Relation between the $2I+1$ Rule and Level Width*

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An additional condition necessary for the $2I+1$ rule to be valid is found: The total level width Γ_μ must be large enough so that the width-fluctuation correction factors F for final states will be approximately equal. Feshbach's formula, corrected by the width-fluctuation correction factor, is used in discussing the dependence of the cross sections on the average total level width, average partial widths of final states, and the spin cutoff parameter. It is shown that the $2I+1$ rule is obeyed, in addition to MacDonald's three conditions, if the F's are approximately equal for all final states, which requires a sufficiently large Γ_μ.

THREE conditions for the $2I+1$ rule have been formulated by MacDonald.[1] They are as follows:

(a) A large number of compound states should be excited; the spin J of compound states should be larger than the spin I of the final states.

(b) The energy of outgoing particles should be large enough to ensure that barrier penetration does not suppress any possible l value.

(c) The spin cutoff parameter σ^2 of compound nuclei should be small.

In the low-energy region, (a) and (b) are not fulfilled, but (c) is always satisfied because σ^2 does not depend on the incident energy. By introducing the penetration effect, Wang *et al.*[2] recently ana yzed the $^{25}\mathrm{Mg}(d,\alpha)^{23}\mathrm{Na}$ reaction and compared the result with the $2I+1$ rule. They calculated the total cross section by using Feshbach's formula with spin cutoff parameter $\sigma^2=\infty$; the agreement of the results of the experiment with the $2I+1$ rule is rather good. Setting the spin cutoff parameter to infinity, however, apparently contradicts condition (c). For small σ^2, it is found that the curve deviates substantially from the curve in the case where $\sigma^2=\infty$; e.g., see curves ① and ⑤ in Fig. 1. To account for the above fact, the width-fluctuation effect[3] is introduced into Feshbach's formula, which is then used to investigate the relation between the level width and the $2I+1$ rule.

In the statistical compound-nuclear theory,[4] the total cross section $\sigma_{\alpha'\alpha}$ of the reaction proceeding from entrance channel α with fragment spins I_α and i_α to the exit channel α' is expressed as

$$\langle\sigma_{\alpha'\alpha}\rangle=\pi\lambda\!\!\!^{-2}\sum\frac{2J+1}{(2I_\alpha+1)(2i_\alpha+1)}\frac{2\pi}{D_J}\times\langle\Gamma_{\mu,\alpha ls}\Gamma_{\mu,\alpha'l's'}/\Gamma_\mu\rangle. \quad (1)$$

Here l, s and l', s' are the relative orbital angular momenta and channel spins in the entrance and exit channels, respectively. J is the total angular momentum, D_J is the mean spacing of resonances with angular momentum J, and Γ_μ is the total width of the resonance μ. The quantity $\Gamma_{\mu,\alpha ls}$ is a partial width.

Introducing the width-fluctuation correction factor F [3] into Eq. (1), we obtain

$$\langle\sigma_{\alpha'\alpha}\rangle=\pi\lambda\!\!\!^{-2}\sum\frac{2J+1}{(2I_\alpha+1)(2i_\alpha+1)}\frac{T_{\alpha ls}{}^J T_{\alpha'l's'}{}^J}{(2\pi/D_J)\langle\Gamma_\mu\rangle}F, \quad (2)$$

where

$$F=\langle\Gamma_{\mu,\alpha ls}\Gamma_{\mu,\alpha'l's'}/\Gamma_\mu\rangle/(\langle\Gamma_{\mu,\alpha ls}\rangle\langle\Gamma_{\mu,\alpha'l's'}\rangle/\langle\Gamma_\mu\rangle)$$

and

$$D_J=D_0\Big/\left[(2J+1)\exp\left(\frac{-J(J+1)}{2\sigma^2}\right)\right].$$

σ^2 is the spin cutoff parameter of the compound nucleus; $T_{\alpha ls}{}^J$ and $T_{\alpha'l's'}{}^J$ are the transmission coefficients of the entrance and exit particles, respectively.

In this paper we consider the reaction $^{25}\mathrm{Mg}(d,\alpha)^{23}\mathrm{Na}$ and evaluate Eq. (2) in the deuteron energy range 2.0–3.0 MeV. The transmission coefficients for deuterons are taken from Ref. 5, and those for α particles have been calculated by using optical-model potentials.[2] The width-fluctuation correction factor F is calculated by using the following expression:

$$\frac{\langle\tau_1{}^{k_1}\tau_2\cdots\tau_n{}^{k_n}/\tau^m\rangle}{\langle\tau_1\rangle^{k_1}\langle\tau_2\rangle^{k_2}\cdots\langle\tau_n\rangle^{k_n}/\langle\tau\rangle^m}=\frac{1}{(m-1)!}\times\int_0^\infty dt\, t^{m-1}\prod_{i=1}^n(2k_i-1)!!\left(1+2t\frac{\langle\tau_i\rangle}{\langle\tau\rangle}\right)^{-k_i-1/2}.$$

For a detailed explanation of the above equation see, Ref. 6. It is assumed that $\Gamma_{\mu,\alpha ls}$ and $\Gamma_{\mu,\alpha'l's'}$ are based on the Porter-Thomas distribution and that $\langle\Gamma_\mu\rangle=\langle\Gamma_{\mu,\alpha ls}+\Gamma_{\mu,\alpha'l's'}+r_\mu\rangle$, where

$$r_\mu\equiv\langle\sum_{\alpha''l''s''}\Gamma_{\alpha''l''s''}\rangle;$$

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TABLE I. Values of the parameters in Eq. (2) that are used to determine the curves of Fig. 1.

Parameters \ Curves	①	②	③	④	⑤
σ^2	6	6	6	6	∞
$\langle\Gamma_\mu\rangle$ (keV)	32	32	16	16	32
$\langle\Gamma_\mu\rangle/D_0$	16	16	16	16	16
$\langle\Gamma_{\mu,\alpha'l's'}\rangle$ for $I=\frac{1}{2}$	no F correction	1 keV	1 keV	1 keV	no F correction
		(0.89)	(0.82)	(0.82)	
$\langle\Gamma_{\mu,\alpha'l's'}\rangle$ $I=\frac{3}{2}$		1.3 keV	1.3 keV	1.3 keV	
		(0.86)	(0.78)	(0.78)	
$\langle\Gamma_{\mu,\alpha'l's'}\rangle$ $I=\frac{5}{2}$		1.9 keV	1.9 keV	2.5 keV	
		(0.825)	(0.73)	(0.675)	
$\langle\Gamma_{\mu,\alpha'l's'}\rangle$ $I=\frac{7}{2}$		1.9 keV	1.9 keV	3.0 keV	
		(0.826)	(0.73)	(0.64)	
$\langle\Gamma_{\mu,\alpha'l's'}\rangle$ $I=\frac{9}{2}$		1.9 keV	1.9 keV	4.0 keV	
		(0.825)	(0.73)	(0.6)	

the double prime denotes the exclusion of the states αls and $\alpha'l's'$ from the summation of possible decay channels. Furthermore, $\langle\Gamma_{\mu,\alpha ls}\rangle=\langle\Gamma_{\mu,\alpha'l's'}\rangle$. The values of the F factors are given within parentheses in Table I. The results of the calculation of $\langle\sigma_{\alpha'\alpha}\rangle$ by Eq. (2), as compared with the $2I+1$ rule, are shown in Fig. 1. Curves ② and ③ in Fig. 1 have the same values for the parameters, except for the value of $\langle\Gamma_\mu\rangle$, we find that the larger $\langle\Gamma_\mu\rangle$ is, the higher the curve is. Curves ③ and ④ also have the same parameter values, except for the values of $\langle\Gamma_{\mu,\alpha'l's'}\rangle$ for $I=\frac{5}{2}$, $\tau=\frac{7}{2}$, and $I=\frac{9}{2}$. If the ratio of $\langle\Gamma_{\mu,\alpha'l's'}\rangle$ for any I to $\langle\Gamma_{\mu,\alpha'l's'}\rangle$ for $I=\frac{1}{2}$ is unity, then curves ③ and ④ will coincide with the one having no F-factor correction, i.e., curve ①; if the ratios are greater than unity, the curves tend toward curve ⑤, and if the ratios are less than unity, the curves are higher than curve ①.

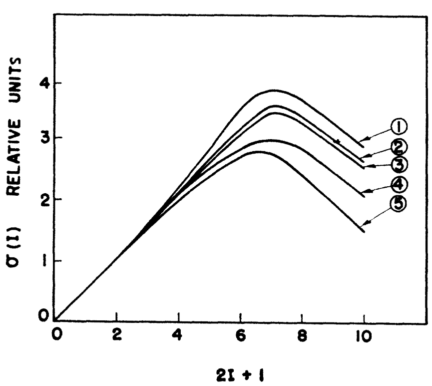

FIG. 1. Total cross sections calculated from Eq. (2) for different final-state I's. The circled numerals correspond to the cases listed in Table I. All curves are normalized so as to coincide at the point $I=\frac{1}{2}$.

From the above discussion, we conclude the following:

(1) If MacDonald's conditions (a)–(c) are fulfilled, but Γ_μ is not large, then the cross sections may deviate from proportionality to $(2I+1)$ because of the differences among the $\langle\Gamma_{\mu\ \alpha'l's'}\rangle$'s for different final-state I's. The $2I+1$ rule thus fails to be satisfied.

(2) The only way to save the $2I+1$ rule is to make $\langle\Gamma_\mu\rangle$ large enough so that all F factors for any final-state I's are approximately equal; then the $2I+1$ rule may be improved.

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