menter's parameter ξ, is

$$J_m = (8e\xi^2/haV)\Delta_0^2, \tag{19}$$

so $J_{\rm crit}/J_m \sim a/\xi$, which is much smaller than 1.

The observed critical currents are actually less than $J_{\rm crit}$. Values of J_c have been determined experimentally[17] for indium samples with a pore size of about 80 Å; the result extrapolated to $T=0$ is $(8\pm2)\times10^4$ A/cm^2. This discrepancy is not surprising. The dimensions of the samples were much larger than λ or ξ, and the currents and fields were not uniform. The theory of vortices in type-II superconductors depends mainly on the London equations.[7] The nonlinear term in the Ginzburg-Landau equation affects only the structure of the core. Parmenter's theory, therefore, predicts that vortices exist in granular superconductors. If $\kappa=\lambda/\xi\gg1$, the differences between Parmenter's equation and the Ginzburg-Landau equations will cause only a small change in the line energy of a vortex and, perhaps, a change in the vortex lattice for fields near H_{c2} where the cores overlap. The condition $\kappa\gg1$ is very well satisfied in our materials; if it is calculated as $H_{c2}/\sqrt{2}H_c$, we find $\kappa=5490/d$ for indium and $2800/d$ for lead, where d is in Å. If vortices exist, we expect the critical current to be determined by the pinning of these vortices. In granular superconductors, regions where the transmission coefficient τ is smaller than average may occur; such regions might make effective pins because κ is inversely proportional to τ and a larger κ would decrease the line energy of a vortex.

[17] J. H. P. Watson, J. Appl. Phys. 39, 3406 (1968).

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Flux Penetration in an Anisotropic Type-II Superconductor*

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Calculations are presented for the magnetic induction B and the free energy of an anisotropic type-II superconductor near its upper critical field H_{c2}. It is shown that this field is not necessarily parallel to the externally applied magnetic field $\mathbf{H}$, its direction depending on the direction of $\mathbf{H}$ relative to the principal axes of the Ginsburg effective-mass tensor. These results suggest that torque measurements, made on geometrically symmetrical samples, should be useful in determining the upper critical field H_{c2} as well as the components of the effective-mass tensor.

I. INTRODUCTION

According to Ginzburg,[1] the free energy per unit volume F_s of an anisotropic superconductor near to its critical temperature T_c should take the form

$$F_s=F_n+\alpha|\psi|^2+\tfrac{1}{2}\beta|\psi|^4+\sum_{ij}\frac{1}{2m_{ij}}(\phi_i\psi)(\phi_j{}^*\psi^*)+\frac{h^2}{8\pi}, \tag{1}$$

where ψ is the order parameter for the superconducting state, ϕ_i is the ith component of the vector operator $-i\hbar\nabla-(e^*/c)\mathbf{A}$, $\mathbf{h}$ is the microscopic magnetic field, e^* is the effective charge on the superconducting electrons, and α and β are the usual temperature-dependent Ginsburg constants.

Equation (1) differs from the usual Ginzburg-Landau (GL) equation for the free energy in having a mass tensor m_{ij} in place of an isotropic mass. Gor'kov and Melik-Barkhudarov[2] have derived the GL equations for an anisotropic superconductor from the microscopic theory of superconductivity.[3–5] They find that a free-energy expression of the form of Eq. (1) is valid in the region of temperature near T_c in which the penetration depth is much larger than the coherence length. The effective charge e^* is twice the electronic charge e. The mass tensor they obtain is given by

$$\frac{1}{m_{ij}}=\frac{3}{2\epsilon_F}\int v_iv_j\phi(\mathbf{P})N\,ds\bigg/\int N\,ds, \tag{2}$$

where ϵ_F is the Fermi energy, v_i is the ith component of the Fermi velocity, and N is the density of states (excluding spin degeneracy) per unit surface area per unit energy range. The integral is over the Fermi surface. The function $\phi(\mathbf{P})$ describes the anisotropy of the energy gap.

The condition for the free energy to be minimum with respect to variations of the order parameter and magnetic field distribution yields the two GL equations.

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[1] V. L. Ginzburg, Zh. Eksperim. i Teor. Fiz. 23, 236 (1952).

[2] L. P. Gor'kov and T. K. Melik-Barkhudarov, Zh. Eksperim. i Teor. Fiz. 45, 1493 (1963) [English transl.: Soviet Phys.—JETP 18, 1031 (1964)].

[3] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957).

[4] L. P. Gor'kov, Zh. Eksperim. i Teor. Fiz. 36, 1918 (1959) [English transl.: Soviet Phys.—JETP 9, 1364 (1959)].

[5] L. P. Gor'kov, Zh. Eksperim. i Teor. Fiz. 34, 735 (1958) [English transl.: Soviet Phys.—JETP 7, 505 (1958)].

These will henceforth be referred to as GL-I and GL-II.

$$\text{GL-I:}\quad \alpha\psi+\beta|\psi|^2\psi+\sum_{ij}\tfrac{1}{2}\mu_{ij}\phi_i\phi_j\psi=0\,,$$

$$\text{GL-II:}\quad J_j=\sum_i ihe\mu_{ij}\left(\psi\frac{\alpha\psi^*}{\alpha x_i}-\psi^*\frac{\alpha\psi}{\alpha x_i}\right)-\frac{4e^2}{c}\mu_{ij}|\psi|^2A_j\,,$$

where $\mu_{ij}=1/m_{ij}$ and A_i is the ith component of the vector potential.

In this paper, we will discuss the solutions to the GL equations in the high-field limit. Tilley[6] has solved the linear anisotropic GL-I equation for the two upper critical fields H_{c2} and H_{c3}, and obtained an expression for the GL κ parameter as a function of crystal orientation. In discussing the mixed state, he makes the assumption that the field due to the supercurrent is parallel to the externally applied field. If this assumption is made, the complete GL-I equation and the expression for the free energy can be reduced to the same form as their corresponding isotropic equation. Since most of the analysis carried out by Abrikosov[7] is independent of the exact form of ψ, and uses only the fact that ψ is approximately a solution of the linear GL-I equation with lowest eigenvalue $|\alpha|$, Tilley concludes that Abrikosov's results for H_{c1}, the free energy, and the slope of the magnetization near H_{c2} may be taken over by replacing the isotropic κ parameter by its anisotropic analog. Section II contains a brief review of Tilley's solution of the linear GL-I equation with a uniform applied field. In Sec. III, the complete GL-I equation is used to obtain expressions for the magnetization and the free energy in the mixed state near H_{c2}. We show that the field produced by the supercurrent is not, in general, parallel to the external field, the exceptions being when the external field is parallel to one of the principal axes of the effective-mass tensor. Finally, we discuss the possible application of torque measurements to the study of anisotropic superconductors.

II. REVIEW OF TILLEY'S SOLUTION OF THE LINEAR GL-I EQUATION

Let the 3 direction be parallel to the uniformly applied magnetic field H_0. One is then free to choose the 1 and 2 axes in any direction in the plane perpendicular to H_0. Geometrically, the effective-mass tensor, being symmetrical, defines an ellipsoid, and the three components μ_{11}, μ_{22}, and μ_{12} define an ellipse at right angles to the magnetic field. Tilley points out that in calculating H_{c2} it is more convenient to choose the 1 and 2 axes as the major and minor axes of the ellipse, in which case μ_{12} is zero. He then chooses as the wave function

$$\psi(r)=g(x_1,k_2,k_3)e^{ik_2x_2}e^{jk_3x_3}\,. \tag{3}$$

[6] D. R. Tilley, Proc. Phys. Soc. (London) **85**, 1177 (1965).

[7] A. A. Abrikosov, Zh. Eksperim. i Teor. Fiz. **32**, 1442 (1957) [English transl.: Soviet Phys.—JETP **5**, 1174 (1957)].

Substituting this wave function into the linear GL-I equation, it is easy to show that the lowest value of the eigenvalue $|\alpha|$, and therefore the highest value of H_{c2}, is found when $k_3=0$, as in the isotropic case. Using the vector potential $\mathbf{A}^0=(0,H_0x_1,0)$, the linear GL-I equation becomes

$$-\frac{\hbar^2}{2}\mu_{11}\frac{\alpha^2 g}{\alpha x_1^2}+\mu_{22}\frac{2e^2}{c^2}H_0^2\left(x_1-\frac{\hbar ck_2}{2eH_0}\right)g=|\alpha|g\,. \tag{4}$$

Equation (4) is analogous to the Schrödinger equation for a harmonic oscillator whose lowest eigenvalue is given by

$$|\alpha|=e\hbar(\mu_{11}\mu_{22})^{1/2}H_0/c\,. \tag{5}$$

This is the highest value of H_0 for which a bound state exists, and thus defines H_{c2}. Writing $|\alpha|$ in terms of the thermodynamic critical field H_c, and defining the GL κ parameter as $H_{c2}/\sqrt{2}H_c$, yields

$$\kappa=[c/2e\hbar(\mu_{11}\mu_{22})^{1/2}](\beta/2\pi)^{1/2}\,. \tag{6}$$

The eigenfunction can be written down immediately, and is given by

$$\psi_{k_2}=e^{ik_2x_2}e^{[-V(x_1-x_0)^2/2]}\,, \tag{7}$$

where

$$V=(2eH_{c2}/\hbar c)(\mu_{22}/\mu_{11})^{1/2}$$

and

$$x_0=\hbar ck_2/2eH_{c2}\,.$$

Here k_2 is an arbitrary parameter. Tilley goes on to show that the surface nucleation field H_{c3} is related to H_{c2} just as it is in the isotropic case (i.e., $H_{c3}=1.69H_{c2}$), and therefore depends on the direction of the magnetic field in the same way as H_{c2}.

III. SOLUTION OF THE GL EQUATIONS IN THE MIXED STATE NEAR H_{c2}

As pointed out by Abrikosov, if H is taken to be only slightly less than H_{c2}, the solution to the complete GL equation must have a strong resemblance to the solutions ψ_{k_2} of the linear equation. Following Abrikosov, we choose as our trial wave functions ψ a linear combination of the ψ_{k_2}'s. Since $|\psi|$ must be periodic in x_1 and x_2, let $2\pi/q$ be the spatial period in the x_2 direction. From Eq. (7), ψ can be written as

$$\psi=\sum_n c_n e^{inqx_2}e^{[-V(x-x_n)^2/2]}\,, \tag{8}$$

where

$$x_n=n\hbar cq/2eH_{c2}\,.$$

In order that $|\psi|$ also be periodic in x_1, it is necessary to impose a periodicity on the c_n's. The form of condition chosen by Abrikosov was $c_{n+\gamma}=c_n$, where γ is a fixed integer.

This trial wave function must satisfy the complete GL-I equation in the presence of the actual vector potential $\mathbf{A}$. Set $\mathbf{A}=\mathbf{A}^0+\mathbf{A}'$, where these potentials have the following meaning:

$$\mathrm{curl}\mathbf{A}^0=\mathbf{H}_{c2},$$
$$\mathrm{curl}\mathbf{A}=\mathbf{H}+\mathbf{h}^s,$$

where $\mathbf{H}$ is the applied magnetic field and $\mathbf{h}^s$ is the field produced by the supercurrents;

$$\mathrm{curl}\mathbf{A}'=\mathbf{h}',$$

where $\mathbf{h}'$ is equal to the modifications due to the fact that (a) the applied field is slightly smaller than H_{c2}, and (b) there exist supercurrents which also contribute to the field. Thus, writing curl$\mathbf{A}$ out in terms of these fields yields

$$\mathbf{H}+\mathbf{h}^s=\mathbf{H}_{c2}+\mathbf{h}'. \tag{9}$$

In the isotropic material all of these fields are assumed to be parallel. However, this assumption cannot be made in the anisotropic material since, as will be demonstrated shortly, the field produced by the supercurrents is not necessarily parallel to the external field. Writing out the complete GL-I equations for ψ and ψ^* in the coordinate system where the x_1 and x_2 axes are parallel to the major and minor axes of the effective-mass ellipse, keeping terms to first order in $\mathbf{A}'$, and using Eq. (4) and the fact that ψ is not a function of x_3, yields

$$\frac{\mu_{11}}{c}\left[2ei\hbar A_1'\frac{\alpha\psi}{\alpha x_1}+ei\hbar\left(\frac{\alpha A_1'}{\alpha x_1}\right)\psi\right]+\frac{\mu_{33}}{c}\left[ei\hbar\left(\frac{\alpha A_3'}{\alpha x_3}\right)\right]\psi+\frac{\mu_{22}}{c}\left[2ei\hbar A_2'\frac{\alpha\psi}{\alpha x_2}+ei\hbar\left(\frac{\alpha A_2'}{\alpha x_2}\right)\psi+\frac{4e^2}{c}A_2{}^0A_2'\psi\right]$$
$$+\frac{\mu_{23}}{c}\left[2ei\hbar A_3'\frac{\alpha\psi}{\alpha x_2}+ei\hbar\left(\frac{\alpha A_3'}{\alpha x_3}+\frac{\alpha A_3'}{\alpha x_2}\right)\psi+\frac{4e^2}{c}A_2{}^0A_3'\psi\right]+\frac{\mu_{13}}{c}\left[2ei\hbar A_3'\frac{\alpha\psi}{\alpha x_1}+ei\hbar\left(\frac{\alpha A_1'}{\alpha x_3}+\frac{\alpha A_3'}{\alpha x_1}\right)\psi+\beta|\psi|^2\psi\right]=0, \tag{10}$$

$$\frac{\mu_{11}}{c}\left[-2ei\hbar A_1'\frac{\alpha\psi^*}{\alpha x_1}-ei\hbar\left(\frac{\alpha A_1'}{\alpha x_1}\right)\psi^*\right]+\mu_{33}\left(-ei\hbar\frac{\alpha A_3'}{\alpha x_3}\psi^*\right)+\frac{\mu_{22}}{c}\left[-2ei\hbar A_2'\frac{\alpha\psi^*}{\alpha x_2}-3i\hbar\left(\frac{\alpha A_2'}{\alpha x_2}\right)\psi^*+\frac{4e^2}{c}A_2{}^0A_2'\psi^*\right]$$
$$+\frac{\mu_{33}}{c}\left[-2ei\hbar A_3'\frac{\alpha\psi^*}{\alpha x_2}-ei\hbar\left(\frac{\alpha A_2'}{\alpha x_3}+\frac{\alpha A_3'}{\alpha x_2}\right)\psi^*+\frac{4e^2}{c}A_2{}^0A_3'\psi^*\right]+\frac{\mu_{13}}{c}\left[-2ei\hbar A_3'\frac{\alpha\psi^*}{\alpha x_1}-ei\hbar\left(\frac{\alpha A_1'}{\alpha x_3}+\frac{\alpha A_3'}{\alpha x_1}\right)\psi^*\right]$$
$$+\beta|\psi|^2\psi^*=0. \tag{11}$$

Multiplying (10) by ψ^*, (11) by ψ, integrating over the volume of the sample, and adding (10) and (11) yields

$$\beta\langle|\psi|^4\rangle_{\mathrm{av}}-(1/c)\langle\mathbf{A}'\cdot\mathbf{J}^s\rangle_{\mathrm{av}}=0, \tag{12}$$

where

$$J^s{}_1=ei\hbar\mu_{11}\left(\psi\frac{\alpha\psi^*}{\alpha x_1}-\psi^*\frac{\alpha\psi}{\alpha x_1}\right), \tag{13a}$$

$$J^s{}_2=ei\hbar\mu_{22}\left(\psi\frac{\alpha\psi^*}{\alpha x_2}-\psi^*\frac{\alpha\psi}{\alpha x_2}\right)-\frac{4e^2}{c}\mu_{22}A_2{}^0|\psi|^2, \tag{13b}$$

and

$$J^s{}_3=\frac{\mu_{23}}{\mu_{22}}J^s{}_2+\frac{\mu_{13}}{\mu_{11}}J^s{}_1. \tag{13c}$$

In the above notation, the integral over the volume $\int|\psi|^2d\mathbf{r}$ is denoted by $\langle|\psi|^2\rangle_{\mathrm{av}}V$. It is easy to verify that Eq. (13) is just the GL-II equation for the supercurrents associated with the unperturbed solution. Integrating the second term in Eq. (12) by parts and setting curl$\mathbf{A}'=\mathbf{h}'$ and curl$\mathbf{h}^s=(4\pi/c)\mathbf{J}^s$ yields

$$\beta\langle|\psi|^4\rangle_{\mathrm{av}}-(1/4\pi)\langle\mathbf{h}'\cdot\mathbf{h}^s\rangle_{\mathrm{av}}=0. \tag{14}$$

It should be mentioned here that Eq. (13) is not exactly the supercurrent since it is $\mathbf{A}^0$, and not $\mathbf{A}$, which enters into Eq. (13). However, as pointed out by de Gennes,[8] both $H_{c2}-H$ and $|\psi|^2$ are of the same order and small, and therefore $A'\langle|\psi|^2\rangle_{\mathrm{av}}$ is of order $|\psi|^4$ and negligible at this stage. Equation (13) can be solved for $\mathbf{h}^s$ with the result

$$\begin{aligned}h^s{}_1&=-(\mu_{13}/\mu_{11})h^s{}_3,\\ h^s{}_2&=-(\mu_{23}/\mu_{22})h^s{}_3,\\ h^s{}_3&=-(4\pi e\hbar/c)(\mu_{11}\mu_{22})^{1/2}|\psi|^2.\end{aligned} \tag{15}$$

This result can be confirmed by substituting it back into Eq. (13). It should be noted from Eq. (15) that $\mathbf{h}^s$, the field due to the supercurrents, will only be parallel to the external field when the external field is along one of the principal axes of the effective-mass tensor (i.e., when the off-diagonal terms of μ_{ij} are zero). It is useful to write $|\psi|=|\psi_0|f$, where $|\psi_0|$ is the equilibrium value of the order parameter in zero magnetic field. Assuming no spatial variation and zero magnetic field, Eq. (1) and GL-I may be used to obtain the following expression for $|\psi_0|^2$ in terms of H_c and β:

$$|\psi_0|^2=H_c/(4\pi\beta)^{1/2}.$$

Then, using the above expression for $|\psi_0|^2$ along with Eq. (6) for κ, Eq. (15) may be written as

$$h^s{}_3=-(H_c/\sqrt{2}\kappa)f^2. \tag{16}$$

[8] P. G. de Gennes, *Superconductivity of Metals and Alloys* (W. A. Benjamin, Inc., New York, 1966), p. 204.

Finally, using Eqs. (9) and (16), Eq. (14) can be written as

$$\langle f^4\rangle_{av}(1-1/2\kappa^2-\epsilon^2/2\kappa^2)-\langle f^2\rangle_{av}(1-H/H_{c2})=0, \quad (17)$$

where $\epsilon^2=(\mu_{23}/\mu_{22})^2+(\mu_{13}/\mu_{11})^2$. This is identical to the expression one would find for an isotropic material except for the term $\epsilon^2/2\kappa^2$. For a given vortex lattice, one may calculate the ratio $\beta_A=\langle f^4\rangle_{av}/\langle f^2\rangle^2_{av}$, which corresponds to this lattice. Before discussing the calculation of β_A, let us continue to obtain expressions for the magnetization and free energy. The magnetic induction $\mathbf{B}=\mathbf{H}+\langle\mathbf{h}^s\rangle_{av}$ can be calculated using Eqs. (15)–(17). The result is

$$B_1=\langle h^s{}_1\rangle_{av}=\frac{\mu_{13}}{\mu_{11}}\frac{H_{c2}-H}{\beta_A(2\kappa^2-1-\epsilon^2)},$$

$$B_2=\langle h^s{}_2\rangle_{av}=\frac{\mu_{23}}{\mu_{22}}\frac{H_{c2}-H}{\beta_A(2\kappa^2-1-\epsilon^2)}, \quad (18)$$

and

$$B_3=H+\langle h^s{}_3\rangle_{av}=H-\frac{H_{c2}-H}{\beta_A(2\kappa^2-1-\epsilon^2)}.$$

The magnetization $\mathbf{M}$, given by $(\mathbf{B}-\mathbf{H})/4\pi$, becomes

$$M_3=-\frac{1}{4\pi\beta_A}\frac{H_{c2}-H}{2\kappa^2-1-\epsilon^2},$$

$$M_2=-(\mu_{23}/\mu_{22})M_3, \quad (19)$$

$$M_1=-(\mu_{13}/\mu_{11})M_3.$$

The free energy per unit volume, F, may be calculated by using Eqs. (4), (12), and (13) to simplify Eq. (1):

$$F_s(\mathbf{B},T)=F_n(0,T)+(1/8\pi)(\langle h^2\rangle_{av}-H_c{}^2\langle f^4\rangle). \quad (20)$$

The Gibbs free energy per unit volume, G, is obtained with the Legendre transformation $G=F-(1/4\pi)\mathbf{B}\cdot\mathbf{H}$ and can be written, with the help of Eqs. (17) and (18), as

$$G_s(\mathbf{H},T)=G_n(\mathbf{H},T)-\frac{1}{8\pi\beta_A}\frac{H_{c2}-H}{2\kappa^2-1-\epsilon^2}. \quad (21)$$

This is identical to the expression for the isotropic case except for the term ϵ^2.

For fixed $\mathbf{H}$, G is an increasing function of β_A if $\kappa>\frac{1}{2}\sqrt{2}(1+\epsilon^2)^{1/2}$. This suggests that the dividing point between type-I and type-II superconductors is $\kappa=\frac{1}{2}\sqrt{2}\times(1+\epsilon^2)^{1/2}$ rather than $1/\sqrt{2}$ for the isotropic case. Assuming $\kappa>\frac{1}{2}\sqrt{2}(1+\epsilon^2)^{1/2}$, the most favorable vortex lattice will correspond to the smallest β_A. Kleiner, Roth, and Autler[9] have shown that, below H_{c2} in an isotropic material, the most favorable lattice is hexagonal with $\beta_A=1.16$. Tilley[6] has extended the work of Kleiner to the case of an anisotropic material. He obtains the following expression for β_A:

$$\beta_A(\phi)=1.16+0.0295\frac{(\mu_{11}-\mu_{22})^2}{\mu_{11}\mu_{22}}\sin^2 2\phi,$$

where μ_{11} and μ_{22} are the principal values of μ_{ij} perpendicular to the magnetic field and ϕ is the angle between these axes and the lattice-symmetry axes. The value of β_A, and therefore the free energy, is smallest when $\phi=0$, that is, when the lattice-symmetry axes lie along the axes with $\mu_{12}=0$. This result should also apply here since our wave function is identical to Tilley's

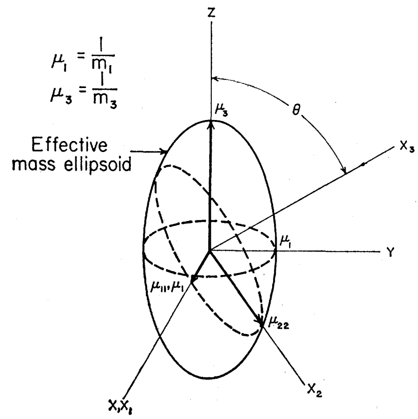

FIG. 1. Coordinate system used in calculating superconducting critical fields in anisotropic uniaxial crystals.

The above results suggest that torque measurements, made on geometrically symmetrical samples, should be useful in measuring anisotropies in H_{c2} as well as the components of the effective-mass tensor. As an example, consider a uniaxial crystal such that the mass tensor has cylindrical symmetry about its crystallographic c axis and can therefore be described by two independent components. Let m_1 and m_3 represent the components of the mass tensor perpendicular and parallel to the c axis, respectively. Let X, Y, and Z define a coordinate system fixed to the crystal, where Z is parallel the crystallographic c axis. Since the effective-mass tensor has been assumed to have uniaxial symmetry about the c axis, X and Y can be chosen as any two orthogonal axes in the basal plane. The effective-mass tensor is then diagonal in this coordinate system.

Sticking to the notation used in the previous sections, let X_1, X_2, and X_3 represent a coordinate system fixed to the externally applied magnetic field, where X_3 is parallel to the external field. As stated earlier, X_1 and X_2 are chosen to be parallel to the principle axes of the components of μ_{ij} perpendicular to the external field (see Fig. 1). Finally let θ be the angle between X_3 and Z. Then in the X_1, X_2, X_3 coordinate system, the

[9] W. H. Kleiner, L. M. Roth, and S. H. Autler, Phys. Rev. **133**, A1226 (1964).

effective-mass tensor becomes

$$\mu=\begin{bmatrix}\mu_{11} & 0 & 0\\ 0 & \mu_{22} & \mu_{23}\\ 0 & \mu_{23} & \mu_{23}\end{bmatrix}, \tag{22}$$

where

$$\mu_{11}=1/m_1,$$
$$\mu_{22}=(1/m_1)\cos^2\theta+(1/m_3)\sin\theta,$$
$$\mu_{23}=(1/m_1-1/m_3)\sin\theta\cos\theta,$$
$$\mu_{33}=(1/m_1)\sin^2\theta+(1/m_3)\cos^2\theta.$$

If the above relations for μ_{ij} are substituted into Eqs. (5) and (6), one obtains the following expressions for H_{c2} and κ as a function of θ:

$$H_{c2}(\theta)=H_{c2}(\tfrac{1}{2}\pi)(1+P\cos^2\theta)^{-1/2}$$

and (23)

$$\kappa(\theta)=\kappa(\tfrac{1}{2}\pi)(1+P\cos^2\theta)^{-1/2},$$

where $P=m_3/m_1-1$. Let the sample be in the shape of a long cylinder with its c axis perpendicular to the cylinder axis. Then, if the sample is suspended along its cylinder axis with the external field in the horizontal direction, Eq. (19) may be used to show that the torque $(\mathbf{T}=\mathbf{M}\times\mathbf{H}V)$ on the sample is in the vertical direction and is given by

$$T(H,\theta)=\frac{\epsilon(\theta)[H_{c2}(\theta)-H]HV}{4\pi\beta_A[2\kappa^2(\theta)-1-\epsilon(\theta)^2+n/\beta_A]}, \tag{24}$$

where $\epsilon(\theta)=P\sin\theta\cos\theta/(1+P\cos^2\theta)$, n is the geometric demagnetization factor $(\approx\frac{1}{2})$, and V is the volume of the sample. The cylindrical geometry has been chosen so that no torque should occur because of demagnetizing effects. This expression for torque depends only on the fact that the effective mass is anisotropic. If one assumes a small anisotropy, then, to first order in P,

$$T(H,\theta)=\frac{P\sin\theta\cos\theta\,[H_{c2}(\theta)-H]HV}{4\pi\beta_A[2\kappa^2(\theta)-1+n/\beta_A]}. \tag{25}$$

Since torsion balances are capable of resolving torques of 10^{-2}–10^{-3}dyn cm,[10] it should be possible to detect very small anisotropies. For example, Eqs. (23) and (25) can be used to show that a superconductor with $\bar{\kappa}\sim2$ and a volume of 1 cm^3 should experience a torque of $\sim10^{-2}\Delta H_{c2}(H_{c2}-H)$ dyn cm for $\theta=\frac{1}{4}\pi$ and H just below H_{c2}. Thus, an anisotropy ΔH_{c2} of 0.1 G could easily be detected at $H_{c2}-H=10$ G. For a superconductor with $H_{c2}=2$ kG, this would correspond to an anisotropy in the effective mass of $10^{-2}\%$.

A further examination of Eqs. (24) or (25) shows that both the magnitude and direction of the torque will depend upon the orientation of the externally applied field with respect to the crystal axes. For example, holding $H_{c2}(\theta)-H$ constant, it is easy to see that (a) the magnitude of the torque will have the symmetry of the crystal lattice (twofold in the above example), (b) as the direction of the applied field passes through a crystallographic symmetry axis, the magnitude of the torque passes through zero while its direction is reversed, and (c) the torque is always directed so as to align the direction of highest H_{c2} parallel to the applied field. This third observation is consistent with Eq. (21) for the Gibbs free energy since G will be a minimum in this case.

This effect could be masked by the geometrical demagnetization effects of a slightly misshaped sample. However, one might be able to cancel out this effect since torque due to demagnetizing effects depends on the geometry of the sample, where as torque due to anisotropies in the effective mass depends upon the orientation of the external field with respect to the crystallographic axes.

Several people[11–14] have used torque measurements to study the superconducting properties of thin films and foils. In these experiments the specimen, which was in the shape of a flat sheet, was suspended along one of its edges. The applied field was horizontal and very nearly in the plane of the specimen. Thus the observed torque was due entirely to demagnetizing effects. The results of these measurements showed, in some cases, large hysteresis effects as well as torques which exist out beyond the upper bulk critical field H_{c2}. These results have been explained as being due to surface states as well as flux trapping.

The experiments proposed here are in marked cnotrast to those described above in that we propose using large bulk samples which are geometrically symmetrical so as to cancel out any effects due to demagnetization. A long cylinder used in the manner described earlier or a sphere would probably be the best sample geometries. These geometries would also cancel out effects due to surface states. Finally, the sample must also be a well annealed single crystal so as to eliminate the problem of trapped flux.

One advantage of torque measurements over magnetization measurements is the fact that the existence of torque at all in a geometrically symmetrical sample indicates that an anisotropy exists, whereas magnetization measurements would depend on an accurate measurement of H_{c2} at various orientations. One could also make torque measurements at various orientations simply by rotating the magnet about the suspended crystal. Thus the crystal need not be removed from the Dewar between measurements.

Thus it might be useful to make torque measurements on geometrically symmetrical samples of several of the known type-II superconductors.

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