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Proton Production from Nuclei Bombarded by Protons of 1, 2, and 3 BeV*

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The momentum spectra of protons emitted from Be, C, Cu, and Pb nuclei bombarded by 1-, 2-, and 3-BeV protons from the Cosmotron have been measured at angles of 0°, 17°, and 32°. After correcting for scattering in the detectors, the spectra show reasonable agreement with Monte Carlo calculations of Bertini. However, the dependence on mass number A requires a much larger value for the exponent of A than is predicted by theory.

1. INTRODUCTION

FEW measurements are available for differential proton-production cross sections from nuclei for protons incident in the BeV region. We wish to compare results for the production of protons of lower energy with Monte Carlo calculations. The comparison is severe, in that the cascade process has run through several stages in reaching the lower energies. However, our results are well above the evaporation region.

2. DISCUSSION OF MONTE CARLO CASCADE CALCULATIONS

At energies above 100 MeV, which is the lower limit in our experiments, the proton wavelength is less than 0.5 F, whereas the internucleon separation in the nucleus is of the order of 1 F. Hence, it is reasonable to assume that the reactions involved can be described in terms of particle-particle collisions. Experimental values of the nucleon-nucleon and pion-nucleon cross sections have been employed by several workers[1–5] to make a semiempirical prediction of the nuclear cascade initiated by an incoming high-energy particle. The life history of each particle is followed by a Monte Carlo calculation. We shall compare our results with two such calculations.[1,4] It is important to understand the underlying assumptions made concerning the intranuclear forces in these calculations, since upon them depends the accuracy of the spectra generated. The model of Metropolis *et al.*[4] assumes a constant density of particles within the nucleus, together with a zero-temperature Fermi energy distribution of the nucleons. The potential for pions within the nucleus was assumed to be zero. This model was improved upon by Bertini.[1] He adjusted the nuclear density distribution of protons in three radial steps to approximate the Fermi charge distribution function obtained from electron scattering data.[6] The neutron and proton region boundaries were assumed to be the same. The effect of these annuli of differing density was to give the nucleus a nonzero Fermi temperature with a kT value of about 15 MeV. The same potential was used for interacting pions as for nucleons. Refraction at the nuclear surface was not included, and Coulomb scattering neglected. Both of these latter approximations are unlikely to affect the results for the energy range in which we were working, although they significantly affect the results at lower energies.[5]

Low-energy experiments (from 60 to several hundred MeV) are in good agreement with the theory.[7] However,

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[2] L. S. Azhgirey *et al.*, Nucl. Phys. **13**, 258 (1958).

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[5] K. Chen *et al.*, Phys. Rev. **166**, 949 (1968).

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multiple production of particles, such as pions, will affect the results at high energies, and these increase the complexity and decrease the accuracy of the calculation. This is one possible reason for the discrepancy between our results and the theory.

There are several interesting features of the production energy spectrum. At the lower end lies the evaporation distribution. This does not extend above 50 MeV, which is below the region we were investigating. The nuclear temperature induced by energetic protons can be quite high; for example, 17 MeV is the average kinetic energy of the evaporated protons for a proton incident with 700 MeV.[8] The nuclear elastic peak lies at the high-energy end of the spectrum, and directly beneath it are peaks produced by inelastic collisions, leaving the residual nucleus in an excited state. These provide interesting information about the interaction of the incoming proton with the nucleus as a whole, but are beyond our energy range. Below this lies the broad quasielastic peak, arising from the elastic collision of the incoming proton with one nucleon, both particles leaving the nucleus without further interaction. Both calculations and experiments differ as to how marked this peak is.[1,4,5,9,10]

The bulk of the secondary protons come from the cascade, in which the incoming proton strikes one or more nucleons in the nucleus, which in turn scatter other particles. The high-energy particles of the cascade are emitted strongly in the forward direction; but, at low energies, the distribution can become almost isotropic. Most of our results lie in the cascade region.

3. EXPERIMENTAL METHOD AND ERRORS

Targets of Be, C, Cu, and Pb, about 7 g/cm^2 thick, were arranged around the periphery of a styrofoam disk, capable of being rotated so that each in turn could be inserted in the external proton beam of the Cosmotron. Particles emitted at angles of 0°, 17°, and 32° entered magnetic spectrometers which deflected the collimated scattered beams, and provided a momentum resolution of approximately 2%. Protons were separated from pions and deuterons by a time-of-flight spectrometer, the spectrum being recorded on paper tape. It provided approximately 100 to 1 discrimination against other particles.

In no case in any experimental result did the statistical standard deviation for errors arising from the number of events exceed 1%. This is too small to be plotted on the figures. Two systematic errors were also important. One arose in obtaining the solid angle to derive the absolute cross section, by tracing particles through the magnet system, using a Monte Carlo calculation and a classical optical method. Uncertainties in this calculation may give rise to errors of as much as 50% in the absolute cross sections. The other occurred in calculating the scattering correction for the detector system. This could be as large as 20% for protons emitted with 100 MeV, but is already very small for 400-MeV protons. These two errors have no effect on the A dependence, since this is a ratio.

4. COMPARISON OF EXPERIMENTAL RESULTS WITH RESULTS OF OTHER WORKERS

We have compared our data with those of Piroue and Smith,[9] for protons incident at 3 BeV and find excellent agreement with their spectral shape for Be and Pt, with ours for Be and Pb. Our results also indicate a peak in the spectrum at approximately the same energy for the larger angles. The ratio of the proton yields at 1 BeV/c which they obtain at 13° for Pt to Be is 6.5, whereas for Pb to Be at 17° and the same momentum and incident energy, we get 10. This represents a factor outside our standard error, but may arise from uncertainties in the efficiency of the circulating beam in their case. Agreement with the absolute cross section is reasonable, considering the additional uncertainties involved. At 30° and 1 BeV/c they get 4.2×10^{-26}, and at 32° we get 6×10^{-26} cm^2/(BeV/c) for Be. Other points show similar agreement at this angle.

We also compared our results with those of King[11] and Lock[12] who employed nuclear emulsions. Although the agreement is quite good at higher emitted energies, our results fall off at lower energies in agreement with Piroue and not with King. Both Piroue and ourselves used magnetic analysis systems which involve large scattering corrections at low energy. It would be interesting to make measurements using a different technique to resolve this point.

5. DISCUSSION OF EXPERIMENTAL RESULTS

The dependence of the differential cross section on the mass number A can be approximated by a power law A^x, where x is plotted as a function of the energy and angle of the emitted proton in Fig. 1.

In order to interpret the significance of the cross-sectional dependence on mass number for different angles, masses and energy ranges, let us consider some extreme cases.

At high energies, elastic scattering from the nucleus is primarily a diffraction process, and hence is proportional to the projected area of the nucleus or $A^{2/3}$. At a somewhat lower energy, quasielastic scattering off individual nucleons in the nucleus occurs. If the incoming proton is likely to make one interaction or less in the nucleus, the cross section will be equal to the

[8] I. Dostrovsky, P. Rabinowitz, and R. Bivins, Phys. Rev. **111**, 1659 (1958).
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[11] D. T. King, Phys. Rev. (to be published); Oak Ridge National Laboratory Neutron Physics Division Report No. ORNL 4280, 1968, p. 150 (unpublished).
[12] W. O. Lock, P. V. March, and R. McKeague, Proc. Roy. Soc. (London) **A231**, 368 (1955).

number of protons Z multiplied by the differential cross section for a single proton, plus the similar relation for neutrons $(A\text{-}Z)$. The roughly constant ratio of neutrons to protons allows us to approximate the cross section to be A multiplied by an average differential cross section $d\sigma/d\Omega$. However, this applies only to very light nuclei since the mean free path for a proton in nuclear matter is approximately 2 F at our energies. It appears[10,13] that the quasielastic peak for all but the lightest nuclei arises from single collisions in the nuclear surface. We may assume there is some depth for such an interaction, a, which is of the order of a mean free path. The incoming proton makes a single collision within this depth, those penetrating more deeply making multiple collisions. Similarly, after scattering, particles emerging from the nucleus must travel a path not greater than a in order to make no further collisions. It is then a simple geometrical calculation to show that the cross section in the forward direction is the average nucleon-nucleon differential cross section multiplied by $\frac{1}{4}\pi a(2a^2+r^2\theta^2)$, where r is the radius of the nucleus and θ the scattering angle. This means that for angles very much less than $\sqrt{2}(a/r)$, the cross section is independent of the mass number A, and for angles larger than $\sqrt{2}(a/r)$, it is proportional to $A^{2/3}$.

This approximation is good only when $a \ll r$; however, it does suggest that at the far forward angles for heavy nuclei, the exponent of A should be small and at large angles, the exponent should be larger, tending to a value near $A^{2/3}$. The effect of the Fermi motion of the particles on the width of the peak has been fully discussed elsewhere.[10,13]

So much for single scattering. Below the quasielastic peak, the cross section arises partly from inelastic scattering from protons and neutrons, with the production of pions, and partly from multiple scattering, or the cascade process. There is some evidence[12] that just below the quasielastic peak the cross section consists largely of single inelastic scattering events, which would have a similar A dependence to the corresponding single quasielastic events. However, as the energy decreases, there is more and more multiple scattering.

A very simple estimate indicates the direction in which the A dependence must go, although only a computer calculation such as the Monte Carlo calculations already discussed will give the exact dependence. The mean distance traveled by the incoming particle through nuclear matter is $\frac{4}{3}\pi r^3/\pi r^2=\frac{4}{3}r$. If λ is the mean free path, the mean number of collisions made in a nucleus by the incoming particle is

$$\frac{4}{3}\frac{r}{\lambda}=\frac{4}{3}\frac{1.4\times10^{-13}A^{1/3}}{2\times10^{-13}}\approx A^{1/3}=n.$$

Now, since on the average the proton loses half its

[13] D. M. Corley (private communication); Ph.D. thesis, University of Maryland, 1968 (unpublished).

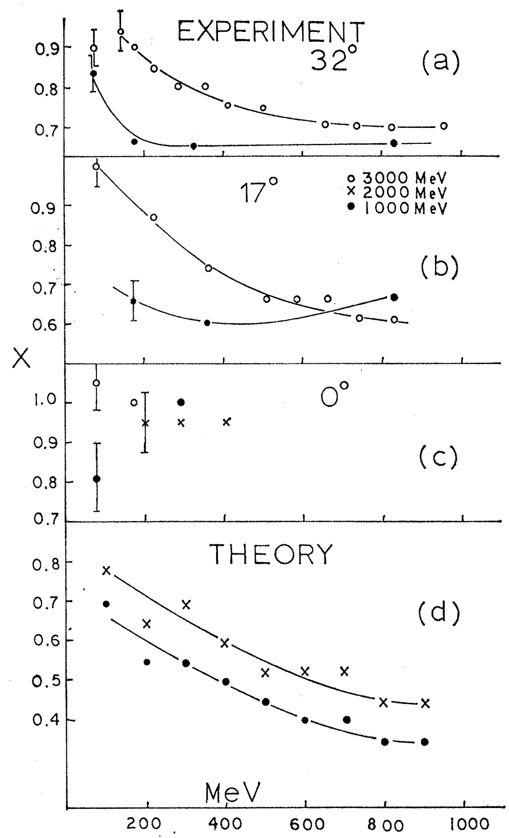

FIG. 1. Value of x in the function A^x, where A is the mass number, plotted against scattered proton energy. (a) Experimental results at 32°, (b) 17°, (c) 0°. The theoretical predictions of Bertini for angles integrated between 10° and 30° are plotted in (d). The lines drawn are to aid in following the points and have no theoretical significance.

energy in a collision, we may assume we have two nucleons per collision, and the number of nucleons goes up by two on each collision. If there are n collisions the number of nucleons leaving the nucleus is 2^n and their average energy would be $E/2^n$, where E is the incident proton energy.

The cross section for the incoming proton, which then produces these 2^n particles will be πr^2, or the area of the nucleus, and hence the cross section for nucleon production would be $2^n(\pi r^2)$ or $r_0^2A^{2/3}2^{A^{1/3}}$, where r_0 is the nuclear radius constant. Only half of the outgoing particles would be protons. In terms of this crude calculation, an A^2 dependence would be expected over the range of nuclei from Be to Pb, indicating that a nuclear cascade would increase the dependence on A. More exact calculations also show a less severe increased A dependence, although it does not take the same form, and is model-dependent. The exponent of A is also strongly dependent on the energy of the outgoing particles. Since the average energy of the outgoing particles is much lower for heavy nuclei, there are correspondingly fewer high-energy particles than for light nuclei. This means that a lower exponent for A would be required at the high energies than at the lower, as can be seen more accurately from the calculations of Bertini and Metropolis.

To summarize, these simple calculations show that a low exponent for A is to be expected at forward angles, and at high energies, in the vicinity of the quasielastic peak. At such energies, but for larger angles, the exponent of A increases but does not excede $A^{2/3}$. As we go from lighter to heavier elements, and as we move farther and farther below the quasielastic peak, the

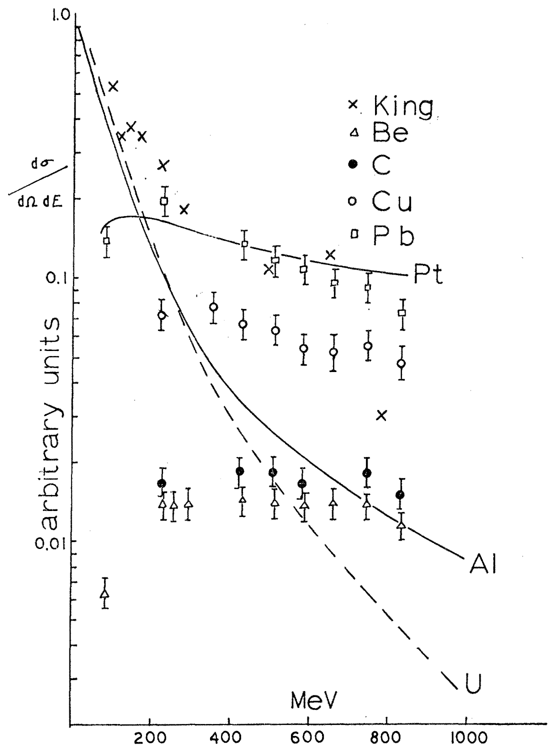

FIG. 2. Our differential cross section at 17° for protons of 3 BeV incident for Be, C, Cu, and Pb. The curve labeled Pt is that of Piroue and Smith (Ref. 11) for 13° with 2.9-BeV protons. The points found by King (Ref. 12) for nuclear emulsions are also plotted. The curves labeled Al and U represent the spectra predicted by Metropolis, on an arbitrary scale, with the cross section integrated over angle for protons incident at 1840 MeV.

single scattering process becomes less important, and multiple scattering, with a stronger A dependence, dominates. Put another way, a dependence of $A^{2/3}$ or less suggests a surface interaction with single scattering. A larger exponent indicates a volume interaction, with multiple scattering. We must now see how our results agree with the findings of this discussion.

Curve (a) of Fig. 1 shows the expected dependence on $A^{2/3}$ for the higher energies, specifically for the lower incident proton energies. At lower scattered energies the cascade process produces a dependence which is closer to 0.9, which is, as anticipated, quite large.

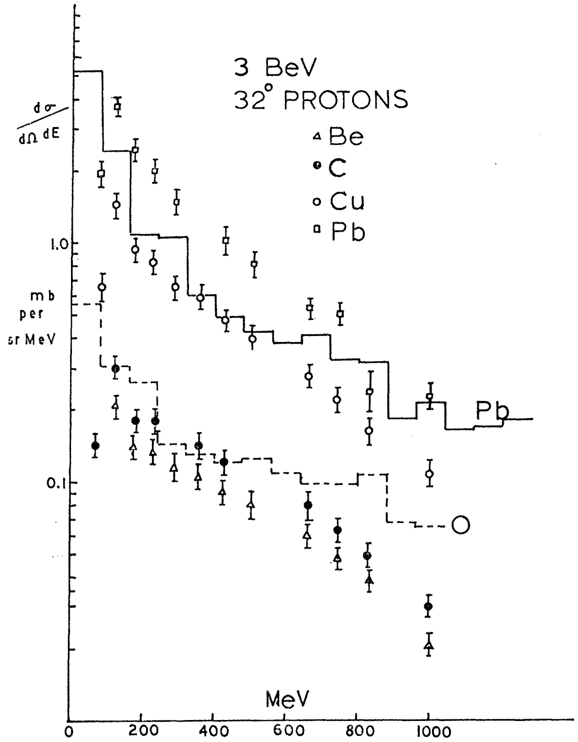

FIG. 3. Our differential cross section at 32° for 3-BeV incident protons. The hodograms represent the Spectra of Bertini, integrating the cross section from 10° to 32°, for 2000-MeV incident protons.

Curve (b) at the smaller angle shows a similar effect but at 0°, the dependence has a higher value for x. This is because particles at low energies and 0° have for the most part come right through the nucleus, and,

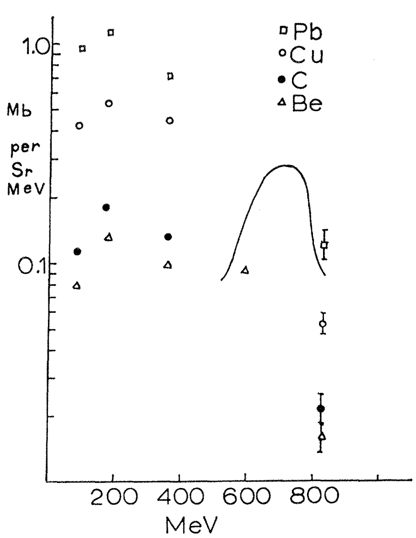

FIG. 4. Our differential cross section at 32° for 1-BeV incident protons. The quasielastic peak predicted by the shell-model calculations of Wall for beryllium is plotted in the same figure.

therefore, the yield is almost entirely cascade. The lower value of x, of 0.3 or 0.4, inferred from Bertini's data is a limit which is not found in practice, although had we examined lower angles for higher scattered energies, lower values for x might have been seen.

The approximation A^x does not provide an exact fit over the entire range of mass number. Thus, if the results for Be^9 and Pb^{208} are used to obtain the value of x, then the C and Cu results fall where masses 14 and 90 would be expected, respectively. This is explainable for C, which has proportionately a larger proton to neutron ratio than Be. Several possible explanations could account for the copper. One is that the lead has proportionately more neutrons, particularly in the surface region than copper, thus reducing the proton output. Fluctuations caused by binding energy, shell configurations, etc., could also cause this effect.

Examples of the energy spectra are plotted in Figs. 2 and 3 for 3 BeV at 17° and 32°, together with the curves of Bertini at 2 BeV and of Metropolis for 1840 MeV, the highest energies which they give.

There is generally reasonable agreement for the spectral shape. The rapid decrease with energy for the results of Metropolis arises from the integration over angle. When corrections are made for this and for extrapolation to 32°, agreement is better.

We have only one or two points which fall where the quasielastic peak is expected, all for 1-BeV protons (Fig. 4). However, Corley and Wall[12] have measured this region very accurately, up to 20°. We have points for Pb and Cu for 17° at 820 MeV. Corley's experimental curves would indicate these fall about three quarters the way to the top of the quasielastic peak. We find our absolute cross sections lie within 20% of this curve, extrapolating the copper results to calcium, and well above the anticipated cross section were there no quasielastic scattering.

At 32° we have one point for Be^9 at 590 MeV, and points for Be, C, Cu, and Pb at 820 MeV. These latter points lie very low, being above the quasielastic peak in energy. However, the point at 590 MeV lies about one-third up the peak, which was very kindly calculated for us by Wall and Roos,[10] and our results are approximately 0.7 of the calculated point, and a factor of 2.5 higher than a line joining the 350- to the 830-MeV points. This is an indication that, for a light element such as Be, the quasielastic peak is present at least as far out as 30°.

6. CONCLUSION

The comparison between these experiments and the cascade theory shows a general agreement in the spectral shape. However, the very large discrepancy in A dependence is surprising, the calculations consistently giving a lower value for the exponent of A, especially at low energies.

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