] H. W. Fulbright, J. A. Robbins, and A. R. Hamann, University of Rochester Department of Physics and Astronomy, Report No. NYO-10261, 1963 (unpublished).

[19] H. W. Fulbright and J. A. Robbins, Nucl. Instr. Methods **71**, 237 (1969).

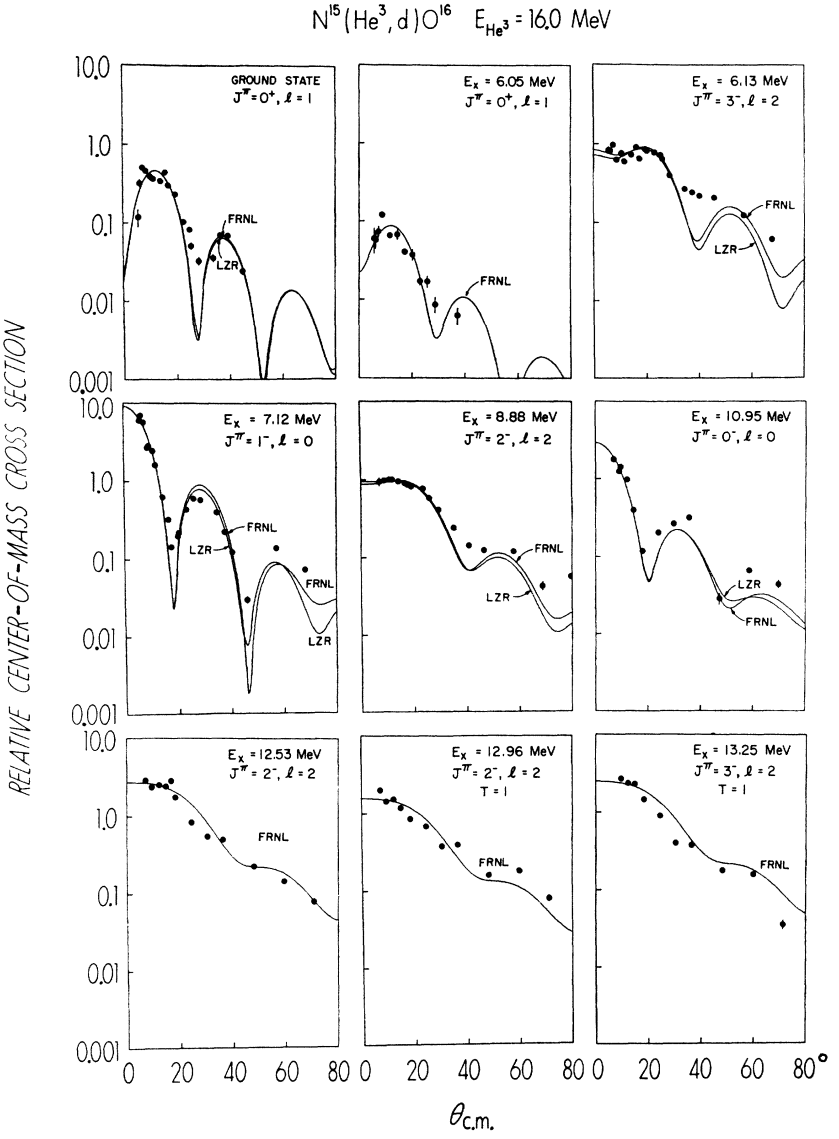

FIG. 3. Angular distributions obtained with a beam energy of 16.00 MeV. The curves represent DWBA results; the parameters used were the CW combination.

Experience showed that great care had to be exercised in preparing these targets in order to minimize the impurity content. The sputtering occurred inside a stainless-steel housing put together with gold gaskets free from ordinary O rings, grease, and other organic materials. Preliminary experience showed that when the sputtering was done with the delicate substrate foils mounted on the anode plate, they were usually broken by the discharge, so they had to be moved out of the direct line of fire. The small aluminum frames carrying the foils were supported against the inner surface of a glass cylinder surrounding the tantalum cathode disk, so that material sputtered from the cathode at angles of about 15° could strike the foils. Before the targets were formed, two preliminary steps were carried out. First, the apparatus was pumped for several hours with an oil-diffusion pump system having a large liquid-nitrogen trap, the stainless-steel housing being baked by external electric heaters during the process. Second, a preliminary sputtering operation was executed with argon gas in the system in order to clean the Ta disk and other surfaces. During this process each of the target foils was separately covered with a small hinged iron flap, which was later swung downward, out of the way, by manipulating an external magnet. Following these steps, the first filling of N^{15} gas was admitted and target sputtering was started. The total sputtering time required was about 30 min, with 10 or 20 mA at about 3000 V. Targets produced had thicknesses estimated at about 10 $\mu g/cm^2$ of N^{15}. Impurities were always present in varying amounts, the ones causing the most difficulty in the experiment being C^{12}, C^{13}, O^{16}, and to a lesser degree, N^{14}.

A gas cell was used several times for absolute cross-

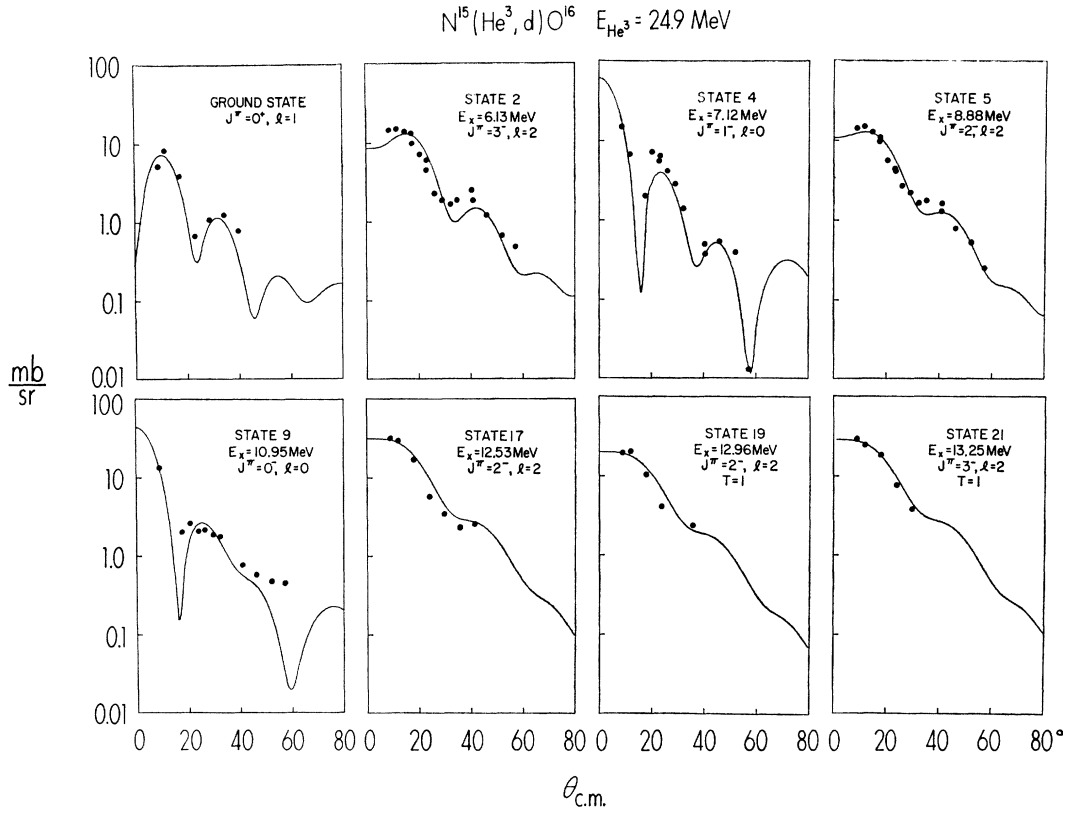

FIG. 4. Angular distributions obtained with a beam energy of 24.90 MeV, based on combined data from counter telescope and from spark counter. The curves represent DWBA fits for the potential combination DV.

section measurements and for verifying the identification of lines as due to N^{15}. It was not useful for the main part of the work because of limitations it imposed on the lower angle of observation (about 12°) and the energy resolution (about 100 keV). The beam window was made of 2 mg/cm² nickel and the exit window for reaction products was made of 0.9 mg/cm² Mylar. Its diameter was 9.0 cm. Gas pressures, measured with an absolute-pressure gauge, were about 0.1 atm.

Frozen NH_3 targets were also used. These were made inside the evacuated scattering chamber by introducing NH_4OH through a thin metal tube having a cup at its free end, the cup being swung to cover a liquid-nitrogen–cooled gold foil. The gas, admitted slowly through a needle valve, condensed on the foil; then the cup was withdrawn. Details of this target procedure will be published separately.[20]

III. EXPERIMENTAL PROCEDURE AND DATA ANALYSIS

A series of observations was first carried out with a 16.00-MeV He^3 beam, runs being made at laboratory angles 5°–80°. A typical spectrum is shown in Fig. 2. Angular distributions, shown in Fig. 3 together with the results of some DWBA calculations, agreed well

[20] H. W. Fulbright (to be published).

TABLE II. Summary of spectroscopic factors found for $N^{15}(He^3, d)O^{16}$ reaction.

State	E,	J^π,	T	16 MeV (CW) LZR	16 MeV (CW) FRNL	24.9 MeV (DV) FRNL
0	0.0	0^+	0	3.52[a]	3.52[a]	3.52[b]
1	6.06	0^+	0	0.15	0.18	...
2	6.13	3^-	0	0.63	0.67	0.59
4	7.12	1^-	0	0.53	0.53	0.48
5	8.88	2^-	0	0.60	0.56	0.50
9	10.95	0^-	0	1.05	1.15	1.35
16	12.44	1^-	0	...	...	0.25
17	12.53	2^-	0	...	2.0	1.45
18	12.80	0^-	0	...	...	...
19	12.96	2^-	1	...	1.2	0.85
20	13.08	1^-	1	...	...	...
20a	13.12	3^-	0	...	...	0.96
21	13.25	3^-	1	...	1.60	0.96

[a] Numbers in this column are normalized to be consistent with the value $S=3.52$ for the ground-state group determined by absolute cross-section measurement in the 24.9-MeV experiment.

[b] Numbers in this column are based on absolute cross-section measurements.

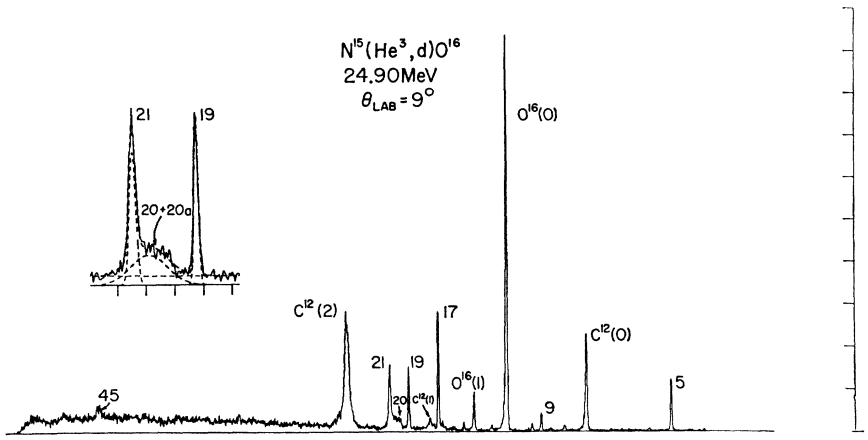

FIG. 5. Deuteron spectrum corresponding to excitation energies from about 8 to 18 MeV in O^{16}. Target: TaN^{15} on thin gold backing. The peak labeled C^{12} (2) is actually two unresolved peaks corresponding to the formation of N^{13} in its second and third excited states; the unusually large width is due largely to the natural widths (about 60 keV in each case), but the fact that the counter was set at the correct position for the reaction on N^{15} rather than on C^{12} also contributes, as it does for all impurity lines. The small peak seen just to the left of peak O^{16} (1) has not been identified. It appears only at small angles. The insert shows the result of a least-squares separation of groups 19, 20 and 20a combined, and 21, and the background from each other. The components are shown (dashed lines) separately and combined.

with theoretical expectations for a pure stripping mode of reaction. In every case, the lowest value of l (capture) consistent with angular momentum and parity conservation gave a good fit. Several observations made at backward angles showed generally small yields. Thus, it was deemed unnecessary to extend the main series of observations to larger angles. A summary of the results given in Table II shows that out of about 35 states which might have been seen, only 10 were clearly observed. A later set of observations was made at 24.90 MeV in order to look for higher states and to check the consistency of the spectroscopic factors which had been extracted from the lower-energy data. Some of the 24.90-MeV data were obtained by use of the gas-cell target, from which absolute cross sections could be calculated, as described below. A summary of the analyzed 24.90-MeV results is given in Table II and the DWBA fits, to be discussed later, are shown in Fig. 4. In spite of the fact that states up to about 22 MeV might have been observed, only one more, at 17.12 MeV, was clearly identified as belonging to O^{16}. An energy spectrum showing part of the higher excitation region is shown in Fig. 5.

A. Treatment of Data

In some spectra, background problems were encountered in treating the data, but fortunately, most lines stood out clearly upon a small background, which was often quite negligible. In the case of the 17.12-MeV state, a large, smooth background was present, but the line appears to be clearly resolved. In a number of other cases, it was difficult to decide whether the quite weak lines seen were due to weakly excited states of O^{16} or to impurities. These cases were decided either by following the kinematic shift of line position with angle or, less often, by recourse to observations made with a gas-cell target. The third-excited-state (6.92-MeV) group happened to coincide almost exactly in energy with the $C^{13}(He^3, d_1)N^{14}$ group, from which, fortunately, it could be separated at angles greater than 30°. It was very weak, sometimes much weaker than the interfering line. The lines corresponding to the 13.12- and 13.25-MeV states could not be resolved from each other because of the 128-keV width of the former. Here, a least-squares-fitting procedure was employed to determine the strengths of the two lines (see Fig. 5). The failure of this procedure to disclose evidence of the expected 1^-, $T=1$ state at 13.08 MeV is discussed below. Background problems prevented us from getting accurate measurements for state 16 [partly because of a line due to $N^{14}(d_6)$] and for state 18 [because of $C^{12}(d_1)$]. The situation was complicated by the relatively low yields expected and by the natural widths of the lines, 89 and 38 keV, respectively.

B. Normalization of Data

In general, each run was made with at least one of two means of monitoring: beam-current integration based on the Faraday-cup current, or monitor counting at 45°. Monitor counting was the more reliable, although results from the two methods were usually consistent. Once the exact shape of the fifth-excited-state (8.88-MeV) angular distribution had been established, the line due to this state could be used for internal normalization of data.

C. Absolute Cross Sections

A direct measurement of the absolute cross sections was made with the 24.90-MeV beam. The method employed involved filling the gas cell with a mixture having known partial pressures of H and N^{15} gases and observing simultaneously the deuterons from the $N^{15}(He^3, d)O^{16}$ reaction and the protons from the $H(He^3, p)He^3$ scattering. Since the cross section for the scattering of protons from He^3 has been measured by

TABLE III. DWBA parameters for the deuteron channel. [a]

	V	r_v	a_v	W	r_w	a_w	r_c	W_s	r_s	a_s	Source
A	85.3	1.25	0.61	12.8	0.96	1.58	1.3	0	...	...	12 MeV on O^{16} [b]
B	120	1.16	0.75	12.4	1.72	0.61	1.3	0	...	...	11.8 MeV on Cu [c]
C	113.4	1.4	0.607	20.0	1.4	0.607	1.3	0	...	...	11.2 MeV on O^{16} [d]
D	92.0	1.053	0.771	0.0			1.3	32.7	1.361	0.772	33.4 MeV on O^{16} [e]
F	85.3	1.11	0.71	0.0			1.3	38.1	1.37	0.67	26 MeV on C^{12} [f]

[a] Parameters are defined according to the equation for the potential $U(r)=Vf(x)+iWf(x_d)+iW_s f'(x_s)+U_c(r)$, where $f(x)$ is the usual Saxon-Wood form factor.

[b] J. L. Alty, L. L. Green, R. Huby, G. D. Jones, J. R. Mines, and J. F. Sharpey-Schafer, Phys. Letters **20**, 664 (1966).

[c] C. M. Perey and F. G. Perey (Ref. 27).

[d] W. R. Smith and E. V. Nash, Phys. Rev. **131**, 304 (1963).

[e] Reference 22.

[f] H. R. E. Tjin, A. Djie, and K. W. Brockman, Jr., Nucl. Phys. **74**, 417 (1965).

Brolley *et al.*[21] to an accuracy of 3% at a proton energy of 8.34 MeV (corresponding to the same c.m. energy as in our case), we could easily obtain the desired cross sections by comparison. Results are believed accurate to about 15%. The absolute cross section for the ground-state (g.s.) transition at 10° obtained in this way is within 10% of the value predicted by dynamic balance from the 34.4-MeV $O^{16}(d, He^3)N^{15}$ data of Hiebert *et al.*[22] No direct measurement of the absolute cross section was made for the 16-MeV data.

IV. DISTORTED-WAVE ANALYSIS

The DWBA analysis was carried out by use of the computer program DWUCK.[23] This program produces results similar to those calculated by the Oak Ridge program JULIE, although there are minor differences in the definitions of input and output quantities. In the case of the (He^3, d) reaction, the normalization factor included in the DWUCK program has the value 4.42 recommended by Bassel.[24]

The cross section for the reaction can be written in the form

$$d\sigma/d\omega=\sum_{lj}\sigma_{lj}(\theta)C^2S_{lj},$$

where l and j refer to the orbit into which the particle is captured, $\sigma_{lj}(\theta)$ is the cross section calculated by the program, C is the isospin factor for the reaction, and S_{lj} is the spectroscopic factor. For all transitions studied here $C^2=\frac{1}{2}$.

Since optical-model parameters appropriate for the analysis of our 16-MeV data were not available, a literature search was made for parameters which would reproduce the experimental angular distributions, starting with values already known to be suitable at other energies and for other target nuclei. Several different combinations were tried. These are listed in Tables III and IV. No spin-orbit potential was included in either the d or He^3 channel, and Saxon-Wood form factors were used throughout. Representative DWBA fits for the g.s. ($l=1$) and 7.12-MeV ($l=0$) states are shown in Fig. 6. It is apparent that satisfactory fits were obtained with at least two combinations of optical potentials. The CW combination was chosen for the spectroscopic analysis of the 16-MeV data (Table II). However, the spectroscopic factors proved to be relatively insensitive to changes in the optical-model parameters, the biggest deviations ($\pm 25\%$) occurring in the case of $l=0$ transfers.

The analysis of the 24.9-MeV data was made using essentially the same parameters which Hiebert *et al.*[22]

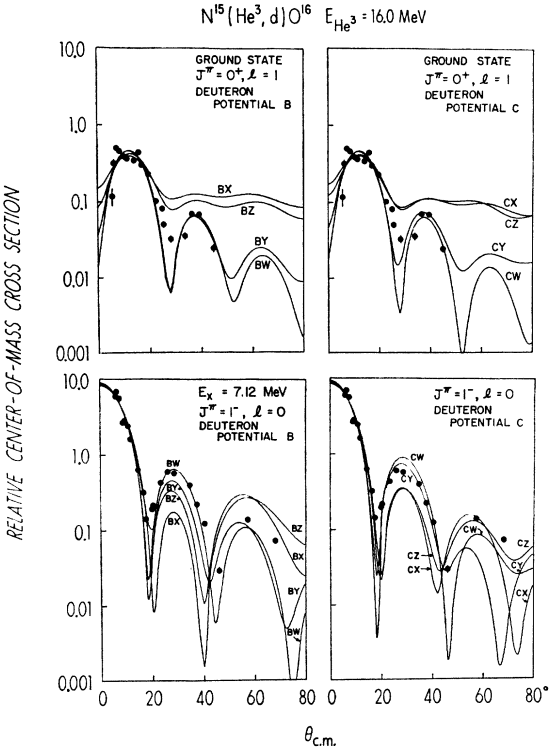

FIG. 6. DWBA fits obtained with various combinations of He^3 and deuteron optical-model parameters, labeled consistently with the entries in Table III.

[21] J. E. Brolley, Jr., T. M. Putnam, L. Rosen, and L. Stewart, Phys. Rev. **117**, 1307 (1960).

[22] J. C. Hiebert, E. Newman, and R. H. Bassel, Phys. Rev. **154**, 898 (1967).

[23] D. Kunz (private communication).

[24] R. H. Bassel, Phys. Rev. **149**, 791 (1966).

TABLE IV. DWBA parameters for the He^3 channel.[a]

	V	r_v	a_v	W	r_w	a_w	r_c	W_s	r_s	a_s	Ref.
T	146.8	1.4	0.55	18.4	1.4	0.55	1.3	0	...	...	12-MeV t on O^{16} [b]
V	180	1.14	0.68	11.2	2.17	0.43	1.3	0	...	...	c
W	166	0.98	0.81	11.0	1.71	0.78	1.3	0	...	...	16-MeV He^3 on K^{39}, averaged[d]
X	157	0.93	0.81	6.8	2.25	0.65	1.3	0	...	...	15-18-MeV He^3 on C^{12} [d]
Y	170	1.03	0.89	20.0	2.06	0.51	1.3	0	...	...	10.5-MeV He^3 on O^{16} [e]
Z	190	1.14	0.68	11.2	2.17	0.43	1.3	0	...	...	29-MeV He^3 on O^{16} [e]

[a] See Ref. a of Table III.
[b] R. N. Glover, A. D. W. Jones, and J. R. Rook, Nucl. Phys. **81**, 289 (1966).
[c] Modification of potential Z.
[d] P. E. Hodgson, Advan. Phys. **17**, 563 (1968).
[e] See Ref. e of Table III.

had found satisfactory for analysis of the data from the inverse reaction $O^{16}(d, He)N^{15}$ at 34.4 MeV; our only modification was to change the real-well depth from 190 to 180 MeV and the Coulomb radii to 1.3 F in both channels. This is the combination DV shown in Tables III and IV. One other potential was also tried (CX), which, for the $l=2$ transfers, yielded essentially the same spectroscopic factors as those shown in Table III for the DV potential. The $l=0$ and $l=1$ transfer cross sections were more sensitive to this change of potentials.

For the 16-MeV analysis, both local-zero-range (LZR) and (simulated) finite-range-nonlocal (FRNL) options were used. The finite-range parameter assumed[23] was 0.770, and the nonlocality range was taken to be 0.850 for the bound state, 0.30 for the He^3 channel,[24,25] and 0.54 for the deuteron channels. Both the LZR and FRNL options yielded essentially the same results, as can be seen in Fig. 3 and in Table II. For the 24.9-MeV analysis, only the FRNL option was employed, yielding the fits shown in Fig. 4. The experimental points in Figs. 3 and 4 have not been corrected for the effect of finite angular resolution. The angular width of the entrance aperture was often 6°, which is large enough to have had an appreciable effect in the regions where sharp valleys and peaks are seen. In all cases the lowest value of l consistent with angular momentum and parity conservation gave a good fit.

TABLE V. Spectroscopic factors from $N^{15}+p$ resonant capture and scattering experiments compared with values from the present work and from the $N^{15}(d, p)N^{16}$ reactions.

No.	J^π,	T	Exc. (MeV)	S ($N^{15}+p$)	S (He^3, d)	S^a $N^{15}(d, p)N^{16}$
16	1^-	0	12.44	0.84	0.25[b]	
17	2^-	0	12.53	1.30	1.45	
18	0^-	1	12.80	0.94	...[b]	0.48
19	2^-	1	12.96	1.50	0.85	0.81
20	1^-	1	13.08	0.84	...[b]	1.15
21	3^-	1	13.26	1.10	0.96	1.00

[a] These values are for the $T=1$ states in N^{16} analogous to those listed for O^{16}. They were obtained by making a DWBA analysis of the data of P. V. Hewka, C. M. Holbrow, and R. Middleton [Nucl. Phys. **88**, 561 (1966)]. They have been normalized to give the best agreement with the (He^3, d) results in states 19 and 21.
[b] These values from our experiment are uncertain or missing, at least partly because of experimental difficulties.

The application of DWBA analysis to the cases of states lying at 12.44 MeV and higher was complicated because they are all unbound against proton emission. Therefore, strictly speaking, ordinary DWBA analysis is not valid. (The usefulness of the more rigorous treatment of this problem proposed by Huby and Mines[26] apparently has not yet been established.) A procedure commonly adopted in the face of this difficulty is arbitrarily to assign a small binding energy to such states, then to make the calculation in the usual way. We have also adopted this questionable procedure, arbitrarily assigning a binding energy of 1 MeV to all unbound states. To get some idea of the uncertainty which might be inherent in the results, a series of calculations was made at 24.9 MeV for the cases of state 19 (12.96 MeV) with an $l=2$ transfer and state 18 (12.80 MeV) with an $l=0$ transfer, for various assumed binding energies. Results showed that for the DV potentials of Tables III and IV the calculated peak cross section for the $l=2$ transfer rises by a factor of 2 as E_B is changed from 2.5 to 0.2 MeV, but that the $l=0$ transfer appears to be quite insensitive. The shapes of the angular distributions in both cases were found not to change significantly. We also tried altering the value of the depth of the real potential well in the deuteron DWBA parameters to see the effect on the cross sections in the belief that it would represent a check on the sensitivity of the results to the energy of the outgoing deuterons. We chose an increase in the real-well depth of 1 MeV per MeV of excitation, with no change in the

[25] F. G. Perey and B. Buck, Nucl. Phys. **32**, 353 (1962); F. G. Perey and D. Saxon, Phys. Letters **10**, 107 (1964); P. J. A. Buttle and L. J. B. Goldfarb, Proc. Roy. Soc. (London) **83**, 701 (1964); J. K. Dickens, R. M. Drisko, F. G. Perey, and G. R. Satchler, Phys. Letters **15**, 337 (1965).

[26] R. Huby and J. R. Mines, Rev. Mod. Phys. **37**, 406 (1965).

TABLE VI. Comparison of experimental and theoretical spectroscopic factors S_{lj}.

No.	State E	J^π	T	Experimental (l,j)	Our work[a]	Seth[b]	$N^{15}(d,n)$[c]	Theoretical (l,j)	Shell[d,e]	ZBM[f]	GVM[g,e]
0	0.0	0^+	0	$1,\frac{1}{2}$	3.52	3.6	3.5	$1,\frac{1}{2}$	4.0	2.83	4.0
1	6.06	0^+	0	$1,\frac{1}{2}$	0.16	<0.8	...	$1,\frac{1}{2}$	...	0.29	...
2	6.13	3^-	0	$2,\frac{5}{2}$	0.63	1.0	1.0	$2,\frac{5}{2}$	0.76	1.03	0.87
4	7.12	1^-	0	$0,\frac{1}{2}$	0.54		0.35	$0,\frac{1}{2}$	0.61	0.91	0.48
								$2,\frac{3}{2}$	0.06		0.21
5	8.88	2^-	0	$2,\frac{5}{2}$	0.55	1.0	0.80	$2,\frac{5}{2}$	0.76	0.91	0.94
								$2,\frac{3}{2}$	0.07		0.01
9	10.95	0^-	0	$0,\frac{1}{2}$	1.20			$0,\frac{1}{2}$	0.98	0.94	1.0
16	12.44	1^-	0	$0,\frac{1}{2}$	0.25			$2,\frac{3}{2}$	0.32		0.02
								$0,\frac{1}{2}$	0.06	0.07	0.15
17	12.53	2^-	0	$2,\frac{3}{2}$	1.45			$2,\frac{3}{2}$	0.38		0.74
								$2,\frac{5}{2}$	0.24	0.03	0.05
18	12.80	0^-	1	$0,\frac{1}{2}$	...			$0,\frac{1}{2}$	1.0	0.94	1.0
19	12.96	2^-	1	$2,\frac{5}{2}$	0.85			$2,\frac{5}{2}$	0.96	0.92	0.96
20	13.08	1^-	1	$0,\frac{1}{2}$	...			$0,\frac{1}{2}$	0.96	0.95	0.98
20a	13.12	3^-	0	$2,\frac{5}{2}$	0.96			$2,\frac{5}{2}$	0.21	0.01	0.11
21	13.25	3^-	1	$2,\frac{5}{2}$	0.96			$2,\frac{5}{2}$	0.96	0.89	1.0

[a] These spectroscopic factors are an average of our 16.0- and 24.9-MeV data, except for the unbound states (16 and above), where only the 24.9-MeV data have been used.

[b] K. K. Seth, Proceedings on Direct Reactions by He^3, Saitama, Japan, 1967, p. 179 (unpublished); private communication.

[c] H. Fuchs, K. Grabish, P. Kraaz, and G. Röschert, Nucl. Phys. **A105**, 590 (1967).

[d] Based on calculations made with the Rochester-Oak Ridge shell-model code.

[e] Calculated assuming a pure $p_{1/2}$ proton hole for the N^{15} ground-state wave function.

[f] Reference 15. We are indebted to Dr. McGrory for a listing of ZBM wave functions.

[g] Reference 8.

imaginary well. This is consistent with very rough extrapolation of previous results.[27] With $E_B=1$ MeV, the peak cross section increased by a factor of 1.5 for the $l=0$ transition of state 18, while the $l=2$ transition for state 19 showed a slight decrease. From all these tests we concluded that the spectroscopic factors obtained for the unbound states might well be in error by as much as a factor of 2, but could be considerably better. Fortunately, reduced proton widths for 6 of the unbound states of interest here are known from earlier experiments on the resonant scattering and absorption of protons.[16,28] A comparison with the results of our analysis is shown in Table V. The S values from the $N^{15}+p$ experiments, deduced from the widths given by Lane,[28] are rough due to the difficulties inherent in the analysis, but it is encouraging that in the three cases where we have good data they agree reasonably well with ours. This tends to support our handling of the unbound-state problem.

Spectroscopic factors averaged from our 16.0- and 24.9-MeV data are shown in Table VI together with experimental values obtained by others who have studied the $N^{15}(d,n)O^{16}$ and $N^{15}(He^3,d)O^{16}$ reactions. In general, the agreement is fairly good. Table VI also shows the expected overlaps with various theoretical calculations. The column labeled "shell" gives results based on our calculation made using the Oak Ridge-Rochester Shell-Model Program[29] which, with the Rosenfeld force, after removal of spurious effects from the 1^- states, gave results essentially identical with those of Elliott and Flowers[6]; the column labeled "GVM" contains results from the calculations of Gillet and Vinh-Mau.[8] In both these cases the calculations of S were made assuming a pure $p_{1/2}$ proton-hole configuration for the N^{15} ground state. The column headed "ZBM" is from the work of Zuker, Buck, and McGrory,[15] which shows a 30% $(2p\text{-}3h)$ component in the N^{15} ground-state wave function.

V. STRUCTURE OF O^{16}

We now discuss the implications of our results with respect to existing theories of the structure of O^{16}. We

[27] F. G. Perey, Phys. Rev. **131**, 745 (1963); C. M. Perey and F. G. Perey, *ibid.* **132**, 755 (1963); L. L. Lee, J. P. Schiffer, B. Zeidman, G. R. Satchler, R. M. Drisko, and R. H. Bassel, *ibid.* **136**, B971 (1964); E. F. Gibson, B. W. Ridley, J. J. Kraushaar, M. B. Rickey, and R. H. Bassel, *ibid.* **155**, 1194 (1967).

[28] A. M. Lane, Rev. Mod. Phys. **32**, 519 (1960).

[29] Our use of the Oak Ridge-Rochester Shell-Model Code; J. B. French, E. C. Halbert, J. B. McGrory, and S. S. M. Wong, in *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum Press, Inc., New York, 1969), Vol. 3.

shall refer to the wave functions of Gillet and Vinh-Mau (GVM),[8] Brown and Green (BG),[10] Brown and Shukla (BS),[30] Kelson (K),[11] Wong (W),[14] and Zuker, Buck, and McGrory (ZBM).[15] In addition, we shall use the results of the shell-model calculation made here.[29]

According to the simplest single-particle shell-model picture, O^{16} in its ground state consists of a doubly-closed-shell structure, and N^{15} in its ground state has the same structure but with a proton hole in the $p_{1/2}$ shell. Accordingly, the spectroscopic factor for the $N^{15}(He^3, d)O^{16}$ ground-state transition would be $S=4$, and for the low-lying odd-parity states the spectroscopic factors would all be $S=1$. More realistic models lead to much more complicated wave functions. Nonetheless, for some states, these have dominant components reflecting the simple picture.

A. Ground State

Most calculations have shown the ground state to be predominantly doubly closed shell in character, with an admixture of $2p$-$2h$ and, to a lesser degree $4p$-$4h$ components. In calculating S, we have used three different ground-state wave functions for N^{15}, the choice in each case being matched with respect to theoretical origin to the choice of the O^{16} wave function. Using BS and BG wave functions yields the estimate $S_0=2.5$ (the subscript refers to the state number given in Table I), in only rough agreement with the average experimental value of 3.5 from Table VI. However, the calculated value is uncertain because it neglects any overlap due to the deformed parts of the wave functions. Using ZBM wave functions for both N^{15} and O^{16} gives $S_0=2.83$, which again is somewhat small. The simple shell-model calculation gives $S_0=4$, which appears to be in best agreement with the data. The experimental value is in quite good agreement with the FRNL results of Hiebert *et al.*[22] for the inverse reaction.

B. 6.05- and 11.26-MeV 0^+ States

Until very recently, pure shell-model calculations restricted to the $1p$ and sd shells have consistently failed to give a satisfactory prediction of the position of the 6.05-MeV state. The wave function is now thought to contain a large amount of $4p$-$4h$ structure, because the most plausible calculations which have managed to put this state at the right energy contain such a component.[9–11,15] The other sizable component is the doubly closed shell, estimated variously to be 2% (K), 8.2% (BG), and 10.7% (ZBM) of the total. Using the BS and BG combination, again neglecting deformed-component contributions, we find a predicted $S_1=0.24$. The ZBM combination gives 0.29. The BS and K combination gives the rough estimate $S_1=0.07$. These numbers are to be compared with our value $S_1=0.16$ and are thus all in reasonable agreement with the data. Wong's calculation[14] on the other hand, predicts a large $(1p^{-1}$-$2p)$ component for this state, which is not supported by the experimental results. According to BG, state 12, 0^+, 11.26 MeV, should be seen about twice as strongly as the 6.05-MeV state. However, the lack of evidence for it in our data is not surprising, because of its great width, 2.5 MeV.

C. 6.92- and 9.85-MeV 2^+ States

The 6.92 state is generally believed to be the second member of a rotational band headed by the 6.05-MeV 0^+ state. It is 100% "collective", according to Kelson,[11] and is 85% $4p$-$4h$ and 15% $2p$-$2h$, according to Brown and Green.[10] Similarly, the 9.85-MeV state is believed to be the first member of a $K=2$ band[4] and the SU_3 calculations of Arima[31] predict it to be also 85% $4p$-$4h$. The shell-model calculations of ZBM give these states an explicit structure having large components of both $4p$-$4h$ and $2p$-$2h$ terms, but necessarily predicting zero overlap in the $N^{15}(He^3, d)O^{16}$ reaction, since a $p_{3/2}$ or $f_{5/2}$ transfer is required. Hence, our observation that these states were almost undetectable is quite consistent with all predictions.

D. Negative-Parity States Stable with Respect to Proton Emission

States observed are number 2(6.13, 3^-), 4(7.12, 1^-), 5(8.88, 2^-), and 9(10.95, 0^-). Because the quality of the fits was generally good when a single l value was assumed, the S factors shown in Table VI are based on pure l transfers. (For the case of the 7.12-MeV level, a least-squares procedure in which both an $l=0$ and an $l=2$ component were allowed produced a best fit with essentially zero $l=2$ amplitude.) The agreement between theory and experiment is best with the simple shell model. The ZBM calculations lead to S values consistently too high. For state 9, however, all the theoretical predictions give essentially the same result, in relatively good agreement with the tabulated experimental value of 1.20. (In fact, the value of S extracted for this state drops to about 0.90 when the real part of the deuteron optical potential is raised from 92 to 100 MeV.) States 6(9.61, 1^-) and 14(11.63, 3^-) were not seen. They are both rather broad (650 and 1200 keV, respectively[1]) so they were not expected to stand out strongly. The background in the neighborhood of state 6, however, was so small that we can state that this level was excited only weakly, at best. The background and impurity structure around state 14 was much more intense. According to Kelson,[11] these states are 84 and 92% collective, respectively, so they should not be strongly excited in the $N^{15}(He^3, d)$ reaction. This result is consistent with shell-model calculations for these states[8,15] and with the calcula-

[30] G. E. Brown and A. P. Shukla, Princeton University Report No. 937-268 (unpublished).

[31] A. Arima (private communication).

tions of Brown and Green,[12] which suggest dominant ($3p$-$3h$) wave functions. The 9.61-MeV 1^- state is thought[4,12] to be the first member of a negative parity rotational band.

E. Negative-Parity States Unstable with Respect to Proton Emission

Beginning with state 16, all the states listed in Table VI are unstable against proton emission. Experimental values for S are uncertain because of the unreliability in the use of the DWBA program for unbound states, as mentioned above. Mostly they are around unity, except for $S_{17}=1.45$, which seems anomalously high. Our simple shell-model calculations show that the wave function of state 17 should be mostly $-0.62\,|\,p_{1/2}^{-1}d_{3/2}\rangle-0.57\,|\,p_{3/2}^{-1}d_{5/2}\rangle+0.49\,|\,p_{1/2}^{-1}d_{5/2}\rangle$; and the GVM result is similar, but with coefficients 0.86, -0.44, and -0.21, respectively. Kelson gives it a predominantly $d_{3/2}$ character, while the ZBM results predict two close-lying 2^- states in this region, both expected to be very weakly excited (their average strength is given in Table VI) in disagreement with observations.

State 16(12.44, 1^-) was observed only in the angular range 5°–25°. Since strong forward peaking was evident, $l=0$ capture was assumed in the analysis. Using our shell-model results for the second 1^- state, predicted to lie at 16.6 MeV, we get $S_{16}=0.38$ (of which 0.32 is due to s capture), in reasonably good agreement with the experimental value 0.25. In using the results for the second rather than the third-predicted 1^- state we are assuming that state 6(9.614, 1^-) has a structure not predictable by the simple shell-model calculation, as mentioned above and as suggested by Kelson's work.[11] The third state would yield $S=0.60$, which disagrees with experiment.

Around 13 MeV lie four $T=1$ states corresponding to the four lowest states of N^{16}. Since the character of these states has long been established,[1] we expected to be able to observe them with ease. States 19 and 21 were strongly populated. State 18 was very weakly excited. In fact, we could not get a reliable measurement of its strength. In the case of state 20(13.08, 1^-), we found an unexpected $l=2$ angular distribution and a yield too large to correspond to the excitation of a spin-1 state. Observation of this state, when the gas-cell target was used, confirmed that it was due to O^{16}, assuming that our N^{15} sample did not contain an impurity having an unusually high (He^3, d) cross section. It was apparent that our results required spin 3, which seemed in contradiction with the established level scheme and with the firm expectations for the $T=1$ state pattern. We now believe that, in all likelihood, two states lie very near each other in this region, one being the expected state 20, the second, here called state 20a, being a 3^-, $T=0$ state. Support for this belief comes from the $N^{15}(p,\gamma)$ work of Earle and Tanner,[32] who found evidence for a substantial amount of d-wave capture in this region of excitation and from the work[33,34] of several groups on the resonant scattering of α-particles from C^{12}. Of the latter, the careful analysis of Morris, Kerr, and Ophel[33] gives strong evidence for a 1^- state at 13.080±0.010 MeV and a 3^- state at 13.120±0.010 MeV, having total widths 113±15 and 128±11 keV, respectively. A 2^+ state with a width of 150 keV which they observed at 13.010±0.010 MeV, we would not expect to see.

In order to separate the counts due to states 20 and 20a (combined) from those due to state 21, it was necessary to use a least-squares peak-fitting program. The decomposition is shown in Fig. 5. The result for states 20 and 20a was a clear $l=2$ angular distribution with no evidence of any $l=0$ contribution from state 20 (the shape and strength were almost identical with those of state 21). Perhaps this should not be considered surprising, in view of the difficulties involved, since DWBA calculations suggest that $d\sigma_{20}$ should be no greater than $0.2d\sigma_{20a}$ at $\theta=10°$. An early plane-wave analysis of $N^{15}(d,p)N^{16}$ data[35] shows that the four analog states in N^{16} are populated with about equal strengths. The results of DWBA analysis of more recent data from the same reaction are shown in Table V. Corresponding to states 19 and 21, they show S values agreeing within 4%, and corresponding to states 18 and 20 they show $S=0.48$ and 1.15, respectively. These values will be used below in discussing sum rules.

The simple shell model predicts additional 3^-, $T=0$ states at about 16 and 22 MeV, but with very little yield in the $N^{15}(He^3,d)O^{16}$ reaction. The ZBM calculation places such a state at 16 MeV, again with a very small yield expected. None of the theoretical results used here has predicted such a state. Interestingly, this same state seems to be strongly excited in the α-capture reaction on C^{12}.[33,34]

Between 13.25 and 22 MeV we have identified only one state of O^{16}, corresponding to an excitation of 17.12 MeV. This could be the 1^-, $T=1$ state listed in Table I at this energy, possibly also corresponding to the 1^- state at 17.2 MeV or the 2^- state at 17.7 MeV, predicted by Kelson[11] to be predominantly $(p_{1/2}^{-1}d_{3/2})$. A possible assignment of 1^+ was reported[36] for a state at 17.14±0.02 MeV, having a width of 80 keV, excited in the $N^{14}(He^3,p)O^{16}$ reaction. If these are the same state, then there is an apparent contradiction in spin assignments. Our results show this state to be fairly strongly excited. Its strength would not be consistent with a 1^+ assignment.

[32] E. D. Earle and N. W. Tanner, Nucl. Phys. **A95**, 241 (1967).
[33] J. M. Morris, G. W. Kerr, and T. R. Ophel, Nucl. Phys. **A112**, 97 (1968).
[34] G. E. Mitchell, E. B. Carter, and R. H. Davis, Phys. Rev. **133**, B1434 (1964).
[35] E. K. Warburton and J. N. McGruer, Phys. Rev. **105**, 639 (1957).
[36] J. R. Comfort, J. E. E. Baglin, and M. N. Thompson, Bull. Am. Phys. Soc. **13**, 608 (1968).

VI. APPLICATION OF SUM RULES

The strength associated with a transition is defined[37] as

$$G_{lj}=(2J+1/2J_0+1)C^2S_{lj}.$$

For the $N^{15}(He^3, d)O^{16}$ reactions the two sum rules which we shall apply are

For $T=0$, $\sum G_{lj}=\langle$(No. of proton holes)$-\frac{1}{2}$(No. of neutron holes)$\rangle$

For $T=1$, $\sum G_{lj}=\frac{1}{2}\langle$No. of neutron holes$\rangle$

For $T=0$, the sum-rule limit for the $1p_{1/2}$ transfer is 1. For $T=0$ or $T=1$, the limit for $1d_{5/2}$ is 3, for $2s_{1/2}$ is 1, and for $1d_{3/2}$ is 2. A summary of the strengths calculated from our results (Table VI) is given in Tables VII and VIII.

The strengths for the $l=2$ transfers have been put into two columns, depending on the orbit believed principally involved. The assignments have been made as follows: Assuming that the N^{15} wave function is a pure $p_{1/2}$ hole, the $d_{5/2}$ assignment must be correct for the 3^- states. In the case of the 2^-, $T=1$ state, all theories show a strongly dominant $d_{5/2}$ component. The assignment is less clear for state $5(2^-, T=0)$; in fact, some small fraction of the strength could be assigned to $d_{3/2}$, depending on the wave function used: about 10% for our shell-model calculations and about 1% according to GVM. For state 17, $(2^-, T=0)$ the case is much less clear, partly because the strength is anomalously large and partly because there is less certainty about the wave function. The $p_{1/2}^{-1}d_{3/2}$ component is "dominant" according to Kelson and is 74% of the wave function according to GVM, whereas our shell-model result is only 38%.

Several interesting conclusions can be drawn from the $T=0$ results of Table VII. First, the ground-state and the first-excited-state transitions together account for over 90% of the $p_{1/2}$ strength. If, as expected according to the mixed model, a substantial amount of strength should be associated with the 0^+ state at 11.26 MeV,[10] which our experiment could not have detected, then those three states together would exhaust the entire strength available. Second, if we consider state 16(12.44 MeV, 1^-) to be due to an $l=0$ transfer, then the $2s_{1/2}$ sum rule for the $T=0$ states would be almost 90% exhausted. Third, even allowing for a factor of 2 decrease in the spectroscopic strengths of the unbound states 17 and 20a (perhaps attributable to uncertainty in the DWBA analysis), the total $l=2$, $T=0$ strength would still be larger than the $1d_{5/2}$ limit, a substantial amount of $d_{3/2}$ strength clearly being required. In fact, if, as shown in Table VII, we attribute state 17(12.53, 2^-) to a $d_{3/2}$ transfer, then its strength is easily absorbed in the $d_{3/2}$ sum. The 1^-, $d_{3/2}$ $T=0$ strength has either not been observed or is a contributor to state 16. The latter possibility is consistent with shell-model expectations[8,29] and with the calculations of Kelson.[11] State 16 was observed at only a few forward angles, so we were not able to determine experimentally the relative $l=0$ and $l=2$ transfer contributions, although our analysis clearly indicates a substantial $l=0$ yield. The $d_{5/2}$ sum-rule limit is slightly exceeded, quite possibly because of uncertainties associated with state 20a.

It is interesting to observe that if we do not attempt to distinguish between the $d_{3/2}$ and $d_{5/2}$ possibilities, we find that the sum of all the $l=2$ strengths is 8.1, very nearly equal to 8, the sum-rule limit in the absence of any $d_{3/2}$, $T=1$ contribution.

Because of our failure to get S values for states 18 and 20, we use the corresponding values from the $N^{15}(d, p)N^{16}$ analysis. Table VIII shows that when this is done about 98% of the $s_{1/2}$ strength is accounted for.

For the $T=1$, $l=2$ transitions, the $d_{5/2}$ strength is sufficient to take care of all the observed total; in fact, if we assign all the $l=2$ strength to $d_{5/2}$ transitions, the sum rule is 90% exhausted, in good agreement with theoretical expectations. For the $d_{3/2}$, $T=1$ strength, the broad level ($\sim$80 keV) observed at 17.12 MeV could well be the 1^- state predicted by Kelson[11] at 17.2

TABLE VII. Application of sum rules ($T=0$ states).

State		C^2S	G_1	G_0	G_2 ($d_{3/2}$)	G_2 ($d_{5/2}$)
0	0^+	1.76	0.88			
1	0^+	0.08	0.04			
2	3^-	0.32				1.12
4	1^-	0.26		0.39		
5	2^-	0.28				0.70
9	0^-	0.60		0.30		
16	1^-	0.12		0.18		
17	2^-	0.73			1.82	
20a	3^-	0.48				1.69
		Totals	0.92	0.87	1.82	3.51

TABLE VIII. Application of sum rules ($T=1$ states).

State		C^2S	G_0	G_2 ($d_{5/2}$)
18[a]	0^-	(0.24)	(0.12)	
19	2^-	0.43		1.08
20[a]	1^-	(0.58)	(0.86)	
21	3^-	0.48		1.69
		Totals	(0.98)	2.77

[a] Assumed from the $N^{15}(d, p)N^{16}$ analysis. See Table V.

[37] M. H. MacFarlane and J. B. French, Rev. Mod. Phys. 32, 567 (1960); J. B. French and M. H. MacFarlane, Nucl. Phys. 26, 168 (1961) (the $T\rangle$ and $T\langle$ of Eq. 7 should be reversed); J. P. Schiffer, J. Phys. Soc. Japan Suppl. 24, 324 (1968).

MeV, as mentioned above. We do not observe the 2^- $d_{3/2}$ state which he predicts to fall at 17.7 MeV, although some of this strength could be contained in the 12.96-MeV transition, which our $l=2$ sum rule could not have detected.

VII. CENTROID-ENERGY CONSIDERATIONS

The values listed in Table VI for the strengths of various transitions can be used in a centroid-energy calculation to get a rough check on the consistency of the assignments of dominant orbitals indicated in the column labels. For example, the $d_{5/2}$ strengths for all the $T=0$ and $T=1$ states can be multiplied by the energies of excitation of the corresponding state. Then, the sum of the products can be divided by the total strength to give an experimental centroid energy for the $d_{5/2}$ level. One gets 11.4 MeV, which can be compared with a rough estimate based on the simple single-particle shell model: taking values for the energies of particles in the various subshells around O^{16} from Wong,[14] we find that raising a $p_{1/2}$ particle to the $d_{5/2}$ level should require about 11.45 MeV, in good agreement with the experimental value. This result supports the assumption that the principal component of the N^{15} ground-state wave function is a $p_{1/2}$ hole. In the case of the $2s_{1/2}$ level we have reliable strength values only for the $T=0$ states, but we can use these together with the value expected from the simple single-particle shell model to predict the centroid energy for the $T=1$ contributions. The result is about 13.4 MeV, which is in satisfactory agreement with the value 13.0 MeV for the centroid energy found if states 18 and 20 are assigned the S values used in the sum-rule discussion. In the $d_{3/2}$ case, a similar estimation puts the expected centroid energy for the $T=1$ states at about 20 MeV, which seems about 2 MeV too high according to the shell-model calculations. However, this estimate is extremely rough, because the assignment of the $T=0$ strength is so uncertain.

VIII. CONCLUSIONS

This investigation has produced information on the nature of 14 observed states of O^{16}, all having odd parity except the ground, first, and third excited states. Many other known states were not observed. In many ways the results tend to support the mixed model, as can be seen with reference to Kelson's[11] table of the dominant components of the wave functions of various states of O^{16}. The table divides the states into two classes: nine with a dominant "collective" character and 12 with a dominant "spherical" character (the latter class giving the shell-model label of the principal configuration). As would be expected, our single-nucleon transfer reaction failed to detect most of the collective levels. The only one for which we did observe a measurable strength was the 6.05-MeV state, which is believed to contain a large enough closed-shell component to account for the observation. We observed all six of Kelson's $T=0$ shell-model states and three of his six $T=1$ states. Of the missing three, one (state 20) was probably not seen because it was masked by another state (20a), and another may well have been seen (state 45). Quantitatively, the strengths of the transitions are mostly in reasonable agreement with expectations from the mixed model and, where applicable, from the simple-shell model, although state 17 seems too strongly excited.

Perhaps the most puzzling aspect of our data is the strong appearance of state 20a, which seems to be a 3^-, $T=0$ state not predicted by any of the theories cited here.

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