Contributions of Spin, Anomalous Magnetic Moment, and Form Factors to the Stopping Power of Matter for Protons and Muons at Extreme Relativistic Energies*

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The stopping power of matter for protons and muons is calculated without the usual restriction that $\gamma m \ll M$, where M is the rest mass of the proton or muon, m is that of the electron, $\gamma=(1-\beta^2)^{-1/2}$, and β is the ratio of the speed of the proton or muon to the speed of light. The calculation includes the effects of the incident proton's spin, anomalous magnetic moment, and form factors, the muon being considered as a point particle that obeys the Dirac equation. The contributions of these effects to the stopping powers of aluminum, copper, and lead are evaluated numerically for protons and muons up to $\gamma=10^4$ ($\sim 10^4$-GeV protons and $\sim 10^3$-GeV muons), where they amount to about 10%. Restricted energy-loss values with cutoff energies of 1.1 and 8.0 MeV are calculated for protons and muons in water.

I. INTRODUCTION

In Bethe's relativistic theory[1] of the stopping power of matter for a heavy charged particle (e.g., muon, pion, proton), explicit use is made of the condition

$$E \ll M^2c^2/m, \quad \text{or} \quad \gamma m \ll M, \tag{1}$$

where m and M are the rest masses of the electron and the heavy particle, c is the speed of light, and $\gamma=(1-\beta^2)^{-1/2}$, $v=\beta c$ being the speed of the particle. When applied to protons, this approximation restricts the theory to energies $\lesssim 100$ GeV and, when applied to muons, $\lesssim 10$ GeV. When condition (1) is satisfied, the maximum energy that can be transferred to an electron, considered initially free and at rest, in a collision is given by $Q_m=2\gamma^2mv^2$. In this case, $Q_m/E \equiv Q_m/\gamma Mc^2 \leq 2\gamma m/M \ll 1$, and so the incident particle can transfer at most only a small fraction of its incident energy in a single close collision with an atomic electron. The differential cross section for scattering of the incident particle by an electron is then independent of the particle's spin, anomalous magnetic moment, and its distributions of charge and magnetic moment (particle form factors). As pointed out by Fano,[2,3] these effects can be included in the theory of stopping power, and we do so here to obtain formulas applicable to protons and muons at extreme relativistic energies.

In the Bethe theory, the average rate of energy loss per unit pathlength in a medium due to distant collisions of a heavy charged particle with atomic electrons is given by[4]

$$(-dE/ds)_{Q<\eta}=\tfrac{1}{2}\kappa[\ln(2\gamma^2mv^2\eta/I^2)-\beta^2]. \tag{2}$$

Here $\kappa=4\pi z^2e^4NZ/mv^2$, where ze is the charge of the incident particle ($-e$ is the electronic charge) and NZ is the number of electrons per unit volume in the medium; I is the mean excitation energy of the medium, and η is an intermediate value of the particle's energy loss Q.[4] When condition (1) is satisfied, the average rate of energy loss due to close collisions is[4]

$$(-dE/ds)_{Q>\eta}=\tfrac{1}{2}\kappa[\ln(2\gamma^2mv^2/\eta)-\beta^2] \quad (\gamma m \ll M). \tag{3}$$

The total stopping power is the sum of (2) and (3):

$$-dE/ds=\kappa[\ln(2\gamma^2mv^2/I)-\beta^2] \quad (\gamma m \ll M). \tag{4}$$

At extreme relativistic energies ($\gamma \gg 1$), Eq. (2) remains valid; Eq. (3) and hence (4) will contain additional terms as shown below.

II. LORENTZ TRANSFORMATION OF ELECTRON-PROTON DIFFERENTIAL SCATTERING CROSS SECTION

Our point of departure is the Rosenbluth formula,[5] giving the differential cross section $d\sigma/d\Omega$ for scattering of an electron at an angle θ from a proton, initially free and at rest:

$$\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_0\left(\frac{1+\tau\mu^2}{1+\tau}+2\tau\mu^2\tan^2\tfrac{1}{2}\theta\right)G_E^2. \tag{5}$$

The quantity

$$\left(\frac{d\sigma}{d\Omega}\right)_0=\left(\frac{ze^2}{2\gamma mv^2}\right)^2\frac{\cos^2\frac{1}{2}\theta}{\sin^4\frac{1}{2}\theta}\left(1+\frac{2\gamma m}{M}\sin^2\tfrac{1}{2}\theta\right)^{-1} \tag{6}$$

is the differential cross section for electron scattering when the spin, anomalous magnetic moment, and form factors of the proton are neglected; $\tau=\hbar^2q^2/4M^2c^2$, where

$$\hbar q=\frac{2\gamma mc\sin\frac{1}{2}\theta}{1+(2\gamma m/M)\sin^2\frac{1}{2}\theta} \tag{7}$$

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[1] H. A. Bethe, in *Handbuch der Physik*, edited by H. Geiger and K. Scheel (Springer-Verlag, Berlin, 1933), Vol. 24/1, pp. 491 ff.

[2] U. Fano, Ann. Rev. Nucl. Sci. **13**, 1 (1963).

[3] U. Fano, in *Studies in Penetration of Charged Particles in Matter*, edited by U. Fano, [National Academy of Sciences–National Research Council (Publication No. 1133), Washington, D. C., 1964], Paper No. 12.

[4] E. A. Uehling, Ann. Rev. Nucl. Sci. **4**, 315 (1954).

[5] M. N. Rosenbluth, Phys. Rev. **79**, 615 (1950).

is the magnitude of the change in the electron's (=proton's) energy-momentum four-vector; μ is the proton's magnetic moment in units of nuclear magnetons, $e\hbar/2Mc$, and G_E is the electric form factor of the proton. In Eq. (5) it is assumed that the proton's magnetic form factor is given by $G_M=\mu G_E$.[6,7] Treating the muon as a point Dirac particle of mass M, we obtain the stopping power for this particle later by setting $\mu=1$ and $G_E=1$.

To calculate stopping power we first Lorentz transform Eq. (5) from the rest system S of the proton before collision to the rest system S' of the electron before collision, the latter being the "laboratory" system for proton slowing down. In Fig. 1 we represent the initial and final momenta of the electron in S by $\mathbf{p}_{01}$ and $\mathbf{p}_1$. The electron is scattered at an angle θ by the proton, which recoils with momentum $\mathbf{p}_2$ at an angle φ. Denoting the respective energies by E_{01}, E_1, and E_2, we write for energy-momentum conservation

$$\begin{aligned} E_{01}+Mc^2&=E_1+E_2\,,\\ p_{01}&=p_1\cos\theta+p_2\cos\varphi\,,\\ p_1\sin\theta&=p_2\sin\varphi\,. \end{aligned}\tag{8}$$

Denoting the corresponding quantities in S' by primed symbols and the proton's initial momentum and energy by $\mathbf{p}_{02}'$ and E_{02}', we have

$$\begin{aligned} E_{02}'+mc^2&=E_1'+E_2'\,,\\ p_{02}'&=p_2'\cos\varphi'+p_1'\cos\theta'\,,\\ p_2'\sin\varphi'&=p_1'\sin\theta'\,. \end{aligned}\tag{9}$$

The energy lost by the proton in the collision is[8]

$$Q\equiv E_{02}'-E_2'=E_1'-mc^2\,,\tag{10}$$

and it follows from (9) and (10) that the maximum energy loss is given by

$$\begin{aligned} Q_m&=\frac{Mc^2[(E_{02}'/Mc^2)^2-1]}{M/2m+E_{02}'/Mc^2+m/2M}\\ &=2\gamma^2Mmv^2/(M+2\gamma m)\,, \end{aligned}\tag{11}$$

the quantity $(m/M)^2$ being neglected compared with unity in the last equality. When condition (1) is fulfilled, we see that $Q_m=2\gamma^2mv^2$, as mentioned above. For very large values of γ, Q_m approaches γMc^2 and the incident particle can lose a large fraction of its KE in a single collision.

The transformation of quantities in the differential cross section into functions of Q is straightforward. The relationships between the components of momentum and the energy in S and S' are

$$\begin{aligned} p_x&=\gamma(p_x'+\beta E'/c)\,,\\ p_y&=p_y'\,,\\ E&=\gamma(E'+\beta c p_x')\,, \end{aligned}\tag{12}$$

x being the direction of motion in Fig. 1 with y perpendicular in the plane of the vectors $\mathbf{p}_1$ and $\mathbf{p}_2$. The scattering angle in S is given by $\cos\theta=p_{1x}/p_1$. It follows from Eqs. (12) and (9) that, in terms of Q,

$$\cos\theta=\frac{\gamma^2Mmv^2-(M+\gamma m)Q}{\gamma m\beta(Q^2-2\gamma Mc^2Q+\gamma^2M^2c^4\beta^2)^{1/2}}.\tag{13}$$

With the help of this equation, we can calculate $\sin^4\frac{1}{2}\theta$, $\cos^2\frac{1}{2}\theta$, $\tan^2\frac{1}{2}\theta$, and $d\Omega=2\pi d(\cos\theta)$ in terms of Q and dQ. From Eq. (7) we find that

$$\hbar^2q^2=2mQ\,.\tag{14}$$

Making these substitutions, using Eq. (11), and carrying through some algebraic simplifications, we obtain from Eq. (5)

$$d\sigma=\frac{2\pi z^2e^4}{mv^2}\frac{dQ}{Q}\left[\frac{1}{Q}\left(\frac{a+\mu^2mQ}{a+mQ}\right)-\frac{\beta^2}{Q_m}\left(\frac{a+\mu^2mQ}{a+mQ}\right)+\frac{\mu^2Q}{2E^2}\right]G_E^2(Q)\,,\tag{15}$$

where $a\equiv 2M^2c^2$ and $E\equiv E_{02}'=\gamma Mc^2$ is the energy of the proton before collision.

III. CALCULATION OF STOPPING POWER

The contribution of close collisions to the stopping power is given by

$$\left(-\frac{dE}{ds}\right)_{Q>\eta}=NZ\int_\eta^{Q_m}Qd\sigma\,.\tag{16}$$

In order to integrate this expression we write G_E in the form

$$G_E(Q)=\sum_{i=1}^{K}\frac{\lambda_i}{1+\nu_iQ},\tag{17}$$

where λ_i and ν_i are constants evaluated explicitly from high-energy electron-proton scattering data.[7] The sum over i can contain any number K of terms. Combining Eqs. (15) and (17) with (16), carrying out a lengthy but straightforward calculation, and adding the result to Eq. (2), we find in place of Eq. (4) that

$$\begin{aligned} -\frac{dE}{ds}&=\tfrac{1}{2}\kappa\left(\ln\frac{2\gamma^2mv^2\eta}{I^2}-\beta^2+\ln\frac{Q_m}{\eta}+T\right)\\ &=\kappa\left(\ln\frac{2\gamma^2mv^2}{I}-\tfrac{1}{2}\beta^2+\tfrac{1}{2}\ln\frac{M}{M+2\gamma m}+\tfrac{1}{2}T\right), \end{aligned}\tag{18}$$

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[6] R. R. Wilson and J. S. Levinger, Ann. Rev. Nucl. Sci. 14, 135 (1964).

[7] L. N. Hand, D. G. Miller, and R. Wilson, Rev. Mod. Phys. 35, 335 (1963).

[8] Although Q is measured in S', we do not use a prime in order that it have the same meaning as in Ref. 4.

TABLE I. Analysis of various contributions to stopping power of aluminum ($I=163$ eV) at extreme relativistic energies.

γ	E_μ (GeV)	E_p (GeV)	ϵ_0	ϵ_μ	ϵ_p	$100(\epsilon_0-\epsilon_\mu)/\epsilon_\mu$	$100(\epsilon_0-\epsilon_p)/\epsilon_p$	ϵ_a	δ	$(-dE/\rho ds)_\mu$ (MeV cm²/g)	$(-dE/\rho ds)_p$ (MeV cm²/g)
10	1.057	9.382	12.35	12.30	12.34	0.37	0.044	12.34	−0.72	1.840	1.846
50	5.285	46.91	15.57	15.39	15.55	1.2	0.088	15.54	−1.9	2.278	2.302
100	10.57	93.82	16.95	16.64	16.90	1.8	0.30	16.92	−2.5	2.464	2.502
250	26.42	234.5	18.79	18.23	18.66	3.0	0.69	18.74	−3.4	2.699	2.762
500	52.85	469.1	20.17	19.38	19.89	4.1	1.5	20.16	−4.1	2.868	2.944
750	79.26	703.5	20.98	20.02	20.52	4.8	2.3	21.03	−4.5	2.964	3.037
1000	105.7	938.2	21.56	20.48	20.92	5.3	3.1	21.68	−4.8	3.031	3.096
2000	211.4	1876	22.94	21.55	21.75	6.5	5.5	23.39	−5.5	3.190	3.220
5000	528.5	4691	24.78	22.95	22.73	8.0	9.0	26.00	−6.4	3.397	3.364
7000	739.9	6567	25.45	23.46	23.07	8.5	10.3	27.08	−6.7	3.472	3.415
10 000	1057	9382	26.16	24.00	23.44	9.0	11.6	28.30	−7.0	3.552	3.469

where

$$T=\sum_i \lambda_i^2\left[\frac{Q_m}{1+\nu_iQ_m}\left(-\frac{\rho_i\nu_i}{\sigma_i}-\frac{\beta^2\rho_i}{\sigma_iQ_m}-\frac{\mu^2}{2E^2\nu_i}\right)-\left(1+\frac{m^2(\mu^2-1)}{\sigma_i^2}+\frac{\beta^2am(\mu^2-1)}{\sigma_i^2Q_m}-\frac{\mu^2}{2E^2\nu_i^2}\right)\ln(1+\nu_iQ_m)\right]$$
$$+m(\mu^2-1)\left(m+\frac{\beta^2a}{Q_m}\right)\ln\left(1+\frac{mQ_m}{a}\right)\sum_{i,j}\frac{\lambda_i\lambda_j}{\sigma_i\sigma_j}+\sum_{i\neq j}\lambda_i\lambda_j\left(\frac{2\nu_i\rho_i}{\sigma_i}+\frac{2\beta^2\rho_i}{\sigma_iQ_m}+\frac{\mu^2}{E^2\nu_i}\right)\frac{\ln(1+\nu_iQ_m)}{\nu_j-\nu_i}. \quad (19)$$

Here $\sigma_i=m-a\nu_i$ and $\rho_i=m\mu^2-a\nu_i$, and use has been made of the inequalities $m\eta\ll a$ and $\eta\ll Q_m$. The new formulas are accurate when $\gamma\gtrsim 10$ and give results that join smoothly with the relativistic formula (4) in the region $\gamma\lesssim 10$.

The physical meaning of the terms in Eqs. (18) and (19) becomes clearer when we expand the form factor (17) and the subsequent equations to first order in the ν_i. For this purpose we set $\lambda_1=1$, $\nu_1=\nu$, and $\lambda_i=0$ for $i>1$. To this approximation, Eqs. (18) and (19) give

$$-\frac{dE}{ds}=\kappa\left[\ln\frac{2\gamma^2mv^2}{I}+\tfrac{1}{2}\ln\frac{M}{M+2\gamma m}-\tfrac{1}{2}\beta^2(1+\mu^2)+\frac{\gamma^2\mu^2m^2\beta^4}{2(M+2\gamma m)^2}+\frac{\nu\mu^2\gamma^2Mmv^2(\beta^2-2)}{M+2\gamma m}-\frac{4\nu\mu^2\gamma^4Mm^3\beta^4v^2}{3(M+2\gamma m)^3}\right.$$
$$\left.-\frac{2\nu M^2v^2}{m}(\mu^2-1)+\tfrac{1}{2}(\mu^2-1)\left(1+\frac{M(M+2\gamma m)}{\gamma^2m^2}\right)\left(1+\frac{4\nu M^2c^2}{m}\right)\ln\left(1+\frac{\gamma^2m^2\beta^2}{M(M+2\gamma m)}\right)\right]. \quad (20)$$

The first term in the square brackets is present in the ordinary relativistic theory. The second term, which is negligible when $\gamma m\ll M$, arises from the factor $M/(M+2\gamma m)$ in Q_m in the ultrarelativistic theory with $\gamma\gg 1$. The third term gives the factor $-\beta^2$ in the relativistic formula when $\mu=1$ (no anomalous magnetic moment). The fourth term arises from the spin $\frac{1}{2}$ of the incident particle. The fifth, sixth, and seventh terms vanish for a point Dirac particle ($\nu=0$), and the seventh and eighth terms vanish when there is no anomalous moment. Equation (20) can be applied to the muon, considered as a point Dirac particle, at all energies by setting $\mu=1$ and $\nu=0$. For protons ($\mu=2.793$) the expansion (20) is valid to the extent that the electric form factor can be represented[9] to first order in Q:

$$G_E(Q)=1-\tfrac{1}{6}\langle r^2\rangle q^2=1-(m\langle r^2\rangle/3\hbar^2)Q=1-\nu Q, \quad (21)$$

$\langle r^2\rangle^{1/2}=8.13\times 10^{-14}$ cm being the rms radius of the proton's charge distribution. Neglect of higher powers of Q implies the smallness, compared with unity, of the parameter

$$\nu Q=\frac{m\langle r^2\rangle Q}{3\hbar^2}\leqq\frac{m\langle r^2\rangle Q_m}{2\hbar^2}\sim\frac{10^{-6}\gamma^2}{1+2\gamma m/M}. \quad (22)$$

The expansion (20) can thus be used for protons of several hundred GeV energy; Eqs. (18) and (19) must be used at higher energies.

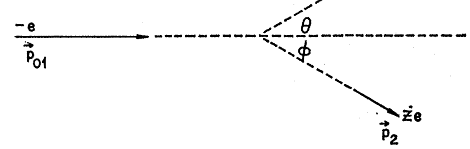

FIG. 1. Representation of collision in initial rest system S of proton.

[9] See, e.g., R. Wilson, in *Particle Interactions at High Energies*, edited by T. W. Preist and L. L. J. Vick (Oliver and Boyd, London, 1967).

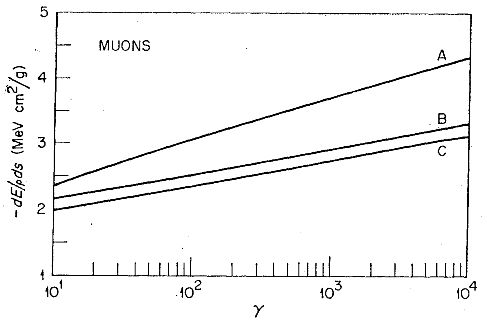

FIG. 2. Curve A: total stopping power of water for muons. Curve B: restricted energy loss with cutoff energy $Q_c=8.0$ MeV (electron range 3.95 cm). Curve C: restricted energy loss with $Q_c=1.1$ MeV (electron range 0.5 cm).

IV. NUMERICAL RESULTS

Table I shows numerical results for aluminum ($I=163$ eV) over the range $\gamma=10$–10 000.[10] The muon and proton energies at each value of γ are shown in columns 2 and 3. The dimensionless quantities

$$\epsilon_0=\ln(2\gamma^2mv^2/I)-\beta^2, \quad (23)$$

$$\epsilon_\mu=\epsilon_0-\tfrac{1}{2}\ln(1+2\gamma m/M_\mu)+\beta^4/2(2+M_\mu/\gamma m)^2, \quad (24)$$

and

$$\epsilon_p=\epsilon_0+\tfrac{1}{2}\beta^2-\tfrac{1}{2}\ln(1+2\gamma m/M_p)+\tfrac{1}{2}T \quad (25)$$

are also given, where M_μ and M_p are the muon and proton rest masses. The differences $\epsilon_0-\epsilon_\mu$ and $\epsilon_0-\epsilon_p$, which are the same for any element because they do not depend on $\ln I$, are due entirely to the extreme relativistic effects. The relative importance of the new terms in the stopping-power formula for muons and protons in aluminum at extreme relativistic energies is given by the percentages $100(\epsilon_0-\epsilon_p)/\epsilon_p$ and $100(\epsilon_0-\epsilon_\mu)/\epsilon_\mu$, which are approximately the same for other elements as well. At 200 GeV, the largest energy of an accelerator currently under construction, the new terms decrease the stopping power by about 6.5% for muons and 0.6% for protons. Whereas the proton's spin and anomalous magnetic moment increase the stopping power, the inclusion of its form factors decreases the energy-loss rate. The quantity ϵ_a, which was calculated in Table I by setting $\mu=2.793$ and $\nu=0$ in Eq. (20), shows the effect of the proton's spin and magnetic moment alone, as though it were a point Dirac particle with an anomalous moment. The decrease in stopping power due to the second term in (20) is larger than the increase due to the spin and anomalous moment when $\gamma\lesssim700$. For larger values of γ, the spin and anomalous magnetic moment make a larger contribution than the second term in Eq. (20). Estimates of the density correction δ to ϵ, shown for comparison, were obtained by applying Sternheimer's formulas[11] at large γ. The mass stopping power of aluminum ($-dE/\rho ds$, where ρ is the density) in columns 11 and 12 of Table I was calculated by means of Eqs. (18) and (19) for protons and Eq. (20) for muons, the density effect not being included.

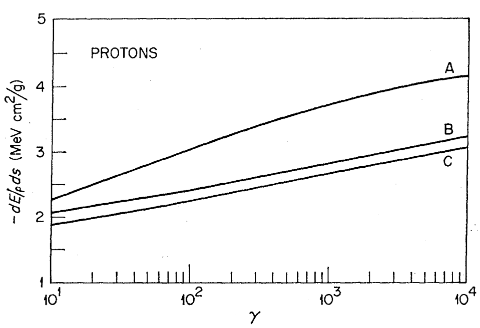

FIG. 3. Curve A: total stopping power of water for protons. Curve B: restricted energy loss with cutoff energy $Q_c=8.0$ MeV (electron range 3.95 cm). Curve C: restricted energy loss with $Q_c=1.1$ MeV (electron range 0.5 cm).

[10] The numerical fit given in Ref. 7 was used with Eq. (17) to represent the proton form factor: $K=3$, $\lambda_1=-1.24$, $\lambda_2=1.34$, and $\lambda_3=0.90$; and, in units of $2m\times10^{-26}/\hbar^2$, $\nu_1=1/30$, $\nu_2=1/14.5$, and $\nu_3=1/15.8$. Although G_E becomes negative with this fit when $\gamma\gtrsim6000$ we have applied it up to $\gamma=10^4$ to estimate G_E^2.

TABLE II. Stopping powers of copper ($I=316$ eV) and lead ($I=825$ eV) at extreme relativistic energies.

	Copper		Lead	
γ	$(-dE/\rho ds)_\mu$ (MeV cm²/g)	$(-dE/\rho ds)_p$ (MeV cm²/g)	$(-dE/\rho ds)_\mu$ (MeV cm²/g)	$(-dE/\rho ds)_p$ (MeV cm²/g)
10	1.649	1.651	1.313	1.318
50	2.065	2.087	1.683	1.702
100	2.241	2.277	1.836	1.867
250	2.464	2.523	2.028	2.080
500	2.624	2.696	2.167	2.230
750	2.715	2.784	2.246	2.306
1000	2.778	2.840	2.301	2.355
2000	2.929	2.957	2.432	2.456
5000	3.125	3.094	2.602	2.575
7000	3.196	3.142	2.663	2.617
10 000	3.272	3.193	2.729	2.661

Values of ϵ for other elements can be readily obtained from Table I. We write

$$\epsilon^{Z,A}=\epsilon^{\mathrm{Al}}+\ln(163/I_Z), \quad (26)$$

where ϵ^{Al} is any one of the values in columns 4, 5, 6, or 9 of Table I, $\epsilon^{Z,A}$ is the corresponding value for an element of atomic number Z and atomic weight A, and I_Z is the mean excitation energy of the element in electron volts. The mass stopping power of the element in MeV cm²/g is given by

$$-\frac{dE}{\rho ds}=\frac{0.3072Z}{\beta^2A}\epsilon^{Z,A}. \quad (27)$$

Values for copper and lead are given in Table II (density correction not included).

[11] R. M. Sternheimer, Phys. Rev. **103**, 511 (1956).

The restricted energy loss,[2] in which the integration over Q is carried out only up through some cutoff value $Q_c < Q_m$, is of interest in some applications. In the present work this quantity, compared with the total energy loss $-dE/\rho ds$, gives a measure of the importance of the relatively infrequent, but very large energy-loss events. Figures 2 and 3 show the total ($Q_c = Q_m$) and the restricted energy losses for protons and muons in water with $Q_c = 8.0$ and 1.1 MeV. (Electrons with these energies have ranges of 0.5 and 3.95 g/cm^2, respectively, in water.) In each figure, the difference between curves A and B or C increases steadily with increasing γ, while the difference between B and C is constant. As an order-of-magnitude estimate, the figures show that δ rays having a range of 0.5 cm or more contribute 28% to $-dE/\rho ds$ when $\gamma = 10$ and 35% when $\gamma = 10^4$.

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Self-Induced Transparency*

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Above a critical power threshold for a given pulse width, a short pulse of coherent traveling-wave optical radiation is observed to propagate with anomalously low energy loss while at resonance with a two-quantum-level system of absorbers. The line shape of the resonant system is determined by inhomogeneous broadening, and the pulse width is short compared to dissipative relaxation times. A new mechanism of self-induced transparency, which accounts for the low energy loss, is analyzed in the ideal limit of a plane wave which excites a resonant medium with no damping present. The stable condition of transparency results after the traversal of the pulse through a few classical absorption lengths into the medium. This condition exists when the initial pulse has evolved into a symmetric hyperbolic-secant pulse function of time and distance, and has the area characteristic of a "2π pulse." Ideal transparency then persists when coherent induced absorption of pulse energy during the first half of the pulse is followed by coherent induced emission of the same amount of energy back into the beam direction during the second half of the pulse. The effects of dissipative relaxation times upon pulse energy, pulse area, and pulse delay time are analyzed to first order in the ratio of short pulse width to long damping time. The analysis shows that the 2π pulse condition can be maintained if losses caused by damping are compensated by beam focusing. In an amplifying inhomogeneously broadened medium an analytic "π pulse area" solution is presented in the limit of a sharp leading edge of the pulse. The dynamics of self-induced transparency are studied for the particular effects of Doppler velocities upon a resonant gas. The analysis of transparency for random orientations of dipole moments associated with degenerate rotational states yields modified forms of self-induced transparency behavior, which indicates a finite pulse energy loss as a function of distance in some cases. The effect of self-induced transparency on the photon echo is considered. Experimental observations of self-induced transparency have been made in a ruby sample at resonance with a pulsed ruby-laser beam. Single and multiple 2π pulse outputs have been observed, and pulse areas measured in the range of 2π. The experimental results are compared with the predictions of the ideal-plane-wave theory. Deviations from the ideal-plane-wave theory are discussed. An analysis is made of the effect of a transverse mode of the propagating beam upon the transparency properties of the pulse.

I. INTRODUCTION

THE development of sources of pulsed coherent radiation has initiated investigations of the behavior of coherent traveling waves as they interact with media which have absorption bands near or at the frequency of the applied pulse. Of particular interest are resonant absorbing media characterized by localized two-level transitions which are excited by optical[1] and phonon[2] radiation. The absorption of low-intensity coherent or incoherent radiation can usually be interpreted in terms of linear dispersion theory, particularly if the ground-state energy levels of the absorbing medium (or excited states in the case of a prepumped active amplifying medium) are only slightly depopulated by the radiation. As the resonant traveling-wave radiation intensity is increased, the linear problem[3] can be perturbed to account for the onset of weak nonlinearities.[4] If damping is not too severe, transient oscillations in state populations can exist during and after the appli-

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[1] A. L. Schawlow and C. H. Townes, Phys. Rev. **112**, 1940 (1958); T. H. Maiman, Nature **187**, 493 (1960); F. J. McClung and R. W. Hellwarth, J. Appl. Phys. **33**, 828 (1963).
[2] E. H. Jacobsen, Phys. Rev. Letters **2**, 249 (1959); H. Bommel and K. Dransfeld, *ibid.* **3**, 83 (1959); N. S. Shiren, Phys. Rev. **128**, 2103 (1962); E. H. Jacobsen and K. W. H. Stevens, *ibid.* **129**, 2036 (1962).
[3] F. J. Lynch, R. E. Holland, and M. Hamermesh, Phys. Rev. **120**, 513 (1962); N. S. Shiren, Phys. Rev. **128**, 2103 (1962).
[4] See Refs. 88–110 in Bibliography by James D. Macomber, IEEE J. Quantum Electron. **QE-4**, 1 (1968).