integration,

$$\gamma_0^2 \approx g\lambda^2(mp_F/2\pi) \sim 4p_F^2, \qquad \text{(D10)}$$

since $\omega_c(k)/k \sim v_F$ is independent of k in this limit. Therefore, only the region around $k=k_c$ contributes significantly giving a factor $\frac{1}{2}\pi$ according to (D8), so that

$$1/\tau \approx (\lambda^2 m/2\pi^2 p_F)(\epsilon_p - \epsilon_F)\tfrac{1}{2}\pi = (\lambda^2/4\pi)(p-p_F), \quad \text{or} \quad 113(p/p_F-1)\ \text{MeV}. \qquad \text{(D11)}$$

The above estimate compares favorably with the value $144(p/p_F-1)$ MeV obtained by numerical integration of Eq. (C1) near $p=p_F$. This agreement indicates that the interpretation of the origin of the unphysical $p-p_F$ behavior of $1/\tau$ near the Fermi surface is indeed correct.

As $\epsilon_p-\epsilon_F$ increases, Fig. 4 suggests that the ratio $\bar{\omega}/k$ increases; the widths γ^2 grow and the instability becomes less important, until at $\epsilon_p-\epsilon_F=(\omega_<)_{\max}=\epsilon_F$ or $p=\sqrt{2}p_F$, when the equation $\bar{\omega}=\omega_c(k)$ certainly has no real roots any more, and the effect of the instability becomes negligible.

Finally, we remark that variational treatments of the unstable ground state have been devised[21] that remove the instability. What role, if any, the root $\omega_c(k)$ has when this is the case has not been investigated yet.

[21] K. Sawada and N. Fukuda, Progr. Theoret. Phys. (Kyoto) **25**, 653 (1961); C. B. Dover, Ph.D. thesis, Massachusetts Institute of Technology, 1967 (unpublished).

PHYSICAL REVIEW VOLUME 183, NUMBER 4 20 JULY 1969

Gamma-Ray Widths in C^{13}, Li^6, and P^{31}

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The width of the 3.68-MeV level of C^{13} has been measured and found to be $\Gamma=0.44\pm0.04$ eV. After correction for the E^3 dependence, this is 0.59 times as strong as the mirror transition in N^{13}, in agreement with Morpurgo's prediction of approximate equality for $M1$, $\Delta T=0$ transitions in mirror nuclei. The C^{13} width was obtained from a comparison of the resonant scattering of bremsstrahlung by C^{13} with that by the 3.51-MeV level of P^{31} and the 3.56-MeV level of Li^6. The widths of the P^{31} and Li^6 levels were measured in self-absorption experiments, and found to be 52 ± 8 meV and 8.1 ± 0.5 eV, respectively. A limit on the energetically allowed but spin- and parity-forbidden decay of the Li^6 level to $\alpha+d$ was established as $\Gamma_{\alpha d}<1.3$ eV.

INTRODUCTION

THE rules of Morpurgo concerning the relative strengths of γ transitions between corresponding states of mirror nuclei follow directly from charge independence or charge symmetry,[1,2] principles so well established on other grounds that the rules seem to be referred to most frequently when used as an aid in correlating mirror levels, with no great experimental effort being devoted to their verification. It would seem, however, that any experimental results might challenge the theorists to construct more accurate and detailed wave functions. We have previously shown,[3] for example, that corresponding $E1$ transitions in C^{13} and N^{13}, which should be equal, according to Morpurpo's predictions, differ by a factor of 2. This difference is qualitatively understandable, but would seem to be worthy of a more detailed study. Warburton *et al.*[4] discuss a similar case in N^{15}-O^{15}.

The subject of the present paper is an $M1$ ($\Delta T=0$) transition in the C^{13}-N^{13} pair. Here Morpurgo has shown one can deduce both the isospin-independent and the isospin-dependent parts of the matrix element from comparison with experiment.

Our measurements are concerned primarily with using resonance-fluorescence techniques to measure the width of the 3.68-MeV level of C^{13}. Since samples of C^{13} large enough for a self-absorption measurement are not easily available, the scattering was compared with scattering by the 3.56-MeV level of Li^6 and the 3.51-MeV level of P^{31}, for which accurate widths could be established by self-absorption measurements.

EXPERIMENTAL DETAILS

The photon beam for our resonance fluorescence measurements is produced when the electron beam

[1] G. Morpurgo, Phys. Rev. **114**, 1075 (1959).
[2] W. M. McDonald, in *Nuclear Spectroscopy, Part B*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960).
[3] S. W. Robinson, C. P. Swann, and V. K. Rasmussen, Phys. Letters **26B**, 298 (1968).
[4] E. K. Warburton, J. W. Olness, and D. E. Alburger, Phys. Rev. **140**, B1202 (1965).

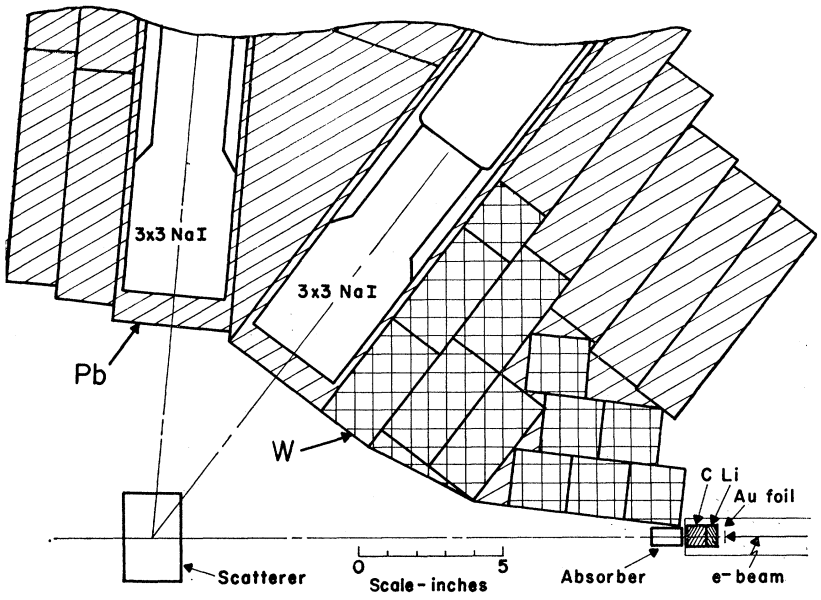

FIG. 1. Experimental arrangement. The energy-analyzed electron beam enters from the right, passes through a nominal 35-mg/cm² Au foil and is stopped in a water-cooled beam dump by 0.82 cm of Li metal and 2.2 cm of graphite. For the Li measurement, the beam was stopped in graphite only.

from the Bartol Van de Graaff accelerator strikes a thin gold radiator. The experimental arrangement, shown in Fig. 1, differs from that previously described[5] mainly in that two detectors are used simultaneously to measure the scattered radiation, allowing us to get two points of an angular distribution. The scattering angles chosen, 96° and 128°, are close to the minimum and zero, respectively, of the Legendre polynomial of order 2. For the cases we have dealt with so far this is sufficient to establish the angular correlations, the observed lifetimes being short enough to limit the quadrupole contribution to values for which the $A_4P_4(\cos\theta)$ term is negligible.

The variation with angle of the intensity and energy of Compton scattered photons is such that the low-energy background is higher in the 96° counter, and more lead is used in front of this counter to reduce the background to that in the 128° counter. Since the 96° counter is also necessarily further away, the counting rate in the region of interest is appreciably lower. The required correction factor is obtained empirically by observations with radioactive sources and with resonant scatterings which are known to be isotropic. Mean values of $P_2(\cos\theta)$ are derived by a program which treats the detectors as points located at their geometric centers, subdivides the scatterer into 360 parts, and calculates P_2 and the relative weight for each subdivision[6] and detector.

The beam current used for any particular measurement is set by the requirement that counting rate losses be limited to ~6–7%.

The phosphorous scatterer and absorber, and the matching comparison absorber, were the same as used in Ref. 5. The carbon scatterer was 7.04 g of C^{13}, in the form of 58% C^{13} soot, packed in an aluminum can and loaned to us by the Oak Ridge National Laboratory. For comparison purposes, we made an approximately identical aluminum can and filled it with normal carbon to the same total weight. For the Li^6 scatterer and absorber we had available a cone of 95.62% Li^6 metal. After cleaning off the protective wax and removing all the nonmetallic inclusions,[7] clean chunks of metal were selected. Pieces totaling 18 g were pressed into a 3.000-in.-diam cylinder. Two 1.000-in.-diam absorbers, one of 1.200 g and the other 1.800 g, were made in the same fashion. All these pieces were dipped in hot Ceresin wax for protection. No comparison scatterers or absorber were needed for the Li^6 measurements, as is apparent from Fig. 2.

RESULTS AND DISCUSSION FOR Li^6

The scattered spectrum observed when the Li^6 scatterer is placed in the bremsstrahlung beam with the maximum energy set at 3.67 MeV is shown in Fig. 2, together with the spectrum when the maximum energy is just below 3.56 MeV. The γ-ray observed results from decay of the 3.562-MeV second excited state of Li^6 to the ground state. The γ-ray width of this level has been measured previously.[8] The results from electron scattering do not seem to agree with each other too well, but the two resonance fluorescence measurements agree

[5] S. W. Robinson, C. P. Swann, and V. K. Rasmussen, Phys. Rev. **174**, 1320 (1968).

[6] We have shown previously that this is an acceptable procedure for our measurements. We also have data for some 0-2-0 transitions, for which the distribution is known to be quite anisotropic, which supports it. A full account will be published shortly.

[7] Formed presumably because of pinholes in the wax. This is a behavior that we usually observe with metallic Li and Na, and is important primarily because it introduces an uncertainty in converting from weight to number of Li or Na atoms present.

[8] T. Lauritsen and F. Ajzenberg-Selove, Nucl. Phys. **78**, 1 (1966).

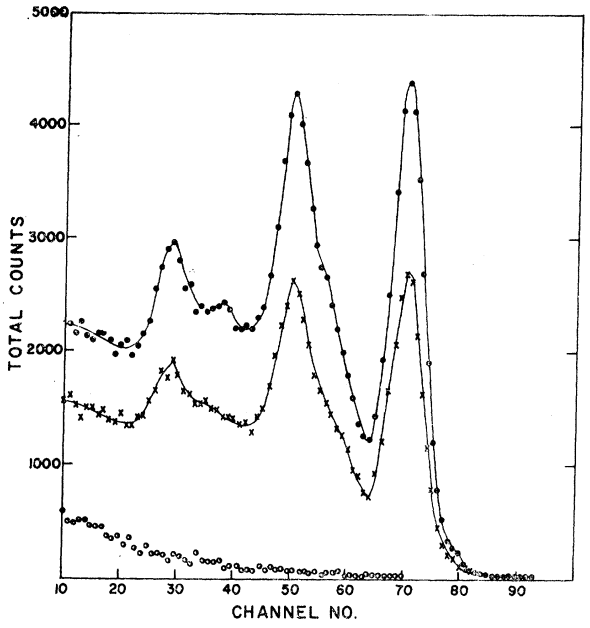

FIG. 2. Spectra observed with a Li^6 scatterer. The upper curve is with no absorber and the middle curve with a 0.356-g/cm² Li^6 absorber, both with a beam energy of 3.67 MeV. The lower curve was obtained at a beam energy of 3.55 MeV. The peaks correspond to the full-energy, one-escape, and two-escape peaks of the 3.56-MeV γ-ray produced by deexcitation of the second excited state of Li^6. The small bump between the one- and two-escape peaks has not been explained with certainty. It may be a statistical fluctuation, or may represent an expected response of the NaI crystal, as shielded, to 3.5-MeV photons. It may correspond, for example, to both annihilation quanta escaping from the NaI, with one being 180° Compton-scattered by the surrounding lead back into the crystal. In any case, it seems safe to say that it has no bearing on the Li^6 nucleus.

on a value of around[9,10] 9 eV. The spin-parity assignment of O^+ to this level follows from its identification with the ground states of He^6 and Be^6 and from failure to observe the energetically possible decay to He^4+d. γ decay to the 3^+ first excited state has not been observed and is assumed to be negligible (competition between a strong $M1$ and a lower energy $M3$).

From Fig. 2, it is clear that one can use all channels from just below the one-escape peak to just above the full-energy peak, after subtraction of a rather small background, to determine the relative intensity of the scattered 3.56-MeV γ ray. Using the results from both counters, we find a self-absorption $R=0.289\pm0.005$ for the thinner absorber and $R=0.385\pm0.003$ for the thicker absorber, where

$$R=\frac{\text{counts without absorber}-\text{counts with absorber}}{\text{counts without absorber}}.$$

A small correction (~1%) for nonresonant (electronic) absorption in the absorbers was made. For these measurements the electron beam was stopped in graphite only, the Li being removed. Values of R calculated for the two counters separately, and values based on the full-energy peak only, were in agreement within statistical error.

It was realized by both Cohen and Tobin[9] and by Skorka *et al.*[10] that the thermal Doppler width of this level $\Delta=11.34$ eV is close to the natural width Γ, so that the calculation of Γ from the measured self-absorption requires integration of the ψ function, the convolution of the Breit-Wigner dispersion formula, and the Gaussian representing the thermal broadening. This problem and our method of approaching it are described in Appendix A. Results are given in Fig. 3. For the thinner absorber we get $t=(\Delta/\Gamma)^2=1.86\pm0.20$ and for the thicker absorber $t=1.98\pm0.10$. The mean value corresponds to $\Gamma=8.1\pm0.5$ eV, where the error includes what we believe to be generous estimates of errors in the ψ-function tables and our integration, uncertainties in Δ, and other experimental errors. This is in agreement with Cohen and Tobin,[11] with Skorka *et al.*, and with some of the electron-scattering results. In a recent publication, Creten *et al.*[12] find $\Gamma=4.6\pm0.4$ eV from a self-absorption measurement. Their method of calculating the self-absorption is, however, only correct for the case $\Delta\gg\Gamma$. We have estimated that a correct calculation with their data would give $\Gamma\simeq9$ eV. Parenthetically, one might note that they also made a mistake in their calculation for the 2.18-MeV level in that they did not take into account the decay of the level to He^4+d. This multiplies their upper limit on the γ-ray width $\Gamma_\gamma<140$ μeV by a factor of 25 keV/Γ_γ.

If we anticipate the results of the next section, then we can calculate the width of the 3.56-MeV level by

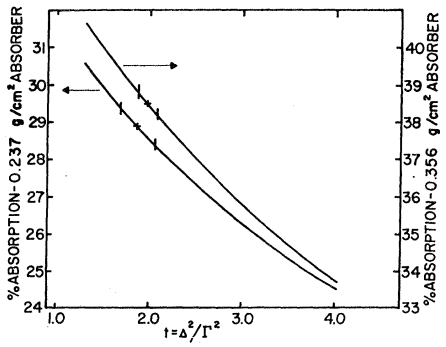

FIG. 3. Plot of the expected self-absorption versus t for the scatterer and absorbers used. The smooth curves connect calculated points (not shown) for $t=1.25$, 1.50, 1.75, 2.00, 3.00, and 4.00. The experimental values and their statistical errors are shown.

[9] Leslie Cohen and Ralph A. Tobin, Nucl. Phys. **14**, 243 (1959/60).

[10] S. J. Skorka, R. Hübner, T. W. Retz-Schmidt, and H. Wahl, Nucl. Phys. **47**, 417 (1963). Comparison of Fig. 2, in which the two upper curves correspond to an integrated electron current of 40 000 μC at ≃8 μA average, with the data of Cohen and Tobin or the data of Skorka *et al.* illustrates nicely the disadvantage of pulsed beams and the difficulties of resonance fluorescence measurements in the presence of neutrons.

[11] Cohen and Tobin give enough experimental details so that we were able to calculate a width from their data. We find good agreement.

[12] W. L. Creten, R. J. Jacobs, and H. M. Ferdinande, Nucl. Phys. **A120**, 126 (1968).

comparing the scattering with that by the 3.51-MeV level of P^{31}. The result, $\Gamma=8.2\pm1.3$ eV, agrees with the self-absorption measurement and sets a limit on the branching of the level. In particular, this is a limit on the parity nonconserving decay to He^4+d of $\Gamma_{ad}<1.3$ eV, not as small as Wilkinson's[13] limit of 0.2 eV, but of interest as an independent measurement.

As expected, the angular distribution of the resonant scattering was isotropic. The limits on any anisotropy are set at around 2% by possible systematic errors involving the measurement of the scattering angles, the placement of the normalizing point source of Co^{56} at the exact center of the Li^6 scatterer, etc.

RESULTS AND DISCUSSION FOR P^{31}

The spectra observed when the phosphorous scatterer and comparison sulfur scatterer were exposed to a 3.66-MeV bremstrahlung beam are shown in Fig. 4. Resonant scattering by the 3.51- and 3.13-MeV levels of P^{31} is indicated. The observed self-absorption for the 3.51-MeV level is 0.235 ± 0.027. Taking the spin sequence for resonant scattering by this level to be[14] $\frac{1}{2}$-$\frac{3}{2}$-$\frac{1}{2}$, the Doppler width to be 4.69 eV and the ground-state decay to be[15] (62 ± 4)% of the total decays, one finds a total width $\Gamma=52\pm8$ meV, or a mean life $\tau=12.7$ fsec. This is in agreement with the Doppler attenuation measurements of Wolff *et al.*[15] and in disagreement with a previous resonance fluorescence measurement by Booth and Wright.[16] One can also obtain from the data of Fig. 4 values of the self-absorption for the 3.13-MeV

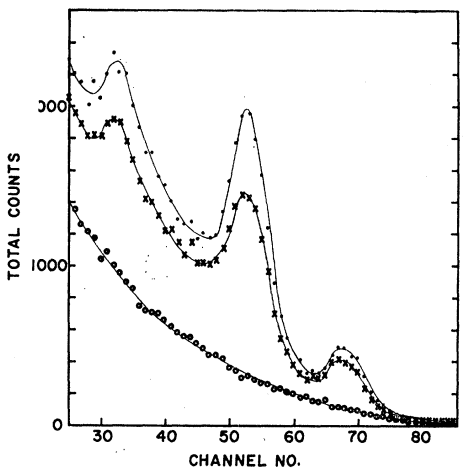

FIG. 4. Spectra observed when P and S scatterers are irradiated with bremsstrahlung of maximum energy 3.66 MeV. The upper curve is for the P scatterer-Al absorber, the middle curve for the P scatterer-P absorber, and the lower curve is the average of the S scatterer, Al and P absorber runs, which were statistically indistinguishable. The observed peaks correspond to 3.51- and 3.13-MeV γ rays from P.

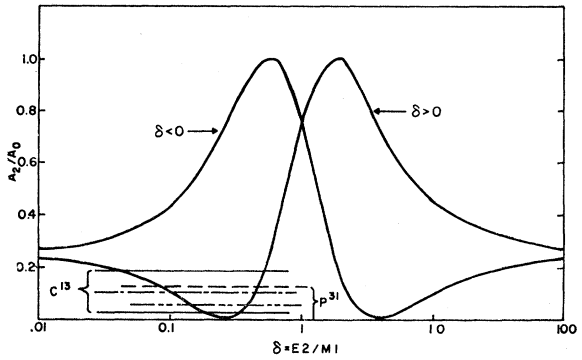

FIG. 5. Plot of the angular distribution coefficient A_2/A_0 for the spin sequence $\frac{1}{2}$-$\frac{3}{2}$-$\frac{1}{2}$. The A_4 term is identically zero for this sequence. The experimental limits for the C^{13} and P^{31} distributions are indicated.

level using the line shape from the Li^6 spectrum to subtract out the 3.51-MeV contribution. The resultant self-absorption, 0.295 ± 0.015, corresponds to a width $\Gamma=61\pm4$ meV compared to our previous value[5] of 66 ± 7 meV. After estimating other sources of error in the present result, we suggest a mean value of 63 ± 5 meV. This changes the mean value quoted[3] for the width of the 3.09-MeV level of C^{13} from 0.44 ± 0.05 to 0.43 ± 0.05 eV.

Taking the form $1+A_2P_2(\cos\theta)$ for the angular distribution of the 3.51-MeV γ ray, we find $A_2=0.057\pm0.070$. A plot of A_2 versus the $E2/M1$ mixing ratio for the spin sequence $\frac{1}{2}$-$\frac{3}{2}$-$\frac{1}{2}$ is given in Fig. 5. This mixing ratio has been measured by Willmes and Harris[17] and has been found to be $x=-0.41\pm0.03$, using, as it happens, a sign convention which is opposite to the one we use. We agree with their result.

RESULTS AND DISCUSSION FOR C^{13}

Spectra obtained with the C^{13} and normal carbon scatterers are shown in Fig. 6, and the lower levels of the C^{13}-N^{13} mirror pair are shown in Fig. 7 for reference. Excitation of the 3.68-MeV level of C^{13}, as well as the previously reported[3] 3.09-MeV level, is observed. Note also the scattering by the 2.21- and 2.98-MeV levels of Al^{27} (12.1-g Al container). The bremsstrahlung energy, 3.79 MeV, was not high enough to excite the 3.86-MeV state; it is known, in addition, that this is a relatively slow transition[18] which we would not expect to see.

The width of the 3.68-MeV level was determined by comparing the scattering with that by the P^{31} 3.51- and Li^6 3.56-MeV levels. Appropriate corrections for self-absorption in the scatterers were made, and corrections for small variations in the incident photon flux at resonant energy were calculated from the Bethe-Heitler formula, as integrated over electron angle by Schiff.[19]

[13] D. H. Wilkinson, Phys. Rev. **109**, 1603 (1958).
[14] P. M. Endt and C. Van der Leun, Nucl. Phys. **A105**, 1 (1967).
[15] A. C. Wolff, M. A. Meyer, and P. M. Endt, Nucl. Phys. **A107**, 332 (1968).
[16] E. C. Booth and K. A. Wright, Nucl. Phys. **35**, 472 (1962).
[17] H. Willmes and G. I. Harris, Phys. Rev. **162**, 1027 (1967).
[18] F. Riess, P. Paul, J. B. Thomas, and S. S. Hanna, Phys. Rev. **176**, 1140 (1968).
[19] L. I. Schiff, Phys. Rev. **83**, 252 (1951).

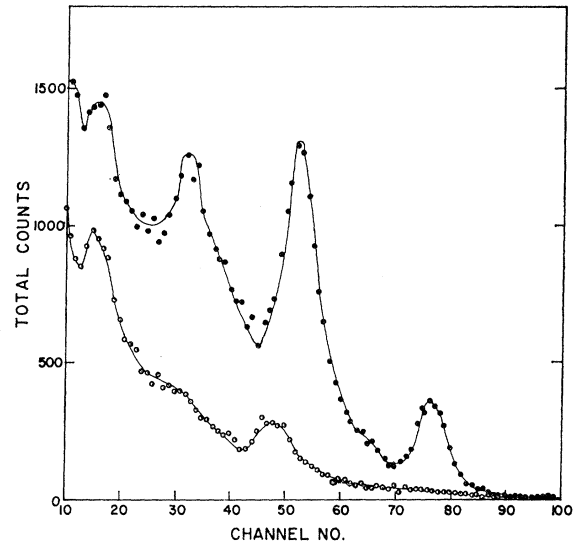

FIG. 6. Scattered spectra with C^{13} (upper curve) and normal carbon (lower curve) scatterers. The peaks for C^{13} correspond to excitation of the 3.68- and 3.09-MeV levels. For normal carbon, the observed peaks result from the Al container.

As will be discussed in more detail in a future publication, the error in this extrapolation seems to be small compared to our statistical errors. Taking the spin sequence to be $\frac{1}{2}$-$\frac{3}{2}$-$\frac{1}{2}$, the Doppler width[20] to be $\Delta=8.4$ eV, and assuming the C and Li decays are directly to the ground state, one finds that the value based on the comparison with Li^6 is only 1% lower than that based on P^{31}. We find, for the width of the 3.68-MeV level of C^{13}, $\Gamma=0.44\pm0.04$ eV, where the error given includes both the statistical contribution and an estimate of other uncertainties.

For the angular distribution we find $A_2=0.104\pm0.080$. As in the 3.51-MeV P^{31} case, this corresponds to a region of Fig. 5 where A_2 is a slowly varying function of the $E2/M1$ mixing ratio δ. This mixing ratio has been measured by Poletti *et al.*[21] They also use a sign convention opposite to the one we use, and find $x=-0.096_{-0.021}{}^{+0.030}$, consistent with our measurements. The partial widths for the ground-state decay of the 3.68-MeV level of C^{13} are then $\Gamma(M1)=0.44$ eV, or[22] 0.42 Weisskopf units (W.u.), and $\Gamma(E2)=4.1_{-1.7}{}^{+3.1}$ meV, or 4.1 W.u.

For the mirror transition in N^{13}, Young *et al.*[23] give

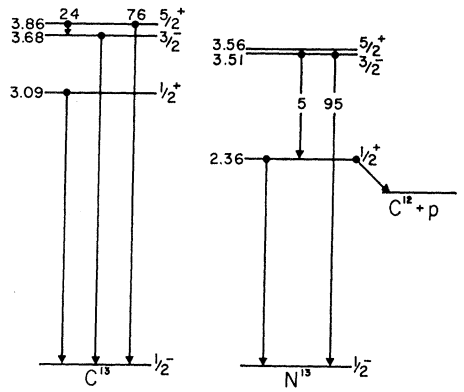

FIG. 7. Lower levels of the mirror pair C^{13}-N^{13} with the Coulomb energy difference of the ground states subtracted out.

$\Gamma_\gamma=0.65$ eV. Poletti *et al.*[21] quote a mixing ratio $x=0.092\pm0.02$, giving $\Gamma(M1)=0.65$ eV or 0.71 W.u. and $\Gamma(E2)=5.5$ meV, or 6.9 W.u. The ratio of the $M1$ strengths of these $\Delta T=0$ mirror transitions is 1.69, in agreement with Morpurgo's prediction[1] of equality within approximately 50%.

Poletti *et al.*,[24] in a paper discussing the relative phases of $E2$ and $M1$ transition amplitudes, have calculated matrix elements for these C^{13} and N^{13} transitions using both intermediate coupling and least-square effective-interaction wave functions. Their results are presented primarily as a plot of $\lambda(M1)$, the reduced transition amplitude versus α/K, the intermediate coupling parameter, and show a smooth decrease of $\lambda(M1)$ for C^{13} from 2.6 at $\alpha/K=0$ to 1.8 at $\alpha/K=9$. For the effective interaction calculation, their Fig. 1 gives $\lambda(M1)=2.25$. Our experimental result for C^{13} is $\lambda(M1)=1.79\pm0.09$. For N^{13}, the corresponding theoretical values are 3.1 to 2.2 and 2.65, and the experimental result corresponds to 2.34 ± 0.2. For both cases agreement between theory and experiment is best for α/K in the range 6–9, although none of the theoretical results is in violent disagreement with experiment.

ACKNOWLEDGMENTS

We are indebted to Dr. H. O. Cohn and Dr. J. K. Bair of Oak Ridge National Laboratory for lending us the C^{13} sample, and to Dr. S. S. Hanna for the communication of results prior to publication. We thank J. M. Moller, a National Science Foundation summer student from the California Institute of Technology, for writing and debugging the computer program which calculates the geometric factor, attenuations, etc., for the various scattering geometries used. Finally, we are indebted to Dr. F. R. Metzger for advice and encouragement.

[20] This width is based on two assumptions, that the C^{13} sample is amorphous and that the appropriate Debye temperature is then around 300°K. Neither of these is too well founded, but fortunately they are not important. Taking the Debye temperature as 1800°K (diamond) gives $\Delta=11.8$ eV and $\Gamma=0.42$ eV.

[21] A. R. Poletti, J. W. Olness, and E. K. Warburton, Phys. Rev. **151**, 812 (1966).

[22] D. H. Wilkinson, in *Nuclear Spectroscopy, Part B*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960).

[23] F. C. Young, J. C. Armstrong, and J. B. Marion, Nucl. Phys. **44**, 486 (1963); J. C. Armstrong, M. J. Baggett, W. R. Harris, and V. A. Latorre, Phys. Rev. **144**, 823 (1966).

[24] A. R. Poletti, E. K. Warburton, and D. Kurath, Phys. Rev. **155**, 1096 (1967).

APPENDIX

The cross sections for resonance scattering and absorption are given, for example, by Metzger[25] as

$$\sigma_{sc}=\sigma_{sc}^{0}\,\psi(x,t), \tag{1}$$

$$\sigma_{abs}=\sigma_{abs}^{0}\,\psi(x,t). \tag{2}$$

Here σ_{sc}^{0} and σ_{abs}^{0} are the peak Breit-Wigner cross sections for scattering and absorption, $x=2(E-E_R)/\Gamma$ is the reduced energy, $\sqrt{t}=\Delta/\Gamma$ is the ratio of the thermal Doppler width of the level to the natural width, and

$$\psi(x,t)=\frac{1}{2(\pi t)^{1/2}}\int_{-\infty}^{\infty}\frac{dy}{1+y^2}\exp\left[-\frac{(x-y)^2}{4t}\right] \tag{3}$$

is the convolution of the Breit-Wigner dispersion formula and the assumed Gaussian distribution of the thermal Doppler shifts.

For nuclear levels which can decay only by electromagnetic processes, one usually finds $\Delta\gg\Gamma$. The resonant transmission in a pathlength l, with n_d nuclei/cm^3, can then be obtained from the series[26]

$$\sum_{m=0}^{\infty}\frac{(-1)^m}{m!(m+1)^{1/2}}\left[\frac{n_d l\sigma_{abs}^{0}\,\Gamma\sqrt{\pi}}{2\Delta}\right]^m. \tag{4}$$

Some cases are found for which $\Gamma\sim\Delta$. Melkonian *et al.*[27] showed that the attenuation can then be obtained from a similar series containing terms

$$\Psi_m(t)=\int_{-\infty}^{\infty}dx\psi^m(x,t).$$

Various tables of $\psi(x,t)$ have been published, and tables of $\Psi_m(t)$ can be derived from these. We decided, however, that it might be more convenient to take a somewhat different approach. The incident beam will reach a small volume element of the scatterer after encountering $n_d(l_{sc}+l_{abs})$ nuclei per cm^2. Neglecting nonresonant attenuation, the scattering from this element will be proportional to

$$\sigma_{sc}^{0}\int_{-\infty}^{\infty}dx\,\psi(x,t)\,\exp[-n_d(l_{sc}+l_{abs})\sigma_{abs}^{0}\psi(x,t)]. \tag{5}$$

Tables of

$$A(B,t)=\int_{-\infty}^{\infty}dx\,\psi(x,t)\,\exp[-B\psi(x,t)] \tag{6}$$

can be prepared by numerical integration and used, together with other appropriate factors such as solid angle, counter efficiency, etc., to predict counting rates with and without absorbers for various values of t.

Three published tabulations of the ψ function were available to us. The oldest and most extensive is that of Rose *et al.*[28] That of Seth and Tabony[29] is more recent. They point out that the tables of Rose *et al.* are apparently in error by as much as 2.6% in, as it happens, a region of particular interest to us. Since, for both of these tabulations, the range of x for different values of t and the interval of x for the same t and different x varied, we performed the numerical integration in the most convenient (but probably not the most accurate) manner by finding the area of the histogram based on the tabulated values. The results can be checked by noting that Eq. (6) can be integrated analytically for $B=0$, and is equal to π. Departures from π of 2–3% depending on the value of t were found. Adding a Breit-Wigner tail, of the form $1/(1+x^2)$ for values of x larger than those tabulated and up to $x\sim1000$ (again using a histogram approximation) reduced this discrepancy to $<0.15\%$. Spot comparisons of the results using Seth's tables for $t=1$, 2, and 4, and various values of B agreed with those from Rose's tables to 0.1% or better, so that the differences seem to integrate out. Our final results were based on the tables of Rose *et al.* We also found that 21 values of B, ranging from 0 to 6 and spaced more closely for small B, covered the range of interest to us, and allowed linear interpolation to better than 0.1%.

In a third tabulation of the ψ function, Cook and Elliot[30] have made the change of variable $\zeta=x/(1+x)$, and give values of ψ for $0\leq\zeta\leq1$, corresponding to $0\leq x\leq\infty$. They also calculate $\Psi_m(t)$ for $m=1$, 10, using their ψ's and Simpson's rule. We calculated our $A(B,t)$ for $t=1$ using Simpson's rule and their ψ's. We find[31] $A(0,1)=3.10844$, departing somewhat from π, and for other values of B, we find disagreements up to $\sim2\%$ with the results from Rose's tables.

In conclusion, we note that while our numerical integration is not as accurate as desirable, it is still adequate for our measurements and probably limited by uncertainties in the tabulated ψ function. We prefer to leave examination of this latter question to someone better qualified.

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