Electron-Magnon Effects in Ferromagnetic Junctions

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(Received 4 February 1969)

We have investigated the effect of magnons on the tunneling characteristics of a metal (A)–oxide of A–metal (B) junction in which metal B is ferromagnetic and metal A is a simple metal. The magnons influence the tunneling conductance in two ways: (1) They produce conduction-electron self-energy effects which modify the tunneling density of states, and (2) they give rise to an additional magnon-assisted inelastic tunneling channel. The calculations were performed using an s-f model. The self-energy corrections give rise to two symmetric peaks located at an energy at which the magnon becomes degenerate with the spin-flip excitations of the conduction band. Using parameters appropriate for Gd, this effect is of the order of 1%. The magnon-assisted tunneling produces an additional conductance whose characteristic depends on whether the barrier is diffuse or specular. This additional conductance contains both an even and an odd contribution. The magnitude of the odd part depends on the ratio of the characteristic magnon energy to the barrier height. A critique of the momentum and energy dependence of the tunneling matrix elements is given in connection with the above calculations.

1. INTRODUCTION

THE electron-magnon interaction in ferromagnetic metals has recently been considered by Nakajima and others.[1,2] As in the electron-phonon problem, it was found that the electron self-energy varies rapidly with energy and a large enhancement of the electronic specific heat at low temperatures was predicted. In this paper, we consider the effect of the electron-magnon interaction on the I-V characteristics of ferromagnetic tunnel junctions. For simplicity we study a normal-metal–oxide-barrier–ferromagnetic-metal junction.[3]

The magnons effect the tunneling characteristics of such a junction in two ways. These are illustrated in Fig. 1. In Fig. 1(a) an electron from metal A tunnels elastically through the oxide into the electron states of metal B, which are renormalized by the electron-magnon interaction in the bulk of the material.

In Fig. 1(b) the electron tunnels inelastically through the oxide into metal B, causing a magnetic impurity[4] near the oxide layer to flip its spin. In contrast to the situation considered by Appelbaum and Anderson,[5] the localized spin will be coupled to the bulk magnetization of the ferromagnetic material. Consequently, its spectral distribution will reflect the magnon density of states in metal B. This process must be distinguished from a somewhat similar process contained in Fig. 1(a) in which the electron first tunnels through the barrier and subsequently flips its spin. In this case the magnon results from the dressing process while in the inelastic tunneling the magnon results from a change in the quantum state of the barrier.

These processes are represented by a Hamiltonian of the form

$$\mathcal{H}=\mathcal{H}_A+\mathcal{H}_B+\mathcal{H}_T. \tag{1}$$

For metal A we assume a one-electron Hamiltonian:

$$\mathcal{H}_A=\sum_{\mathbf{k},\alpha}\epsilon_{\mathbf{k}}{}^a a_{\mathbf{k}\alpha}{}^\dagger a_{\mathbf{k}\alpha}, \tag{2}$$

while for metal B we assume the magnetic electrons localized, and adopt the usual s-f (d) model for their coupling to the conduction electrons.

$$\mathcal{H}_B=\sum_{\mathbf{k},\alpha}\epsilon_{\mathbf{k}}{}^b b_{\mathbf{k}\alpha}{}^\dagger b_{\mathbf{k}\alpha} -\frac{J}{N}\sum_{\mathbf{k},\mathbf{k}'}\sum_{\alpha,\alpha'}\sum_i \mathbf{S}(R_i)\cdot\boldsymbol{\sigma}_{\alpha,\alpha'}e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{R}_i}b_{\mathbf{k}\alpha}{}^\dagger b_{\mathbf{k}'\alpha}. \tag{3}$$

In these equations $a_{\mathbf{k}\alpha}{}^\dagger$ ($b_{\mathbf{k}\alpha}{}^\dagger$) creates an electron with energy $\epsilon_{\mathbf{k}}{}^a$ ($\epsilon_{\mathbf{k}}{}^b$) and spin α in metal A (B). The localized spin at site i is denoted by $\mathbf{S}(\mathbf{R}_i)$ and $\boldsymbol{\sigma}_{\alpha\alpha'}$ are the conventional Pauli spin matrices.

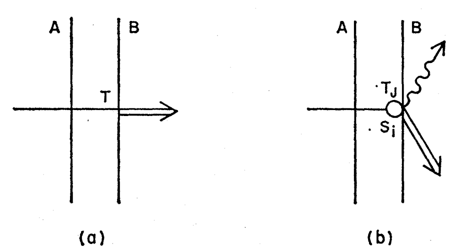

FIG. 1. Magnon-electron interaction effects on the tunneling current. (a) The electron tunnels elastically into the renormalized electron state in metal B (⇒). (b) The electron tunnels inelastically interacting with a spin located in the barrier emitting a spin wave (wiggly line).

[1] S. Nakajima, Progr. Theoret. Phys. (Kyoto) 38, 23 (1967).
[2] H. S. D. Cole and R. E. Turner, Phys. Rev. Letters 19, 501 (1967); L. C. Davis and S. H. Liu, Phys. Rev. 163, 503 (1967).
[3] J. A. Appelbaum and W. F. Brinkman, Bull. Am. Phys. Soc. 13, 442 (1968).
[4] The magnetic impurities we are referring to are ions from the ferromagnetic metal. Their presence near the metal-oxide interface results from the lack of atomic sharpness in the definition of this interface. A certain fraction of these are more appropriately thought of as belonging to the oxide barrier; it is these that are responsible for the "assisted tunneling." This definition is, of necessity, somewhat imprecise, and results from the partially phenomenological tunneling Hamiltonian formalism, which requires a separation of tunneling processes from many-body interactions in the metals.
[5] J. A. Appelbaum, Phys. Rev. Letters 17, 91 (1967); P. W. Anderson, *ibid.* 17, 95 (1967).

The tunneling between the two metals is described by

$$\mathcal{H}_T=\sum_{\mathbf{k},\mathbf{k}',\alpha} T(a_{\mathbf{k}\alpha}{}^\dagger b_{\mathbf{k}'\alpha}+b_{\mathbf{k}'\alpha}{}^\dagger a_{\mathbf{k}\alpha}) +\sum_i \sum_{\mathbf{k},\mathbf{k}'} \sum_{\alpha,\alpha'} T_J \mathbf{S}_i\cdot\boldsymbol{\sigma}_{\alpha,\alpha'} e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}_i} \times(a_{\mathbf{k}\alpha}{}^\dagger b_{\mathbf{k}'\alpha'}+b_{\mathbf{k}'\alpha'}{}^\dagger a_{\mathbf{k}\alpha}). \quad (4)$$

The first term of Eq. (4) describes ordinary tunneling, while the second term describes magnetic-impurity-assisted tunneling. $\mathbf{S}_i$ here refers to a localized spin at position $\mathbf{r}_i$ in the barrier.

In treating the Hamiltonian we use the Holstein-Primakoff transformation[2] to replace both $\mathbf{S}(\mathbf{R}_i)$ and $\mathbf{S}_i$ by magnon creation and annihilation operators for metal B. The above transformation enables one to treat this problem by conventional many-body techniques. Using the electron self-energy calculated in Ref. 2 we study its effect on the conductance. We find that at small voltages the conductance has a peak whose width is the order of $\frac{1}{10}$ the maximum magnon energy. This effect, discussed in Sec. 2, is due to a logarithmic term in the self-energy. The magnitude of the effect is of the order of 1% and should be observable.

In Sec. 3 we discuss the assisted tunneling assuming both specular and diffuse boundary conditions for this process. In the latter case, the extra conductance is just proportional to the total number of magnons with energy less than $|eV|$. For specular transmission the situation is more complicated but qualitatively similar to the diffuse result. In Sec. 4 we give our conclusions and discuss certain of the experimental aspects for Ni and Gd.

2. RENORMALIZATION EFFECTS

For the considerations of this section we ignore the last term in Eq. (4). If the first term in $\mathcal{H}_T$ is treated by standard second-order perturbation theory the tunneling current is given by[6]

$$I=4\pi e \sum_{\alpha',\alpha\mathbf{k},\mathbf{k}'} |T_{\mathbf{k}\mathbf{k}'}|^2 \int_{-\infty}^{\infty}\frac{d\epsilon}{\pi}\int_{-\infty}^{\infty}\frac{d\epsilon'}{\pi}\operatorname{Im}G_{\mathbf{k}\alpha}{}^a(\epsilon) \times \operatorname{Im}G_{\mathbf{k}'\alpha'}{}^b(\epsilon')[f(\epsilon)-f(\epsilon')]\delta(\epsilon'-\epsilon-eV). \quad (5)$$

Here $G_{\mathbf{k}\alpha}{}^{a(b)}(\omega)$ is the one-electron Green's function for metal $A(B)$ and f is the Fermi function. Electrons flow into metal B when a voltage V greater than zero is applied to metal B. The current is defined positive when it flows from B to A.

If we assume specular transmission and make an effective-mass approximation for metal A, Eq. (5) can be reduced to

$$I=4\pi e\int_{-\infty}^{\infty} d\epsilon \sum_{\mathbf{k}_{\parallel},k_\perp} \mathrm{A}(\mathbf{k}) \sum_\alpha \operatorname{Im}G_{\mathbf{k}\alpha}{}^b(\epsilon) \times[f(\epsilon)-f(\epsilon+eV)], \quad (6)$$

where

$$A'(\mathbf{k})=|T_{\mathbf{k}\mathbf{k}'}|^2/ (d\epsilon_{k'}{}^a/dk_\perp')\,|_{k_\perp'=[2m^a(\epsilon_F{}^a+eV+\epsilon)-\mathbf{k}_{\parallel}{}^2]^{1/2};\,\mathbf{k}_{\parallel}'=\mathbf{k}_{\parallel}}, \quad (7)$$

and $k_\perp$ ($\mathbf{k}_\parallel$) is the component of $\mathbf{k}$ perpendicular (parallel) to the barrier.

We measure energies from the Fermi energy $\epsilon_F{}^a$ of metal A. Differentiating the current with respect to voltage and replacing the momentum sums by integrals over energy, we obtain for the conductance G at zero temperature

$$G(V)=\int_{-\infty}^{\infty} d\epsilon\,\delta(\epsilon+eV)\int_0^\infty d\epsilon_k \sum_\alpha \operatorname{Im}G_{k\alpha}{}^b(\epsilon) \times\left[\frac{m^b e^2}{\pi}\int_0^{\epsilon_k} d\epsilon_{k_\parallel} A(\epsilon_{k_\parallel},\epsilon_k)\right], \quad (8)$$

where

$$A(\epsilon_{k_\parallel},\epsilon_k)=|T_{\mathbf{k}\mathbf{k}'}|^2 \bigg/ \left(\frac{d\epsilon_{\mathbf{k}'}{}^a}{dk_\perp'}\frac{d\epsilon_{\mathbf{k}}{}^b}{dk_\perp}\right)\bigg|_{\mathbf{k}_\parallel'=\mathbf{k}_\parallel;\,k_\perp'=[2m^a(\epsilon_F{}^a+eV+\epsilon)-k_\parallel{}^2]^{1/2}}. \quad (9)$$

In the usual treatment of superconducting tunneling[6] the expression in brackets, $F(\epsilon_k)$, in Eq. (8) is assumed constant. This is justified because $\operatorname{Im}G_{k\alpha b}(-eV)$ varies rapidly over an energy scale (gap$\simeq$1 meV) in which $F(\epsilon_k)$ varies by 1%, at most. In the present problem $\operatorname{Im}G_{k\alpha}{}^b(-eV)$ varies much less rapidly on a considerably larger energy scale ($\sim$20 meV) over which $F(\epsilon_k)$ may change by 50%. This means we must consider with much more care the tunneling matrix elements.

There exists a number of prescriptions in the literature for calculating the matrix elements.[7,8] In the tunneling-Hamiltonian approach,[8,9] used to obtain Eq. (8), $T_{\mathbf{k}\mathbf{k}'}$ is assumed obtainable from a calculation of the one-electron wave functions of the junction. In the literature, the expressions usually quoted for $T_{\mathbf{k}\mathbf{k}'}$ assume that $\epsilon_k{}^a=\epsilon_{k'}{}^b$. This will generally not be the case because real, not bare, energy is conserved in the tunneling process. An expression for $T_{\mathbf{k}\mathbf{k}'}$ not subject to the above restriction can be calculated following Prange.[10] This expression is rather complicated; the dominant part has the form

$$T_{\mathbf{k},\mathbf{k}'}\propto\frac{e^{-\kappa^a d}-e^{-\kappa^b d}}{-\kappa^a+\kappa^b}, \quad (10)$$

where

$$\kappa^{(a,b)}=[2m(U-\epsilon_k{}^{(a,b)})]^{1/2}. \quad (11)$$

This form can also be obtained using Bardeen's expression for $T_{\mathbf{k}\mathbf{k}'}$.[7]

[6] J. R. Schrieffer, *Theory of Superconductivity* (W. A. Benjamin, Inc., New York, 1964), pp. 78–87.

[7] J. Bardeen, Phys. Rev. Letters **6**, 57 (1961); **9**, 147 (1962).

[8] R. E. Prange, Phys. Rev. **131**, 1083 (1963).

[9] M. H. Cohen, L. M. Falicov, and J. C. Phillips, Phys. Rev. Letters **8**, 31 (1962).

[10] J. A. Appelbaum (unpublished).

We wish to point out that using Eq. (10) in Eq. (8) for the conductance can lead to serious errors. For example, if one used for

$$\sum_{\alpha} \mathrm{Im} G_{k\alpha}{}^{b}(-eV)=\frac{2\Gamma}{[eV+\epsilon_k+\Sigma(-eV)]^2+\Gamma^2}, \quad (12)$$

one finds that, in evaluating Eq. (8), $T_{\mathbf{kk}'}{}^2$ can weight the high-energy tail of the spectral function so that this region dominates the conductance. This is clearly not physical. The reason for this high-energy catastrophe is that the tunneling problem cannot be solved in the order that the tunneling Hamiltonian implies, i.e., solving for the tunneling matrix elements prior to solving the many-body problem. This can be seen by switching on a weak effective potential in the semi-infinite metal. The wave functions in the metal can be calculated quite accurately by perturbation theory but the tail of the wave function outside the metal is not given correctly. This means that the tunneling Hamiltonian is not invariant with respect to the turning on of the many-body interactions. As recognized by several people,[11,12] the electron tunnels at its many-body energy ϵ, *not* its bare energy. The tunneling matrix element must be

$$T_{\mathbf{kk}'}(\epsilon) \propto \exp(-\{2m[U(V)-\epsilon+\epsilon_{k_{11}}]\}^{1/2}d), \quad (13)$$

where we have suppressed any preexponential factors. If the boundaries of the barrier vary slowly compared to a Fermi wavelength, the density-of-states factors in Eq. (9) cancel against the preexponential factors. Then

$$A(\epsilon_{k_{11}},\epsilon_k)=\exp(-2\{2m[U(V)-\epsilon+\epsilon_{k_{11}}]\}^{1/2}d), \quad (14)$$

and the upper limit of the integral over $\epsilon_{k_{11}}$ in Eq. (8) is $\epsilon+\epsilon_F$. This integral then depends only on ϵ and explicitly on voltage through the voltage dependence of the average barrier height. Therefore,

$$G(V)=F(eV)\int_0^\infty d\epsilon_k \sum_{\alpha} \mathrm{Im} G_{k\alpha}{}^{b}(-eV), \quad (15)$$

where $F(eV)$ describes the background conductance characteristic of the barrier. If the self-energy is purely frequency-dependent the structure does not appear in the conductance.[13] If the metal-barrier boundaries are sharp then the preexponential factors do not cancel, Eq. (14) then depends on $\epsilon_{\mathbf{k}}$, and structure in conductance proportional to the self-energy appears.[14] The calculation of the conductance is thus reduced to calculating the tunneling density of states of the metal. To this end we turn to a discussion of the electron self-energy.

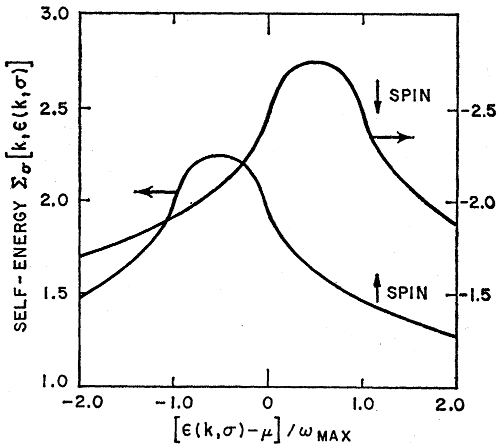

FIG. 2. The self-energy $\Sigma_\alpha(k,\epsilon_{\mathbf{k}\alpha})$ versus $\epsilon_{\mathbf{k}\alpha}$.

The self-energy[2] is spin-dependent and, if the localized f spins are assumed up (+), is given by

$$\Sigma_+(\mathbf{k},\omega)=\frac{2SJ^2}{N}\sum_q \frac{f_{\mathbf{k}+\mathbf{q}-}}{[\omega-\epsilon_{\mathbf{k}+\mathbf{q}-}+\omega_s(\mathbf{q})]}, \quad (16a)$$

$$\Sigma_-(\mathbf{k},\omega)=\frac{2SJ^2}{N}\sum_q \frac{(1-f_{\mathbf{k}-\mathbf{q}+})}{[\omega-\epsilon_{\mathbf{k}-\mathbf{q}+}-\omega_s(\mathbf{q})]}, \quad (16b)$$

where $\omega_s(q)$ is the energy of a magnon with wave vector $\mathbf{q}$ and $\epsilon_{\mathbf{k}\alpha}=\epsilon_{\mathbf{k}}{}^b-\alpha SJ$ is the spin-dependent electron energy. The exchange coupling is taken to be ferromagnetic. Assuming a quadratic spin wave spectrum $\omega_s(q)=q^2/2m_s$, the integrals in Eq. (16) are found to be accurately represented by the expression

$$\Sigma_\alpha(k,\omega)=\frac{SJ^2N(0)m_s}{kk_F}\times\biggl\{[\omega+\alpha\omega_s(k-k_{F-\alpha})]\ln\frac{\omega_s(k-k_{F-\alpha})+\alpha\omega}{\epsilon_F}-[\omega+\alpha\omega_s(q_m)]\ln\frac{\omega_s(q_m)+\alpha\omega}{\epsilon_F}+\alpha[\omega_s(q_m)-\omega_s(k-k_{F-\alpha})]\biggr\}. \quad (17)$$

In this expression, $N(0)$ is the density of states at the Fermi surface, $k_{F\alpha}$ is the Fermi momentum of spin α,

[11] J. W. Wilkins, Ph.D. thesis, University of Illinois, 1963 (unpublished); D. J. Scalapino, J. R. Schrieffer, and J. W. Wilkins, Phys. Rev. **148**, 263 (1966).

[12] W. L. McMillan (private communication).

[13] A number of recent papers have assumed that the bare energy enters into the tunneling matrix element. Using the resulting expression, they obtain structure in the conductance proportional to the self-energy. See L. C. Davis and C. B. Duke, Solid State Commun. **6**, 193 (1968); H. Hermann and A. Schmid, Z. Physik **211**, 313 (1968). Experimentally, structure proportional to the self-energy law has been observed in the conductance. See J. M. Rowell, W. L. McMillan, and W. L. Feldmann, Phys. Rev. **178**, 897 (1969); E. L. Wolf, Phys. Rev. Letters **20**, 204 (1968). We believe, however, that this structure does not have its origins in the exponential factor in the tunneling matrix element. On the contrary, the experimental observations may have their origins in the weak momentum dependence of the self-energy or in the momentum dependence of the prefactors of the tunneling matrix elements as discussed in the text.

[14] W. A. Harrison, Phys. Rev. **123**, 85 (1961).

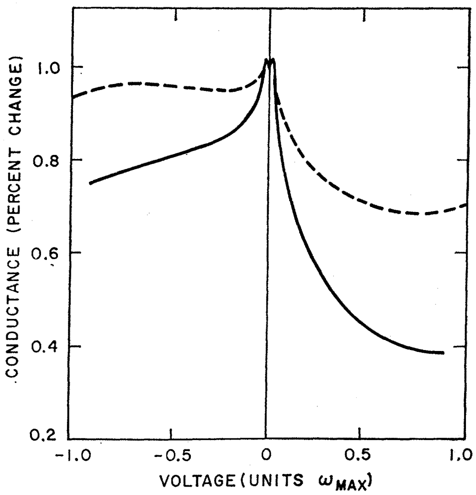

FIG. 3. Renormalization effect on the conductance. The dashed line is obtained using the pole approximation [Eq. (18)], whereas the solid line was obtained by performing the integration over the ϵ_k in Eq. (15).

and ϵ_F is the Fermi energy. Except where important the $k_{F\alpha}$ is replaced by its average value. Although in actual calculations we used the complete expression obtained by carrying out the integrations in Eq. (16) exactly, the corrections were small. The full functions $\Sigma_\alpha(k,\epsilon_{k\alpha})$ are plotted versus energy in Fig. 2.

The important feature is the logarithmic singularity at $\omega=-\alpha\omega_s(k-k_{F-\alpha})$. In order to see the effect of this singularity on the conductance, we suppose that the $\mathrm{Im}\Sigma$ can be ignored for the moment so that

$$G(eV)=4\pi e^2A(k_F)\pi m_b \times\sum_\alpha\left(1+\frac{m_b}{k}\frac{\partial\,\mathrm{Re}\Sigma_\alpha(k,-eV)}{\partial k}\bigg|_{E_k=-eV}\right)^{-1}. \quad (18)$$

The notation $E_k=-eV$ means that we solve the equation

$$+eV+\epsilon_{k\alpha}+\mathrm{Re}\Sigma_\alpha(k,-eV)=0 \quad (19)$$

for k. Differentiating Eq. (17) with respect to k and substituting into Eq. (18), we obtain the dashed curve in Fig. 3 for the conductance. The two symmetric peaks near zero bias reflect the logarithmic singularity at $\omega=\pm\omega_s(k_{F+}-k_{F-})\equiv\pm\omega_m$, while the broad asymmetric background comes from the k in the denominator in Eq. (17). In plotting G in Fig. 3 we have used parameters appropriate for Gd; i.e., $J=0.077$ eV, $m_b=3m_e$, and $m_s=300m_e$, where m_e is the bare electron mass.[15] These values are consistent with a conduction-band magnetization of 0.5 μ_B.

In Eq. (18) we have ignored the imaginary part of the self-energy. For $|eV|<\omega_m$ this is an excellent approximation because electrons with momentum near $k_{F\pm}$ cannot emit a real magnon if their energy is less than ω_m. However, since an electron can tunnel with arbitrary momentum, the incoherent part of the Green's function also contributes in this energy range. This part describes a process in which an electron tunnels into metal B with spin down and subsequently decays into a spin-up state, emitting a magnon. For $|eV|>\omega_m$, the coherent-pole contribution and the incoherent part of the Green's function become mixed and there is no clear separation. We have calculated the total integral in Eq. (15) numerically, and the results are plotted as the solid curve in Fig. 3. It appears that the additional contribution from the incoherent part of G is quite small, and the dominant effect is the smearing out of the pole of G for voltages greater than ω_m. This tends to sharpen the peak at small voltage.

[15] J. O. Dimmock and A. J. Freeman, Phys. Rev. Letters 13, 25 (1964).

3. MAGNON-ASSISTED TUNNELING

We now consider magnon-assisted tunneling that is described by the last term in Eq. (4). We assume that a voltage V is applied so that the potential of the electrons in metal B are raised by an amount $|eV|=eV$. The electron current will then flow from B to A. Since we again assume the magnetization to be up, only electrons of spin down can undergo a spin flip and tunnel into spin-up states on the A side.

As a first approximation we assume that we can ignore the energy and momentum dependence of the tunneling matrix element $|T_J|^2$ in calculating the current except insofar as it assures us that we will not see, in the absence of any particularly violent behavior for the electron density of states on sides A and B, density-of-states effects. (Magnon-renormalization effects presumably occur for magnon-assisted tunneling as well as for the regular tunneling processes calculated previously; however, here they are a 1% effect on top of a 1% effect.) Furthermore, since the tunneling matrix element T_J decreases exponentially with the angle the tunneling electron makes with the normal to the barrier, we make the assumption that the magnon emitted carries off all the transverse momentum of the tunneling electron. We assume no restriction on the normal momentum of the magnon. We define the current as positive when it flows from B to A. The temperature is assumed to be zero. Treating T_J in second-order perturbation theory we obtain for the current

$$I^e\propto\sum_{l_z}\sum_k\sum_{k'}|T_J(\epsilon_{k'+}{}^a)|^2 f(\epsilon_{k-}{}^b)[1-f(\epsilon_{k'+}{}^a)] \times\delta(\epsilon_{k-}{}^b+|eV|-\omega_{k\perp}-\omega_{l_z}-\epsilon_{k'+}{}^a). \quad (20)$$

If the barrier is assumed symmetric we can take

$$T_J(\epsilon)\simeq a\exp[-\alpha(v_B-\epsilon+|eV|/2)^{1/2}], \quad (21)$$

where a is a constant, v_B is the barrier height, and $\alpha=2(2m)^{1/2}d/h$. The oxide thickness is given by d. Furthermore, we have replaced $U(V)$ by $v_B+\frac{1}{2}|eV|$. This corresponds to replacing the actual barrier by one with a rectangular barrier of the same average height. Note that the emission process occurs on the B side, so that the electron tunnels at the energy $\epsilon_k{}^a$. If we expand this for small ϵ and eV, we obtain

$$T_J(\epsilon)\simeq T_J(1+\beta(\epsilon-\tfrac{1}{2}|eV|)), \tag{22}$$

where

$$\beta=(d/\hbar)(2m/v_B)^{1/2}. \tag{23}$$

The leading term in (22) gives rise to a conductance which is even in V, while the first-order term leads to a conductance that is odd. Thus β serves as a measure of the ratio of the odd part of the conductance to that of the even part. These two contributions to the current will be considered separately.

We consider the even part first. The $\mathbf{k}'$ integration can be carried out immediately to eliminate the delta function. Then using $\omega_{l_z}=l_z^2/2m_s$ the integral over l_z can also be performed, giving

$$I\propto 2A\sum_{\mathbf{k}} f(\epsilon_{\mathbf{k}}{}^b)[\theta(\epsilon_{\mathbf{k}}{}^b+|eV|-\omega_{\max})(\omega_{\max}-\omega_{\mathbf{k}-})^{1/2}$$
$$+\theta(\epsilon_{\mathbf{k}-}{}^b+|eV|-\omega_{k\perp})\theta(-\epsilon_{\mathbf{k}-}{}^a-|eV|+\omega_{\max})$$
$$\times(\epsilon_{\mathbf{k}-}{}^b+|eV|-\omega_{k\perp})^{1/2}]. \tag{24}$$

Here A is simply a constant factor. The integration over the direction of $\mathbf{k}$ can also be carried through, giving

$$I\propto 2A\left(\frac{V}{(2\pi)^2}\right)\int_0^\infty k^2dk f(\epsilon_{k-}{}^b)[\theta(\epsilon_{k-}{}^b+|eV|-\omega_{\max})$$
$$\times\mathcal{J}_+(\omega_{\max})\theta(\epsilon_{k-}{}^a+|eV|-\omega_k)\theta(-\epsilon_{-k}{}^b-|eV|+\omega_{\max})$$
$$\times\mathcal{J}_+(\epsilon_{k-}{}^b+|eV|)+\theta(\epsilon_{k-}{}^b+eV)\theta(-\epsilon_{k-}{}^b-|eV|+\omega_k)$$
$$\times\theta(-\epsilon_{k-}{}^b-|eV|+\omega_{\max})\mathcal{J}_-(\epsilon_{k-}{}^b+|eV|)], \tag{25}$$

where

$$\mathcal{J}_\pm(\omega)=\omega^{1/2}\pm\frac{\omega-\omega_k}{\omega_k{}^{1/2}}\ln\frac{\omega_k{}^{1/2}+\omega^{1/2}}{[\pm(\omega-\omega_k)]^{1/2}}. \tag{26}$$

At this stage, a simplifying assumption can be made. For small voltages of the order of a few $\omega_{\max}$, the values of k that come in are those close to the Fermi surface. Therefore, we replace $\omega_k\sim\omega_{kF}\sim\omega_{\max}$ everywhere it occurs, since the magnon energy varies slowly with k in comparison to $\epsilon_k{}^b$. We then see that the second term is negligible and

$$I\propto B\int_{-D}^{0} d\epsilon[\theta(\epsilon+|eV|-\omega_{\max})\mathcal{J}_+(\omega_{\max})$$
$$+\theta(\epsilon+|eV|)\theta(-\epsilon-|eV|+\omega_{\max})\mathcal{J}_-(\epsilon+|eV|)]. \tag{27}$$

Here D is the bottom of the conduction band measured relative to the Fermi surface. The conductance is easily obtained by differentiating:

$$G(|V|)\propto B(\omega_{\max})^{1/2}\left[\left(\frac{eV}{\omega_{\max}}\right)^{1/2}+\frac{|eV|-\omega_{\max}}{\omega_{\max}}\times\ln\frac{\omega_{\max}{}^{1/2}+(|eV|)^{1/2}}{(\omega_{\max}-|eV|)^{1/2}}\right]\quad(|eV|<\omega_{\max})$$
$$\propto B(\omega_{\max})^{1/2}\quad(|eV|>\omega_{\max}). \tag{28}$$

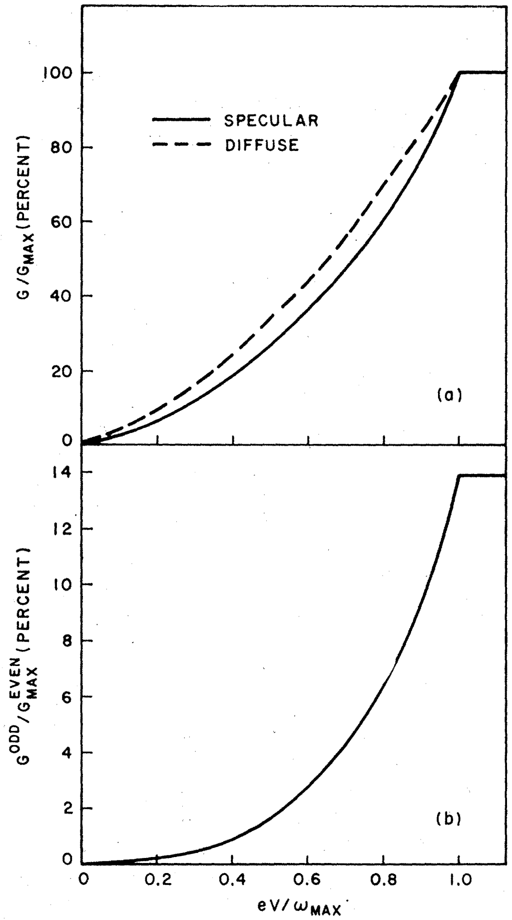

FIG. 4. Magnon-assisted tunneling. (a) Even part of the conductance. The dashed line gives the conductance for diffuse boundary conditions. The solid line was calculated assuming that the magnon must carry off the transverse momentum of the electron. (b) Ratio of the odd to even part of the conductance, assuming specular boundary conditions.

This function is plotted in Fig. 4. It is interesting to compare it with the result obtained without the specular condition that the transverse momentum be taken up by the spin wave. One then finds

$$G_e\propto\int_0^{|eV|}\rho_s(\omega)d\omega\propto\left(\frac{eV}{\omega_{\max}}\right)^{3/2}\quad|eV|<\omega_{\max}. \tag{29}$$

This is given as the dashed line in Fig. 4. The net effect of the specular condition is to shift the weight to the short-wavelength magnons by a small amount. In actual experiments, the boundary condition will be something between these two extremes, but in any case the result will strongly reflect the integrated density of states for the magnons.

The odd part of the conductance can be calculated by following the same steps and making the same approximation as for the even part. The integrals are complicated by the factor $\epsilon-\frac{1}{2}|eV|$ but can be carried out to the same extent as before. The resulting expression for the current is

$$G_0(V)=B(\omega_{\max})^{1/2}\beta\left\{\frac{1}{3}\left(\frac{|eV|}{\omega_{\max}}\right)^{3/2}-\frac{1}{2}\left(\frac{|eV|}{\omega_{\max}}\right)^{1/2}+\left(\frac{|eV|}{\omega_{\max}}-1\right)\left(\frac{|eV|}{\omega_{\max}}-\frac{1}{2}\right)\ln\frac{1+(|eV|/\omega_{\max})^{1/2}}{[1-(|eV|/\omega_{\max})]^{1/2}}-\frac{1}{2}\int_0^{|eV|/\omega_{\max}}\left[x^{1/2}+(x-1)\ln\frac{1+x^{1/2}}{(1-x)^{1/2}}\right]dx\right\}\quad(|eV|<\omega_{\max})$$

$$=\tfrac{1}{3}B(\omega_{\max})^{3/2}\beta\quad(|eV|>\omega_{\max}).\tag{30}$$

Comparing $G_0(V)$ and $G_e(V)$ for $|eV|>\omega_{\max}$, one finds the relative magnitude of these terms is of the order of 15% for a 20 Å barrier 1-V high and assuming $\omega_{\max}\sim300$°K. Using these parameters Eq. (30) is plotted in Fig. 4 as the dashed line.

4. CONCLUSIONS

We would now like to discuss the relevance of our calculations to experiment, considering in turn the renormalization effects in elastic tunneling and magnon-assisted inelastic tunneling.

The main feature of the renormalization effect is the logarithmic singularities at $\pm\omega_m$. These peaks should be relatively insensitive to the details of the theory, since they result from a sharp threshold for the decay of an electron into an electron-magnon pair. The peaks may be broadened by the variation of the exchange splitting over the Fermi surface as well as by various lifetime effects, i.e., thermal and impurity broadening. For Ni where ω_m may be of the order of 2–5 meV these effects should not prevent the experimental resolution of the two peaks.[16] A two-peak structure has been observed by Rowell for Ni tunnel junctions, but it appears sensitive to application of a magnetic field at the substrate during the growth of the Ni film. As a result, it is too early to identify this two-peak structure with that calculated in this paper. For Gd, where $\omega_m\simeq0.1$ meV, the broadening effects are probably sufficient to broaden the two peaks into a single zero-bias conductance peak. The other noticeable feature in Fig. 2, the broad asymmetric background, depends on the details of the band structure and the density of states. It is therefore characteristic of the model that we have chosen.

In our discussion of magnon-assisted tunneling, the main approximation is that the spectral distribution of the spins in the barrier is given by the bulk magnon spectrum. In doing this, we have ignored the contribution of localized or surface magnons[17] to the spectral distribution. The contribution of the localized modes to the spectral function would lead to the usual steps in the conductance at their characteristic energy. The surface modes give structure at their critical points which may show up as steps in the derivative of the conductance.

ACKNOWLEDGMENTS

The authors would like to express their gratitude to J. M. Rowell and W. L. McMillan for several useful discussions.

[16] Although the spin-flip model is certainly not appropriate as a description of Ni, the electron-magnon interaction will have essentially the same structure as described here.

[17] D. L. Mills and A. A. Maradudin, J. Phys. Chem. Solids 28, 1855 (1967).