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Charge-Dependent Nuclear Forces and the Damping of Analog States*

A. Z. MEKJIAN
Department of Physics, Rutgers—The State University, New Brunswick, New Jersey 08903
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The spreading width of an analog state due to a charge-dependent nuclear force is calculated. The mixing with the surrounding different-isospin states via the configuration states and also direct mixing are investigated. The restrictions imposed by the requirement of charge symmetry on these mixings are discussed. The mixing arising from a charge-asymmetric force is also calculated.

THE coupling of an analog state to the dense spectrum of states which surround it is inhibited by isospin selection rules, since these states are characteristized by different isospin quantum numbers. The coupling must therefore proceed through charge-dependent forces. The mixing by the charge-dependent Coulomb force has been considered in detail by several authors.[1] However, in addition to the Coulomb force, the nuclear force may have a small charge dependence. The possible importance of a charge-dependent nuclear force in the mixing of analog states has been suggested,[2] but no detailed considerations have been given. This paper reports an estimate of the mixing of analog states arising from a charge-dependent nuclear force.

For simplicity, we assume that the parent configuration of the analog state consists of a core with two excess neutron levels with quantum numbers β and γ, plus an additional neutron in the level α. We take this state to be a $1p$-$0h$ state with $N+1$ neutrons and Z protons. A particular group of $1p$-$0h$ and $2p=1h$ states can then be obtained from the parent configuration by applying an isospin-lowering operator to each individual neutron state in this configuration. There exist $N-Z+1$ such states which are also degenerate. From this set can be constructed the state

$$\phi_a=[\phi_{pc}+(N-Z)^{1/2}\phi_{na}]/(N-Z+1)^{1/2}, \qquad (1)$$

called the analog, which has isospin $T_>=(N-Z+1)/2$. This state is the linear coherent superposition of these particle-hole states with all the signs the same. It is obtained by simply applying the total isospin-lowering operator to the parent state. The ϕ_{pc} represents a state of a bound proton in the last orbital plus the core of the target, while ϕ_{na} represents the configuration in which a neutron exists in the last orbital, and in which the analog of the target is excited.

Besides the analog state, a group of $T_<=(N-Z-1)/2$ states can be constructed out of the $1p$-$0h$ and $2p$-$1h$ states. These states are called the configuration states,[3] since they are obtained from the same particle-hole configurations from which the analog is derived. Since the target considered here consists of two excess neutron shells, two configuration states can be constructed. One of these is the state which we call the antianalog

$$\phi_{\bar{a}}=[(N-Z)^{1/2}\phi_{pc}-\phi_{na}]/(N-Z+1)^{1/2} \qquad (2)$$

because of its special relation to the analog. The other configuration state is

$$\Phi_c=[(\sqrt{N_\gamma})\Phi_{na_\beta}-(\sqrt{N_\beta})\Phi_{na_\gamma}]/(N-Z)^{1/2}. \qquad (3)$$

Here Φ_{na_β} and Φ_{na_γ} are the wave functions for a neutron in the last orbital and a proton-particle–neutron-hole in the levels β and γ, respectively. The N_β and N_γ are the number of neutrons in the levels β and γ in the parent. The two configuration states lie several MeV below the analog and are approximately degenerate in energy.

In addition to the configuration states, there are other $T_<$ states. These states can be constructed from linear combinations of other $2p$-$1h$ and also more complicated $3p$-$2h$ states. They are of mixed particle-hole hierarchy, because the charge-exchange term in the two-body nuclear Hamiltonian mixes $2p$-$1h$ and $3p$-$2h$ states. These states form the dense spectrum of $T_>$ states which surround the analog. In fact, the coupling of the analog to these states causes the analog to share its strength among these nearby $T_<$ states, thereby giving rise to the spreading width[4] describing on the average its decay into these states.

In calculating the mixing of an analog state into the different-isospin states which surround it, two mechanisms for coupling seem to be important:

(1) The analog state is coupled to the surrounding different-isospin states directly by a charge-dependent two-body interaction.

(2) The analog is coupled to these states by an intermediate isospin-forbidden coupling to the configuration states.

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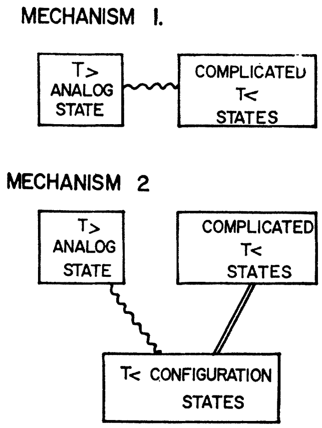

FIG. 1. Schematic illustration of the two coupling mechanisms. The wiggly lines are charge-breaking couplings; the double solid line is an isospin-allowed coupling.

The two mechanisms are schematically shown in Fig. 1. In the first mechanism, the coupling of the analog to the $T_<$ states is through the two-body nature of the charge-dependent force because of the particle-hole structure of these $T_<$ states. For the second mechanism, the analog is coupled to the different-isospin configuration states by the central part of the charge-dependent force, since the analog and configuration states are obtained from the same basis of particle-hole states. The configuration states are then coupled to the $T_<$ states which surround the analog through the charge-independent nuclear two-body interaction. In the case of Coulomb mixing, the latter mechanism is assumed to be larger than the former.[1,5] However, we will see that in the case of a nuclear charge-dependent force, the first mechanism is larger than the second.

To calculate the mixing matrix elements between the analog and configuration states through a charge-dependent nuclear interaction, we first assume charge symmetry and take a charge-dependent two-body potential of the form[6]

$$V_{12}^{\rm C.D.}=a_\rho V(r_{12})\tfrac{1}{2}(1+4\tau_{1\rho}\tau_{2\rho}). \qquad (4)$$

Here $\tau_{1\rho}$ is the third component of the isospin operator and gives $+\frac{1}{2}$ $(-\frac{1}{2})$ when operating on a neutron (proton) state. The $V(r_{12})$ is the radial shape of the two-body charge-dependent interaction and is of short range. The coefficient a_ρ is determined by the magnitude of the deviation from charge independence. From Eq. (4) we have $|V_{pp}-V_{pn}|/V_{pp}=a_\rho$. Although the magnitude of a nuclear charge-breaking potential is uncertain, a deviation from charge independence of the order of 4%[7] is possible. This gives for a_ρ a value of $\sim$0.04.

The difference in energy between a proton and a neutron in a state ν arising from this charge-dependent nuclear interaction is[8]

$$(\epsilon_\nu^{1/2}-\epsilon_\nu^{-1/2})_{\rm C.D.}=a_\rho \sum_{\mu=Z+1}^{N} [(V_{\nu\mu\nu\mu}-V_{\nu\mu\mu\nu}) = (U_{\nu\nu}^{1/2\,1/2}-U_{\nu\nu}^{-1/2\,-1/2})_{\rm C.D.}, \qquad (5)$$

where $V_{\nu\mu\nu\mu}$, $V_{\nu\mu\mu\nu}$ are the direct, exchange matrix elements of $V(r_{12})$. Here $(U_{\nu\nu}^{1/2\,1/2}-U_{\nu\nu}^{-1/2\,-1/2})_{\rm C.D.}$ is the difference in the central potential for neutrons and protons arising from the charge-dependent nuclear interaction. The $+\frac{1}{2}$ refers to neutrons while the $-\frac{1}{2}$ refers to protons. The sum μ in Eq. (5) is over the excess neutron states.

We now approximate the relative shift $\epsilon_\nu^{1/2}-\epsilon_\nu^{-1/2}=(\Delta\epsilon_\nu)_{\rm C.D.}$ of proton and neutron levels as follows. The Hartree-Fock central potential for protons or neutrons arising from charge-independent Wigner forces is

$$U_{\nu\nu}^{\rm C.I.}=\sum_{\alpha=1}^{N,Z} a_0(V_{\nu\mu\nu\mu}-V_{\nu\mu\mu\nu}), \qquad a_0=1 \qquad (6)$$

which we assume has the form $U(r)=\frac{1}{2}m\omega^2r^2$. Since the difference between proton and neutron potentials in Eq. (5) involves only matrix elements over the neutron shells, we have

$$(\Delta\epsilon_\nu)_{\rm C.D.}\approx a_\rho[(N-Z)/A]\tfrac{1}{2}\hbar\omega(2n_\nu+l_\nu-\tfrac{1}{2}). \qquad (7)$$

Here n_ν and l_ν are the principal and orbital angular momentum quantum numbers in the level ν. The $(N-Z)/A$ factor arises because the excess neutrons only participate in the shift effect when the nuclear force is charge-symmetric. It should be noted that the $(N-Z)/A$ dependence is also a characteristic of the symmetry potential[9] which also arises from the excess neutrons.

The matrix elements of the analog to the configuration states from the charge-breaking interaction of Eq. (4) are then given by the expression[8]

$$M_{a\bar{a}}=\langle\phi_a|\tfrac{1}{2}\sum_{i,j}V_{ij}^{\rm C.D.}|\phi_{\bar{a}}\rangle = \frac{(N-Z)(\Delta\epsilon_\alpha)_{\rm C.D.}-N_\beta(\Delta\epsilon_\beta)_{\rm C.D.}-N_\gamma(\Delta\epsilon_\gamma)_{\rm C.D.}}{(N-Z+1)(N-Z)^{1/2}},$$

$$M_{ac}=\frac{(N_\beta N_\gamma)^{1/2}[(\Delta\epsilon_\beta)_{\rm C.D.}-(\Delta\epsilon_\gamma)_{\rm C.D.}]}{(N-Z)^{1/2}(N-Z+1)^{1/2}}. \qquad (8)$$

[5] For Coulomb forces a third mechanism has to be considered, which is called *external mixing* by D. Robson, Phys. Rev. **137**, B535 (1965). This mixing is important for Coulomb forces because of the long-range character of the Coulomb interaction. It can therefore be neglected for a short-range charge-dependent nuclear interaction.

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For comparison, the coupling matrix due to the charge-dependent Coulomb force is obtained from Eq. (8) by substituting $\Delta_c(\alpha)$ for $(\Delta\epsilon_\alpha)_{\text{C.D.}}$, where $\Delta_c(\alpha)$ is the Coulomb energy of a proton in the level α arising from its interaction with all the target protons, i.e., $\Delta_c(\alpha)\sim Z/A^{1/3}$.

The spreading width of the analog $\Gamma_{a\downarrow}$ into the more complicated $T_<$ states which surround it through its coupling to the configuration states is given by the equation[1,8]

$$\Gamma_{a\downarrow}=\frac{M_{a\bar{a}}^2\Gamma_{\bar{a}\downarrow}(E_a)}{(E_a-E_{\bar{a}})^2+[\Gamma_{\bar{a}\downarrow}(E_a)/2]^2}+\frac{M_{ac}^2\Gamma_{c\downarrow}(E_a)}{(E_a-E_c)^2+[\Gamma_{c\downarrow}(E_a)/2]^2}. \quad (9)$$

The $\Gamma_{\bar{a}\downarrow}(E_a)$ and $\Gamma_{c\downarrow}(E_c)$ are the nuclear spreading widths of the antianalog and the other configuration state into the more complicated $T_<$ states evaluated at the analog energy. The above expression for the spreading width can be interpreted as follows.

The coupling of the analog to the configuration states by the central part of the charge-dependent interaction introduces an isospin impurity into the analog wave function. This $T_<$ impurity in the analog is then coupled to the more complicated $T_<$ states which surround the analog by the charge-independent nuclear force. From this interpretation we can easily see that the nuclear spreading of the configuration states into the $T_<$ states which surround the analog reflects the level density of the more complicated states at the analog energy and not at the configuration-state energy. These densities are considerably different because of the large energy separation of the two states.

In evaluating the nuclear spreading widths of the antianalog and the other configuration state, we take for the spreading width of a $1p$-$0h$ state the giant-resonance[10] value of 5 MeV and for a $2p$-$1h$ state the intermediate-structure value of 1 MeV.[11] For the p-wave resonance in K^{41},[12] the coupling matrix elements $M_{a\bar{a}}$ and M_{ac} are 12 and 24 keV, respectively. In obtaining these values, we have taken $\hbar\omega$ to be 15 MeV. (The energy splitting of the analog from the configuration states is about 4 MeV.[1]) The values for $M_{a\bar{a}}$ and M_{ac} for other nuclei are also of this order of magnitude or are an order of magnitude smaller if all the excess neutrons in the parent belong to the same harmonic-oscillator shell. The configuration-state impurity in intensity is $\sim10^{-5}$, and the spreading width of the analog for the second mechanism is $\Gamma_{a\downarrow}=80$ eV. For comparison, the spreading width of the analog state in K^{41} from this mechanism by Coulomb forces is $\sim$10 keV, while the impurity in intensity is $\sim1\times10^{-3}$.[8]

One of the main reasons why a charge-symmetric force produces little mixing by this mechanism is that only the excess neutrons are involved in the mixing. By contrast, for a charge-asymmetric force all the nucleons in the target participate in shifting the proton energies with respect to the neutron energies. In fact, a charge-asymmetric force of the form[6]

$$V_{12}^{\text{C.D.}}=b_\rho V(r_{12})\tfrac{1}{2}(\tau_{1\rho}+\tau_{2\rho}) \quad (10)$$

will lead to an energy difference between a proton and a neutron in the state ν given by the expression

$$(\epsilon_\nu^{1/2}-\epsilon_\nu^{-1/2})=\tfrac{1}{2}b_\rho\sum_{M=1}^{N,Z}(V_{\nu\mu\nu\mu}-V_{\nu\mu\nu\mu})\approx b_\rho\tfrac{1}{4}\hbar\omega(2n_\nu+l_\nu-\tfrac{1}{2}). \quad (11)$$

However, the deviations from charge symmetry are much smaller than the deviations from charge independence. In particular, the experimental data show consistency with charge symmetry to within 1%.[7] This gives for $b_\rho\sim0.01$. In K^{41}, the matrix elements of the analog to the configuration states for this charge-asymmetric force are $M_{a\bar{a}}\sim8$ keV and $M_{ac}\sim15$ keV. The spreading width of the analog is then $\sim$100 eV and still insignificant compared to the Coulomb spreading. Even if charge symmetry were violated by as much as 4%, the spreading width of the analog would be only 1 keV.

Therefore, if a charge-dependent nuclear force is to produce significant spreading, it must operate through mechanism 1. Under this mechanism, a two-body charge-dependent nuclear force will couple the analog directly to the $T_<$ states which surround it, reaching both proton and neutron particle-hold excitations in these states. For Coulomb forces, only certain proton particle-hold excitations are reached from the analog. For this reason, mechanism 1 can be larger for charge-dependent nuclear forces.

To estimate the spreading due to this mechanism, we take the spreading of the $1p$-$0h$ component of the analog to be $(\frac{1}{2}a_\rho)^2$ times the giant-resonance value of 5 MeV. For the $2p$-$1h$ component we take $(\frac{1}{2}a_\rho)^2$ times the intermediate-structure value of 1 MeV. The factor $\frac{1}{2}$ arises from the $\frac{1}{2}(1+4\tau_{1\rho}\tau_{2\rho})$ term in Eq. 4. The 5 and 1 MeV are the spreading widths of $1p$-$0h$ and $2p$-$1h$ states arising from Wigner forces. The values $(\frac{1}{2}a_\rho)^2\times5$ MeV and $(\frac{1}{2}a_\rho)^2\times1$ MeV are, however, overestimates, since a charge-symmetric force of the form of Eq. (4) will not reach some of the particle-hole excitations which can be reached from the analog by a charge-independent Wigner force, since $V_{pn}^{\text{C.D.}}=0$. However, if we take a charge-symmetric force of the form $a_\rho 2\tau_{1\rho}\tau_{2\rho}$, the Wigner and charge-symmetric forces can reach the same particle-hole states starting from

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the analog. The spreading width from mechanism 1 is then

$$\Gamma_{a\downarrow}\sim 1/(N-Z+1)\times(2\text{ keV})+(N-Z)/(N-Z+1)\times(0.4\text{ keV}). \quad (12)$$

For K^{41} we obtain a value of $\Gamma_{a\downarrow}\sim 1$ keV, while for heavier nuclei we obtain a smaller value because of the growth of $N-Z$. Even if we had taken the spreading width of a $1p$-$0h$ and $2p$-$1h$ state to be as large as 10 MeV for Wigner forces, the spreading width of the analog only be $\sim$4 keV.

We therefore conclude that charge-dependent nuclear forces can contribute at most a few keV to the spreading of an analog state. This should be compared with the spreading width arising from Coulomb mixing, which is an order of magnitude larger. Consequently, the possibility of observing the effects of a charge-dependent nuclear force from the spreading width of an analog state is washed out by the charge-dependent Coulomb force.

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Deuteron Production in Nuclei by Protons of from 1 to 3-BeV

R. D. Edge and H. H. Knox
University of South Carolina, Columbia, South Carolina 29208

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The deuteron yield and momentum spectrum for targets of Be, C, Cu, and Pb bombarded by 1-, 2-, and 3-BeV protons from the Cosmotron has been found at angles of 0, 17, and 32°, using a magnetic spectrometer and time-of-flight telescope. The results cannot be explained in terms of a simple pickup, evaporation, or nucleon-nucleon statistical production process. Two other models involving two-stage processes have been proposed, namely, the indirect pickup process and a model in which nuclear matter "catalyzes" the coalescing of two cascade nucleons into a deuteron. Our results show that, at higher momenta in the deuteron spectrum, agreement can be obtained with the cascade theory. At the lower end of the momentum spectrum, deuterons may arise by the indirect pickup process.

I. INTRODUCTION

SOME eight years ago, experimenters on the Brookhaven and CERN alternating gradient synchrotrons[1,2] noted that the production of deuterons from nuclei bombarded by high-energy protons was much greater than could be accounted for on any simple theory, such as the direct pickup process occurring at low energies.[3,4] It seemed intuitively surprising that the weak binding of the deuteron should remain intact in such energetic processes. In order to explain this anomaly, several theories have been proposed.

The first of the more important theories was put forward by Bransden in 1952[5] to explain results at lower energies. He suggested that an indirect pickup process was probable for protons above 200 MeV, in which the incoming proton struck a nucleon, producing a low-energy secondary particle, which in turn picked up a nucleon to form a deuteron on the way out of the nucleus.

At high energies Butler and Pearson[6–9] proposed that deuteron production occurred in the cascade of nucleons ejected by the incoming proton. The angular distribution of the knock-on "shower" nucleons has a strong forward peak, and pairs of particles having small relative momenta could coalesce to form a deuteron with the nuclear matter around them acting as a "catalyst" and providing the momentum balance.

In another process Hagedorn[10–12] suggested that statistical nucleon-nucleon interactions at high energies might yield sufficient deuterons to explain the anomaly. The reaction occurring would be of the type $p+$ nucleon$\rightarrow d+$pions$+$others. The process would occur entirely within the small region of phase space surrounding one nucleon.

Other processes, such as evaporation,[13] or the knockout of a deuteron formed by fluctuations in the nuclear

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