Large-Amplitude Ion Acoustic Waves in a Plasma

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Experiments are performed on collisionless damping of large-amplitude ion acoustic waves excited externally in a thermally ionized cesium plasma. The wave damping of small-amplitude ion acoustic waves is independent of the wave amplitude, and the waves are damped by Landau damping. For large-amplitude ion acoustic waves, the wave damping is not exponential along the plasma column. Over a few wavelengths from the wave exciter, the waves are damped almost exponentially, though the damping rate increases with the increase of the exciting voltage. After this strong damping, the wave damping begins to saturate and the damping rate becomes smaller than that of the small-amplitude waves. The results are in reasonable agreement with those of the numerical computation carried out by Armstrong on large-amplitude electron plasma waves. Amplitude oscillations are observed for $e\varphi/KT_i \gtrsim 0.4$, where $e\varphi/KT_i$ denotes fluctuating electric energy normalized with thermal energy of ions. The distance between the amplitude maxima becomes shorter by increasing the exciting voltage or frequency, and is several times longer than the wavelength. The phenomena are well interpreted by the same mechanism as predicted by the nonlinear theories of Al'tshul' and Karpman, and O'Neil on electron plasma waves.

I. INTRODUCTION

Ion acoustic waves are low-frequency longitudinal compression waves in which electrons and ions are constrained to move in phase. They were first predicted on the basis of the two-fluid model by Tonks and Langmuir[1] in 1929. More recently, basing on the linearized collisionless Vlasov equation, a number of authors[2] have shown that ion acoustic waves can exist in a plasma even in the absence of collisions. They have also predicted collisionless damping (Landau damping) of ion acoustic waves due to the interaction of the waves with charged particles moving with the velocities close to its phase velocity. This damping was first predicted by Landau[3] for electron plasma waves. For ion acoustic waves, the Landau damping becomes stronger by decreasing the ratio of the electron temperature T_e to the ion temperature T_i, and the damping is very strong when the electron and ion temperatures are comparable.

The first experimental work on ion acoustic waves was reported with rather inconclusive results by Revans,[4] who observed the waves excited spontaneously in a mercury discharge. Recently, a number of authors[5] made detailed experiments on those self-excited waves in gaseous discharge tubes.

Since excitation methods of ion acoustic waves by an external source with a grid (Hatta and Sato)[6] or a small coil (Little)[7] were published, the propagation and damping of ion acoustic waves have been studied more conveniently by a number of authors.[8] The Landau damping of plasma waves was first verified by Wong, Motley, and D'Angelo,[9] who made an experiment on propagating ion acoustic waves excited by a grid immersed in thermally ionized cesium and potassium plasmas. More recently, Alexeff, Jones, and Montgomery[10] succeeded to measure the dependence of the Landau damping on the ratio T_e/T_i by cooling electrons in discharge tubes. This dependence was also measured in a clear-cut fashion by cooling ions by ion-neutral collisions.[11] The Landau damping of electron plasma waves was examined in detail by the recent experiments of several authors.[12] Since the experimental works mentioned above were all restricted to the small-amplitude waves, the linear theories[1-3] were adequate in describing the experimental results.

Recently, Andersen, D'Angelo, Michelsen, and Neilsen[13] made an experiment on shock formation as a next step in this line of investigation. Large-amplitude pulses were observed to develop into shock when the Landau damping was removed by increasing the ratio T_e/T_i.

On the other hand, the quasilinear and nonlinear theories[14–21] have predicted many interesting features of the collisionless damping of large-amplitude plasma waves. According to them, there appear many particles trapped in the potential troughs of the large-amplitude waves. When the effects of those trapped particles become appreciable, the wave amplitude ceases to damp exponentially and the saturation of damping occurs, as shown numerically by Knorr[16] and Armstrong.[17] Recently, Dawson and Shanny[19] have pointed out the initial strong damping which is increased with the wave amplitude. This enhanced damping lasts only while the particles are being accelerated, and the saturation ultimately occurs. The damping of the large-amplitude waves has also been treated by Al'tshul' and Karpman,[20] and O'Neil.[21] They have predicted oscillations in the wave amplitude, which are attributed to the oscillations of charged particles in the potential troughs of the waves. In the recent study of Malmberg and Wharton[22] on large-amplitude electron plasma waves, the amplitude oscillations have been observed for the case of very small damping.

In this paper, we describe the damping of large-amplitude ion acoustic waves excited by a grid in a thermally ionized cesium plasma. The small-amplitude waves are damped strongly by the Landau damping. The large-amplitude waves are excited by modulating the plasma density by large-amplitude sinusoidal voltages applied to the grid. The large-amplitude waves do not follow simple exponential damping along the plasma column. The wave damping begins to saturate after initial strong damping. Amplitude oscillations are observed for the larger-amplitude waves. Preliminary experimental results have already been reported.[23,24]

In Sec. II, the experimental apparatus and methods are described. Section III presents the experimental results. In Sec. IV, the results are discussed on the basis of the nonlinear theories. Section V contains conclusions.

II. EXPERIMENTAL APPARATUS AND METHODS

The experiments were performed on the TP-C Machine[25] at the Institute of Plasma Physics, Nagoya University. This Machine is a "single-ended Q Machine" (alkali plasma source) described by Rynn and D'Angelo.[26]

A schematic diagram of the experimental apparatus is shown in Fig. 1. The plasma is produced by surface ionization of cesium atoms on a tantalum plate heated up to 2300°K and is confined radially by an external magnetic field up to 2.5 kG. The plasma column, about 2 cm in diameter and about 130 cm long, is terminated at the opposite end from the hot plate by a stainless-steel wall. The plasma density, in the range 10^9-10^{10} cm^{-3}, is measured by Langmuir probes to be uniform within a variation of 5% along the column where the experiments are performed. The electron temperature T_e measured also by the Langmuir probes is 1.5–3 times higher than the temperature of the hot plate T_p. Such a high temperature is estimated also by the wave damping of ion acoustic waves. The ion temperature T_i is determined by the ion sensitive probe[27] to be close to but a little higher than T_p. The background gas pressure is kept to be in the range $(1-5)\times 10^{-6}$ Torr. The mean free paths between charged particles and neutral atoms are much longer than the length of the plasma column.

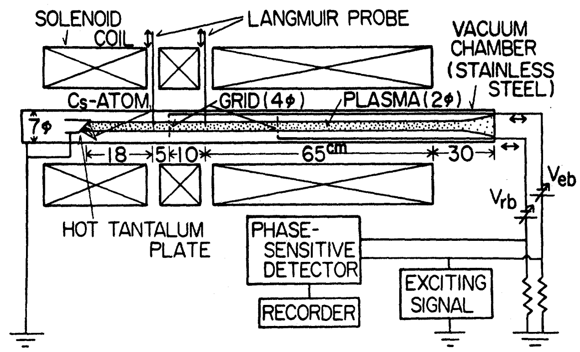

FIG. 1. Schematic diagram of the experimental apparatus. All dimensions are in centimeters.

A grid 4 cm diam made of 0.1-mm-diam molybdenum wires spaced 1.5 mm apart is used for wave excitation. The grid, with its plane normal to the axis, is immersed in the plasma about 20 cm from the hot plate. This grid is kept at negative bias with respect to the hot plate, and it absorbs a part of ions from the hot plate. Continuous sinusoidal voltage V_{ex} between 0.2 and 50 V peak to peak is applied to the grid, which varies the transmission rate to the plasma from 1 to 90%. The frequency of the sinusoidal signals f is varied from 10 to 150 kc/sec. The waves excited are detected by a similar grid. It is also biased negatively and is able to be moved axially for collecting the ion density fluctuations at the various positions along the plasma column. The signals picked up by the receiving grid are fed to a phase-sensitive detector, together with the reference signals from the exciting source. Phase velocity and wave damping are obtained from the phase-sensitive detector output plotted as a function of the grid separation on a recorder. The propagation and damping of the waves are checked roughly using a dual-beam oscilloscope. The measurements are made mainly on the downstream waves, i.e., the waves propagating along the plasma flow, the direction of which is from the hot plate to the opposite wall.

III. EXPERIMENTAL RESULTS

The excited waves propagate along the plasma column and are damped out before reaching the plasma-column end, without any reflection back to the plasma. The measurements are made over 3–8 wavelengths downstream and 1–3 wavelengths upstream. A typical record of the phase-sensitive detector output is shown as a function of the grid separation in Fig. 2.

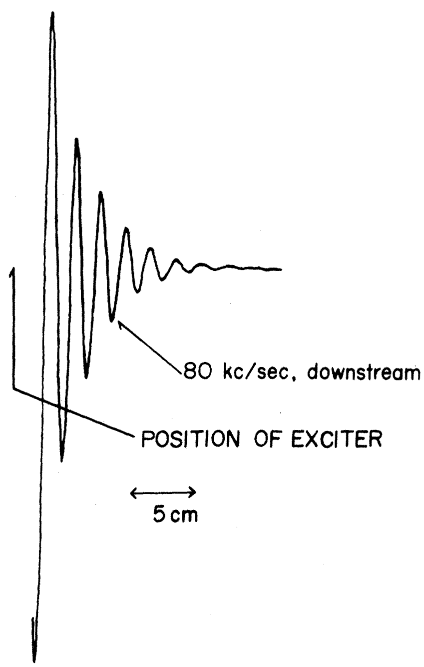

FIG. 2. A typical record of the phase-sensitive detector output plotted as a function of the grid separation, which demonstrates the propagation and damping of the ion acoustic wave excited by the sinusoidal voltage of 1 V peak to peak.

For small-amplitude ion acoustic waves ($V \lesssim$ a few volts peak to peak), the phase velocity and the wave damping do not depend on the exciting voltage. The waves decay by the Landau damping as observed by Wong, Motley, and D'Angelo.[9] Typical examples of the phase delay and the relative wave amplitude are shown as a function of the the grid separation d in Figs. 3 and 4, respectively. The phase velocities v_p of 1.65×10^5 cm/sec (downstream) and of 1.45×10^5 cm/sec

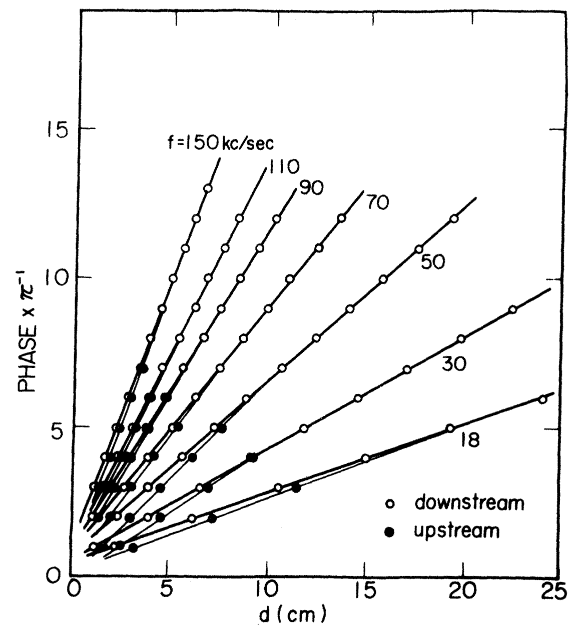

FIG. 3. Relative phase delay of small-amplitude ion acoustic waves as a function of the grid separation d. The exciting voltage is 1 V peak to peak.

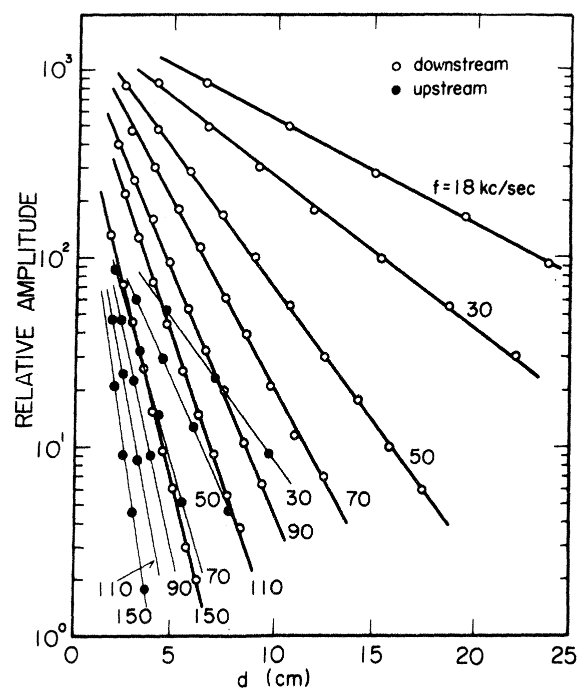

FIG. 4. Relative wave amplitude of small-amplitude ion acoustic waves as a function of the grid separation d. The exciting voltage is 1 V peak to peak.

(upstream) are independent of the wave frequency f as shown in Fig. 5. Thus, the velocity of the

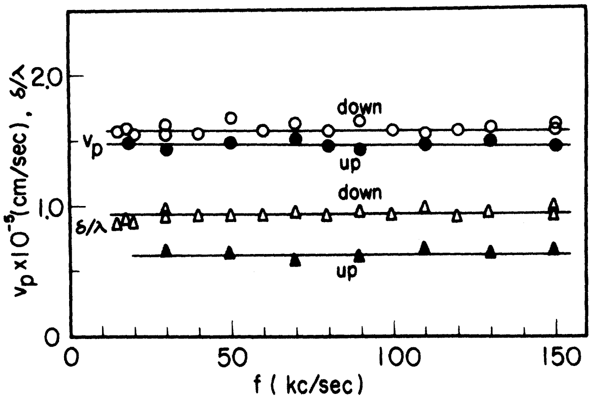

FIG. 5. Phase velocity v_p and damping distance normalized with the wavelength δ/λ as a function of the wave frequency f. The exciting voltage is 1 V peak to peak.

plasma flow is estimated to be about 10% of the phase velocity. The damping distances normalized with the wavelength δ/λ of 0.95 (downstream) and of 0.60 (upstream) are also found to be independent of the wave frequency. Those values depend slightly on the bias of the exciting grid, because the grid bias gives small changes to the plasma parameters. The results shown in Figs. 3–5 are measured for the grid bias of − 20 V with respect to the hot plate (the plasma potential is about − 4 V with respect to the hot plate). Both the phase velocity and the normalized damping distance are larger than those found in the results of Wong *et al.*[9] This difference is attributed to the fact that $T_e > T_i \gtrsim T_p$ in our experiments. Such a high electron temperature is justified by the experimental conditions under which electrons are accelerated by the ion sheath formed in front of the hot plate.

According to the recent study of Hirshfield and Jacob,[28] the values of δ/λ not somewhat larger than about 0.4 represent a decay dominated by the free streaming of noninteracting particles rather than the spatial Landau damping. In our experiments, the values of δ/λ are found to be about 0.8 by averaging out the effect of the plasma flow. Thus, they satisfy the criterion for avoiding a significant contribution due to the free-streaming particles.

The wave frequencies f are not always smaller than the ion cyclotron frequency f_{ci}(~ 26 kc/sec) in contrast with the experiments of Wong *et al.*,[9,29] in which the frequencies are adjusted to be smaller than the ion cyclotron frequency in order to avoid the possible ion cyclotron resonance. In spite of our efforts, however, no different propagation of the waves with $\lambda \gtrsim R$ (plasma radius) is observed at or above the ion cyclotron frequency compared with the case of $f < f_{ci}$.[30] The results show that a plane-wave analysis is also adequate for ion acoustic waves with $f \gtrsim f_{ci}$ and $\lambda \gtrsim R$ excited by a plane grid in a plasma produced by surface ionization and confined by an axial magnetic field.

In the experiments on ion acoustic waves excited by means of a small coil[7] in low-pressure discharges under a weak axial magnetic field ($f \gg f_{ci}$), Crawford and Kuhler[31] observed the waves propagating down to zero frequency, together with the waves with a low-frequency cut-off. The latter (radially varying mode[32]), excited by the same method, was also treated by the recent work of Levin and Oleson.[33] By decreasing f so that $\lambda \to R$, the waves with $f \gtrsim f_{ci}$ were observed to have a low-frequency cut-off depending on an axial magnetic field, as predicted by their theory. On the other hand, the former was shown to be a principal mode,[32] the phase velocity of which was equal to the group velocity just as in an unbounded plasma. Our results are well understood by supposing the waves as the principal mode, for which any different behavior is not expected at $f \gtrsim f_{ci}$ compared with the case of $f < f_{ci}$ even if $\lambda \gtrsim R$.

For the large-amplitude waves, the measurements are made mainly on the waves propagating along the plasma flow. Along this direction, the large-amplitude waves are easily excited and the wave damping is able to be measured correctly over a long distance. The phase velocity of the large-amplitude waves is slightly increased by increasing the exciting voltage, though it does not depend on the local wave amplitude for the fixed value of the exciting voltage. The same results are also obtained from the experiments on the waves excited by the pulse voltages applied to the grid.

The damping of the large-amplitude waves is rather interesting. As shown in Fig. 6, the wave amplitude no longer follows the exponential damping along the plasma column. Over a short distance from the exciting grid, the wave damping is almost exponential. In this region, however, the wave damping is observed to become stronger by increasing the exciting voltage. After this enhanced damping, the wave amplitude deviates from the exponential damping. The wave damping begins to saturate and becomes slower than the Landau damping of the small-amplitude waves. The distance d_s, beyond which the wave amplitude deviates from the exponential damping, is inversely proportional to the square root of the exciting voltage $V_{ex}{}^{1/2}$ as shown in Fig. 7, where the inverse of this distance normalized with the wavelength $2\pi/d_s k$ is plotted as a function of the square root of the exciting voltage. The dependence of the phenomena on the wave frequency f is also examined in the range 10–150 kc/sec. As the frequency is decreased, the deviation from the exponential damping disappeared gradually.

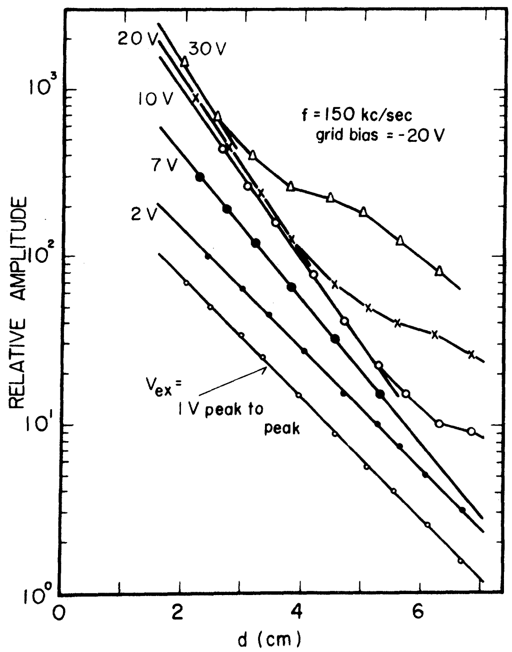

FIG. 6. Relative wave amplitude as a function of the grid separation d for various exciting voltages V_{ex}. The wave frequency is 150 kc/sec. The grids are biased at −20 V with respect to the hot plate.

The deviation is recognized down to about 50 kc/sec, though d_s is not definitely determined for $f \lesssim 80$ kc/sec. Nevertheless, it is roughly said that d_s is almost proportional to the wavelength.

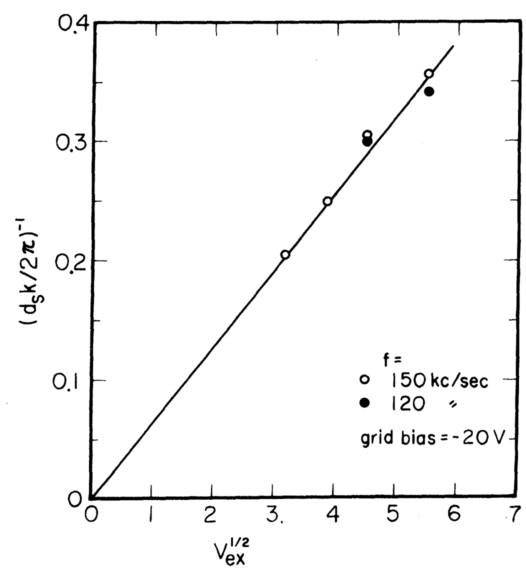

FIG. 7. The inverse of the normalized distance with the wavelength $(d_s k/2\pi)^{-1}$, at which the deviation from the exponential damping starts, as a function of the square root of the exciting voltage $V_{ex}^{1/2}$.

When the bias of the exciting grid is increased negatively, the unperturbed density is decreased in the propagation region. Thus, the ratio of the initial density perturbation to the unperturbed density becomes larger for the fixed values of the exciting voltage, though less decrease of the initial density perturbation is also observed. The dependence of the wave damping on the grid bias is shown in Fig. 8. The waves begin to show more interesting behavior when the grid bias is increased toward more negative values. It is to be noted that, for the grid bias of − 40 V, there appears a region where the wave grows. A typical example of the phase-sensitive detector output, which also demonstrates the growth and subsequent peaking in the wave amplitude, is shown as a function of the grid separation in Fig. 9.

More distinct features of the phenomena are observed for the grid bias of − 45 V as shown in Fig. 10, where the wave amplitude is plotted as a function of the grid separation d for various exciting voltages V_{ex}. The wave amplitude is observed to be damped accompanied by oscillations in the wave amplitude for the large-amplitude exciting voltage. With the increase of the exciting voltage, the positions of the amplitude maxima are observed to shift toward the exciting grid ($A_a \to A_b \to A_c$, $B_b \to B_c$ in Fig. 10). The dependence of the amplitude oscillations on the wave frequency f is shown in Fig. 11. The positions

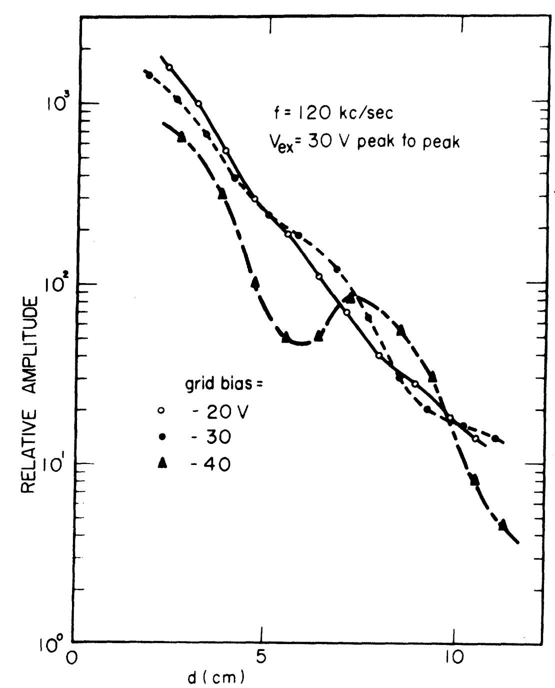

FIG. 8. Relative wave amplitude as a function of the grid separation d for various grid biases. The exciting voltage is 30 V peak to peak. The wave frequency is 120 kc/sec.

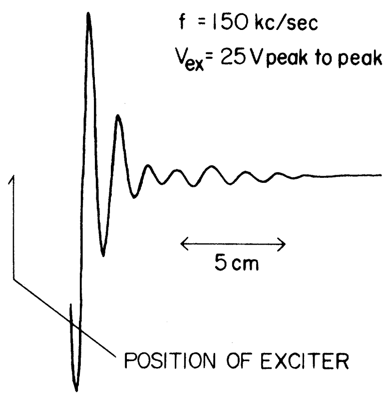

FIG. 9. A typical record of the phase-sensitive detector output as a function of the grid separation, which demonstrates the growth and subsequent peaking in the wave amplitude of the large-amplitude ion acoustic wave. The grids are biased at −30 V with respect to the hot plate.

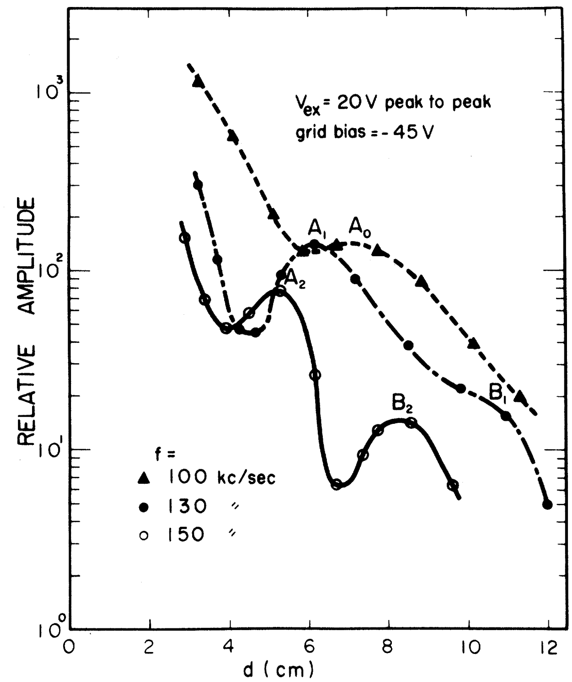

FIG. 11. Relative wave amplitude as a function of the grid separation d for various wave frequencies f. The grids are biased at −45 V with respect to the hot plate. The exciting voltage is 20 V peak to peak.

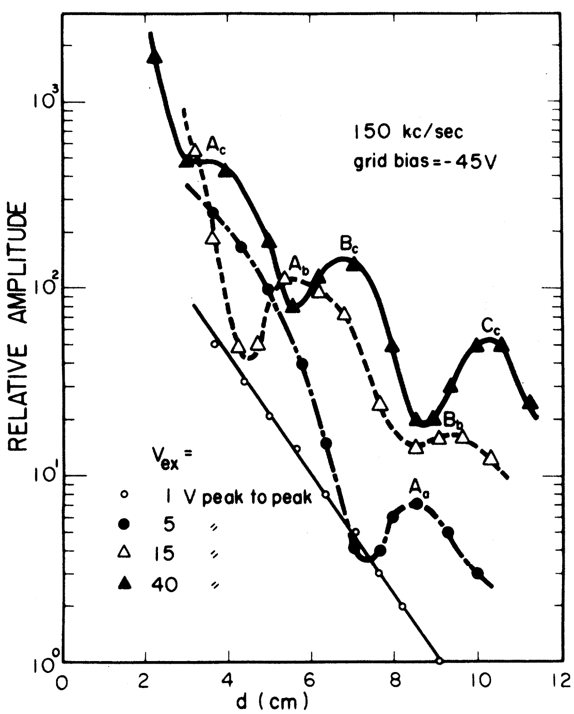

FIG. 10. Relative wave amplitude as a function of the grid separation d for various exciting voltages V_{ex}. The grids are biased at −45 V with respect to the hot plate. The wave frequency is 150 kc/sec.

of the amplitude maxima also shift toward the exciting grid by increasing the frequency as shown in this figure ($A_0 \to A_1 \to A_2$, $B_1 \to B_2$). The distance d_p, at which the amplitude maxima appear, is almost inversely proportional to the square root of the exciting voltage and to the wave number as shown in Fig. 12, where the inverse of this distance normalized with the wavelength $2\pi/d_p k$ is plotted as a function of the square root of the exciting voltage $V_{ex}{}^{1/2}$. As shown in Fig. 13, the distance between the amplitude maxima $2\pi/k_{osc}$ is found to become shorter by increasing the exciting voltage or frequency. This distance is several times longer than the wavelength. The amplitude oscillations become less appreciable by decreasing the wave frequency. The positions of the amplitude maxima are not definitely determined for $f < 100$ kc/sec. When f is smaller than several tens of kilocycles, the amplitude oscillations are not recognized. The data shown in Figs. 10–13 are obtained when the plasma density is about 3×10^9 cm^{-3} and the electron temperature is about two times higher than the ion temperature in the propagation region. The amplitude oscillations disappear gradually as the plasma density is increased for the fixed values of the exciting grid bias. Around the plasma density of 1×10^{11} cm^{-3}, the amplitude oscillations are not observed.

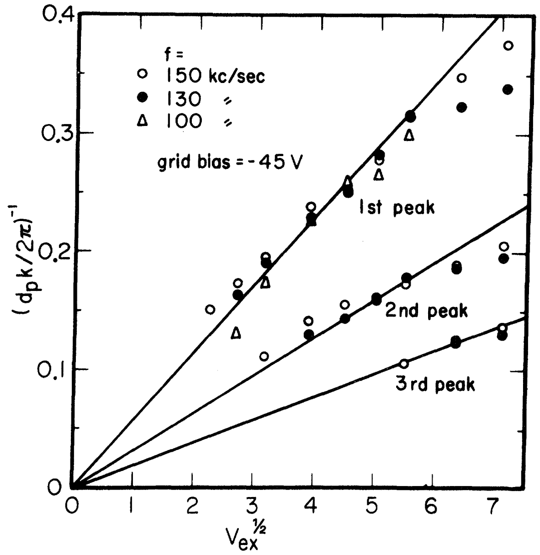

FIG. 12. The inverse of the normalized distance with the wavelength $(d_p k/2\pi)^{-1}$, at which the amplitude maxima appear, as a function of the square root of the exciting voltage $V_{ex}^{1/2}$.

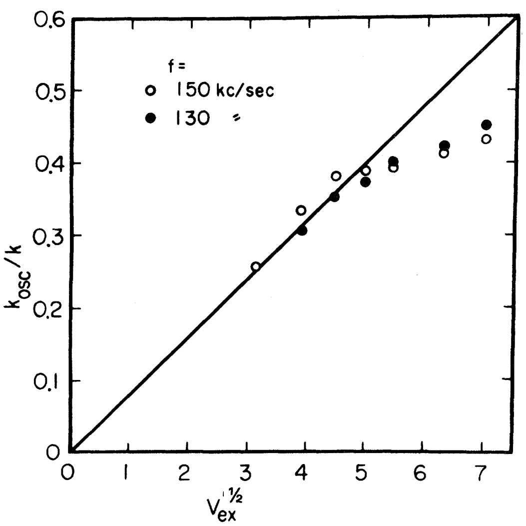

FIG. 13. Amplitude oscillation wave number normalized with the wave number k_{osc}/k as a function of the square root of the exciting voltage $V_{ex}^{1/2}$.

IV. DISCUSSIONS

There have been many theoretical works[14–21] on large-amplitude plasma waves. According to those works, for the large-amplitude waves, there exist an appreciable number of particles trapped in the potential troughs of the waves. As the wave amplitude is increased, the collisionless damping is predicted to deviate from the linear behavior owing to the effects of the trapped particles.

As derived from the quasilinear theories,[14,15] the zero-order velocity distribution is flattened by the waves in the vicinity of the phase velocity. The wave amplitude is no longer damped exponentially. The slower damping shown in Fig. 6 is able to be explained on the basis of the trapped ions. The positions, at which the waves start following the slower damping, are observed to shift toward the exciting grid by increasing the exciting voltage. This dependence is well understood taking into account that an appreciable deformation of the ion velocity distribution is achieved at the shorter distance for the larger-amplitude waves. It is well known that the linear theory should break down after the position corresponding to the oscillation period of trapped particles in the potential troughs of the waves.[21] This period is inversely proportional to the square root of the wave amplitude and to the wave number, as will be discussed later about amplitude oscillations. Thus, it is reasonable that the inverse of d_S normalized with the wavelength is found to be proportional to the square root of the exciting voltage in Fig. 7.

The same phenomena were investigated by Knorr[16] on large-amplitude electron plasma waves by carrying out a numerical integration of the nonlinear Vlasov equation, though the problem was treated as an initial-value problem. The calculation was repeated and extended by Armstrong[17] who obtained similar results. The problem was considered analytically by Gary[18] whose results resembled those of Knorr and Armstrong. According to their results, over a short time the waves are damped at the damping rate approximately equal to that predicted by the linear (Landau) theory. For sufficiently long times, however, the waves decay at a rate smaller than that of the linear theory. The time at which the wave damping deviates from the linear theory becomes shorter by increasing the initial amplitude of the perturbation.

In order to compare our results with their computation, we must replace the time by the position, since the spatial damping is measured in our experiments. The experimental results shown in Figs. 6 and 7, the ratio of the initial density perturbation to the unperturbed density is varied from 0.2 ($V_{ex} = 10$ V) to 0.5 ($V_{ex} = 30$ V). For those values, the positions at which the wave amplitude deviates from the exponential damping are found to be given by $d_S k/2\pi \sim 2.9$ ($V_{ex} = 30$ V)$-$4.8 ($V_{ex} = 10$ V) in Fig. 7. They are in reasonable agreement with the numerical computation of Armstrong,[17] in which the wave damping is almost exponential over several oscillation periods for the initial perturbation of

0.25, though his work was restricted to the temporal damping of electron plasma waves.

The enhanced initial damping observed in the experiments is also found in the results of Armstrong's computation.[17] Recently, the enhancement of the initial damping has been pointed out by Dawson and Shanny.[19] According to them, for a Maxwellian velocity distribution, the number of the trapped particles which cause the damping will rapidly increase with the wave amplitude, because the velocity distribution function increases very rapidly as we move toward lower velocities. Thus, the damping of a large-amplitude wave is much stronger than that predicted by the linear theory. The increased damping lasts only while the particles are being accelerated, and the saturation ultimately occurs. This enhancement of the initial damping was also demonstrated in their computer experiments on electron plasma waves. In our experiments, the enhanced damping is clearly observed for the initial density perturbation larger than about one tenth of the unperturbed density.

The ratio of the initial density perturbation to the unperturbed density is able to be controlled by the bias of the exciting grid for the fixed value of the exciting voltage. The ratio is confirmed to be larger for the more negative bias by measuring the ion currents collected by the receiving grid. Thus, more distinct effects of the trapped particles are expected to appear for the relatively larger negative bias. In fact, as mentioned in the previous section, oscillations in the wave amplitude are observed for the values of the grid bias more negative than a few tens of volts in the experiments. For those cases, the plasma density ($\sim 10^9$ cm^{-3}) is measured in the propagation region to be so large that the wave frequency is much smaller than the ion plasma frequency. The waves of the second and third harmonics are also observed to propagate along the plasma column. The amplitude of those waves, however, have such a higher order of smallness that the propagation and damping of the large-amplitude waves are not influenced by those higher harmonics.

Amplitude oscillations have been treated theoretically by Al'tshul' and Karpman,[20] and O'Neil[21] on large-amplitude electron plasma waves. For ion acoustic waves, the distance between the amplitude maxima $2\pi/k_{osc}$ is predicted to be given by the relation

$$k_{osc}/k = (e\varphi/m_i)^{1/2}/v_p \quad , \qquad (1)$$

where m_i is the ion mass, e the ionic charge, and φ the fluctuating electric potential. In a plasma with the electron temperature two times higher than the ion temperature, which corresponds to our experimental condition, the phase velocity of ion acoustic waves is given by $v_p = 2.5(KT_i/m_i)^{1/2}$. Hence, Eq. (1) is rewritten as follows:

$$k_{osc}/k = 0.4(e\varphi/KT_i)^{1/2} \quad . \qquad (2)$$

It is to be noted that $e\varphi/KT_i$ denotes the fluctuating electric energy normalized with the thermal energy of ions.

In order to compare the experimental results with Eq. (2), the experimental values of k_{osc}/k are plotted as a function of the square root of the exciting voltage $V_{ex}{}^{1/2}$ in Fig. 13, since, roughly speaking, φ can be assumed to be proportional to V_{ex}. In this figure, the observed values of k_{osc}/k are proportional to the square root of the exciting voltage as predicted by Eq. (2). The saturation for larger values of the exciting voltage is mainly due to the fact that the wave amplitude is not proportional to the exciting voltage in that region. By substituting the experimental data of k_{osc}/k into Eq. (2), $e\varphi/KT_i$ is estimated to be 0.4–1.2; in other words, the amplitude oscillations are observed for $e\varphi/KT_i \gtrsim 0.4$ in the experiments.

Theoretically, amplitude oscillations should occur when $k_{osc}/ki \gtrsim (2\pi)^{1/2}$, where k_i is the imaginary part of the wave number of the small-amplitude waves. When the damping of the small-amplitude waves is slow enough so that $k_{osc}/k_i \gg (2\pi)^{1/2}$ is easily satisfied, the effects of the trapped particles become more remarkable and amplitude oscillations are well predicted to occur.[20,21] Malmberg and Wharton[22] observed the strong amplitude oscillations of electron plasma waves under this condition. In our case, the waves are observed to be damped slowly, and to be accompanied by amplitude oscillations, which are not so strong as those observed in their experiments. The difference is mainly due to the fact that the experimental values of k_{osc}/k_i, 1.8–3.2, are comparable with $(2\pi)^{1/2}$. In our case, the mean free path of the ion-ion collisions is several tens of centimeters and is only about ten times as large as the values of $2\pi/k_{osc}$. Thus, the gradual disappearance of the amplitude oscillations with the increase of the plasma density is due to those collisions by which the trapped ions are scattered.

There is, to our knowledge, no theoretical work on the spatial damping of large-amplitude ion acoustic waves. For this reason we have qualitatively discussed our results on the basis of the nonlinear theories and the numerical computations of the nonlinear Vlasov equation for the temporal case of electron plasma waves. It is needless to say that details of their results cannot be applied to our case. The physical mechanisms and their qualitative behaviors predicted by those works, however, can be extended in a straightforward manner to the spatial case of ion acoustic waves by the simple expedient of replacing the time by the position and including the ion dynamics.[34] The theories of Al'tshul' and Karpman,[20] and O'Neil[21] were restricted to the case of $k_{osc}/k_i \gg (2\pi)^{1/2}$ and so their results cannot be extended in a straightforward manner to our case. But it should be justified that amplitude oscillations occur under the condition of $k_{osc}/k_i \gtrsim (2\pi)^{1/2}$, though the theoretical treatment for this case will be more complicated. From an elementary physical argument for the ion motion in a potential well, the oscillation period is easily understood to be given by Eqs. (1) and (2).

At high negative biasses used on the exciting grid, it is no wonder that the particle velocity distribution downstream is changed by those biasses. The change of the velocity distribution in the vicinity of the phase velocity can be experimentally checked by measuring the linear wave damping at those biasses. In fact, at the grid bias of − 45 V, the normalized damping distance δ/λ is observed to be larger by 10% than that at the bias of − 20 V. Thus, the change of the velocity distribution by the grid bias is too small to have appreciable influence such a large change of the wave damping along the plasma column as presented in the previous section.

The slight increase of the phase velocity with the exciting voltage is not well interpreted now. In the TP-C Machine, in addition to ion acoustic waves, we have observed fast propagation signals excited by the positive pulses applied to the grid biased negatively.[35] This fast propagation mode was first observed by Lonngren, Montgomery, Alexeff, and Jones[36] in discharge tubes. This is not a wave, but a burst of accelerated plasma by the grid. In our case, the propagation velocity of this pseudo wave is given by $(2e\delta V/m_i)^{1/2}$, where δV is the excess part of the pulse voltage from the plasma potential. When the pulse voltage does not exceed the plasma potential, the pseudo wave cannot be observed. In our experiments on ion acoustic waves, the negative bias of the grid is kept large enough so that the grid potential does not exceed the plasma potential even when the exciting voltage is applied. Thus, the results reported in the previous section are not related to the pseudo waves.

Finally, we remark some experiments on the damping of a small-amplitude ion acoustic wave of frequency f_0 superposed to a large-amplitude wave of frequency f. The two waves are excited simultaneously by one grid in contrast with the echo experiments.[37] The small-amplitude wave shows amplitude oscillations in the presence of the large-amplitude wave. It may be reasonable to conclude that the results obtained are attributed to the deformation of the velocity distribution by the large-amplitude wave, because the phase velocity of the small-amplitude wave is equal to that of the large-amplitude wave. But the positions of the amplitude maxima and the distance between them do not always coincide with those for the large-amplitude wave. This discrepancy may be attributed to the fine structure of the deformed velocity distribution, because the wave amplitude of the small-amplitude wave is much smaller than the deformed region of the velocity distribution. The waves with various combination frequencies, $mf \pm nf_0$, are observed to propagate along the plasma column. It is needless to say that those signals result partly from the excitation due to the nonlinearity of the ion sheath surrounding the grid. The damping of those waves, however, is different from that of the small-amplitude wave whose frequency is set to be equal to $mf \pm nf_0$. The results show that those signals are created partly due to the coupling of the externally excited waves. Anyhow, those waves are damped also accompanied by amplitude oscillations, as in the case of the small-amplitude wave excited externally. The amplitude oscillations of the small-amplitude wave superposed to the large-amplitude wave are to be discussed in detail elsewhere.

V. CONCLUSIONS

We have studied experimentally large-amplitude ion acoustic waves propagating in a thermally ionized cesium plasma. Three distinct features of the collisionless damping of the large-amplitude waves are verified in the experiments. They are the enhancement of the initial damping with the wave amplitude, the saturation of the damping after this strong damping, and the amplitude oscillations of the larger-amplitude waves. The results are well interpreted by the nonlinear theories on the plasma waves.

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