Annihilation of Positrons in Argon II. Theoretical

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The results of measurements of the electric field dependence of the direct annihilation rate of positrons in argon are used to test the validity of simple models representing the interaction between a positron and an argon atom. Despite the fact that such models yield results in agreement with electron scattering data, they are inadequate in the case of positrons. By comparing this data with recent measurements of the temperature dependence of the lifetime of positrons in argon, a value of $(33 \pm 5)\pi a_0^2$ is obtained for the scattering cross section of thermal positrons in argon.

I. INTRODUCTION

The interaction of low-energy positrons with atoms has been a subject which has recently received attention both theoretically[1–3] and experimentally.[4, 5, 3] Although the simplest situation for theoretical consideration is the interaction of positrons with atomic hydrogen,[6, 3] this process is extremely difficult to investigate experimentally because of the high densities of gas required using current experimental techniques.[4, 7, 8] These techniques involve investigations of the annihilation of positrons in a gas when subjected to uniform electric fields. Since the velocity distribution of the positrons in the gas depends on the strength of the electric field, it is possible to check certain velocity-dependent effects predicted by the theories.

Although the theory of the interaction of positrons with helium atoms has also been investigated recently,[1–3] most of the experimental information available[9, 10] was obtained under zero electric field conditions and so is of limited use for testing the theories. For this reason, we are currently involved in measurements of positron lifetimes in helium as a function of electric field. For the other noble gases, the measurements are easier to obtain. Such measurements for the case of argon, for example, are presented in the preceding Paper I. The theoretical situation is much less tractable for these gases, however, and most work has employed rather crude models for the interaction.

The simplest approximation to this many-particle problem involves the use of an effective two-body interaction. For low-energy electron scattering by noble-gas atoms, this approximation has met with considerable success.[11–13] In this model, scattering of the electron is represented by an interaction which is simply the sum of the potential due to the average electron charge distribution of the unperturbed atom and that arising from the electric dipole moment induced in the atom by the field of the electron. This electric polarization of the atom by the incoming electron is taken into account by an attractive term which at large separations, behaves as $\frac{1}{2}\alpha/R^4$, where α is the classical electric polarizability of the atom. Early work by Holtsmark[11] showed that a two-body interaction consisting of such a potential was able to reproduce the Ramsauer effect in argon and other noble gases in some detail. Much of the recent work on the scattering of electrons by noble gases has confirmed this point of view.[12, 13]

In this paper, the relationship between the measured electric field dependent positron lifetimes and the velocity-dependent momentum-transfer cross section and annihilation rate is treated in detail. In addition, such cross sections are calculated for the case of positrons in argon according to the simple model discussed above. Several variations of this model were examined in order to determine the applicability of this method for the case of positron scattering.

The diffusion analysis used for relating the theoretical momentum-transfer cross sections and annihilation rates to the experimental results is presented in Sec. III. Values of the velocity-dependent cross sections, calculated using the simplified model discussed above, are given in Sec. IV. In Sec. V the results of a diffusion analysis based on these cross sections is compared to the experimental results and the observed discrepancies are discussed. Finally, it is shown that use of the experimental results from both electric field and temperature-dependent measurements yield a model-independent estimate of the low-velocity momentum-transfer cross section for positrons in argon.

II. OUTLINE OF PROCEDURE

In order to compute the effect of the electric field on the annihilation rate of positrons in a gas both the velocity-dependent annihilation rate $\nu_a(v)$ and the momentum-transfer cross section $\sigma_d(v)$ are needed. These may be obtained for some specific model of the positron-atom interaction by solving the appropriate Schrödinger equation. The velocity distribution of positrons under the influence of an electric field in a gas at temperature T is then determined by solving a diffusion equation. The differential equation describing such a situation has been described by a number of authors,[14–16] and its numerical solution for the case of positrons in argon have been described in preliminary reports[4,17] from this laboratory.

Once the velocity distribution is obtained for a particular $\sigma_d(v)$, $\nu_a(v)$ and electric field, the mean annihilation rate λ is found by averaging the velocity-dependent annihilation rate, ν_a over the entire velocity distribution. It should be emphasized that the specific model considered here for obtaining $\nu_a(v)$ and $\sigma_d(v)$ neglects any contribution to the annihilation rate from radiative capture of the positron into a bound Ar-e^+ system, since neither the existence nor probability of formation of such bound states has been discussed in any quantitative way in the literature. Such a process would, in our experiment, be indistinguishable from the direct annihilation mechanism considered.

III. SOLUTION OF THE DIFFUSION EQUATION

A. General Discussion

The differential equation describing the velocity distribution of positrons in a gas under the influence of a uniform electric field is[14,17,16]

$$\frac{\partial f(v,t)}{\partial t}=\frac{1}{v^2}\frac{\partial}{\partial v}\left[\left(\frac{a^2v^2}{3\nu_d}+\frac{\mu v^2\nu_d KT}{m}\right)\frac{\partial f(v,t)}{\partial t}+\mu v^3\nu_d f(v,t)\right]-\nu_a f(v,t), \tag{1}$$

where $f(v,t)$ is the probability density in velocity space at time t, $a=eE/m$ is the acceleration experienced by a positron of mass m and charge e in an electric field of strength E, $\nu_d(v)=vn_s\sigma_d(v)$ with $\sigma_d(v)$ the momentum-transfer cross section for a collision between a positron of velocity v and a gas atom, n_s is the density of scattering atoms, $\nu_a(v)=\pi r_0^2 cn_s\zeta(v)$ [see Eq. (23)], r_0 is the classical electron radius, c is the velocity of light, $\mu=m/M$ with M the mass of a scattering atom, T is the absolute temperature of the host gas, and K is Boltzmann's constant.

Equation (1) is applicable in the energy range in which only elastic collisions between the positron and gas atom need be considered. The function

$$y(v,t)=v^2f(v,t) \tag{2}$$

is the velocity distribution of positrons per unit velocity interval. The boundary conditions for $y(v,t)$ are

$$y(0,t)=0,\quad y(\infty,t)=0, \tag{3}$$

for all physically meaningful momentum-transfer and annihilation cross sections.[15] It was found convenient when performing numerical integration of these equations to have the equations expressed in terms of y rather than f. With this change of variable, Eq. (1) becomes

$$\frac{\partial y(v,t)}{\partial t}=\frac{\partial}{\partial v}\left[\left(\frac{a^2}{3\nu_d}+\frac{\mu\nu_d KT}{m}\right)\frac{\partial y(v,t)}{\partial v}+\left(\mu v\nu_d-\frac{2a^2}{3\nu_d v}-\frac{2\mu\nu_d KT}{mv}\right)y(v,t)\right]-\nu_a y(v,t). \tag{4}$$

If the initial positron velocity distribution $y(v,0)$ is known, Eq. (4) can be solved to yield the positron velocity distribution at any time t.

$$\lambda(t)=\int_0^\infty y(v,t)\nu_a(v)dv\left[\int_0^\infty y(v,t)dv\right]^{-1} \tag{5}$$

is the velocity-averaged direct annihilation rate as a function of time. It is the quantity which corresponds to the experimentally determined annihilation rate.[4,5] Although preliminary analyses based on this time-dependent rate have been reported,[4] the results depend on the assumptions made regarding the $y(v,0)$ for which there is no experimental data. For this reason, most of our attention has been devoted to an examination of the equilibrium solutions of (4). As these have no implicit time dependence, they are thus independent of $y(v,0)$.

B. Equilibrium Solution

While the velocity distribution relaxes from its initial shape $y(v,0)$ to that corresponding to thermal equilibrium, the value of $\lambda(t)$ varies, this variation being responsible for the shoulder in the experimental lifetime results. When equilibrium is attained, λ becomes a constant, the value of which depends on the shape of the distribution. This is assumed to be the situation characterizing the exponential portion of the free positron annihilation spectrum; this portion is described by a velocity distribution of constant shape, but with an amplitude that decreases exponentially with time.

The shape of the equilibrium distribution $Y(v)$ is obtained by solving (4) after the separation of variables

$$y(v,t)=T(t)Y(v) \tag{6}$$

which, on insertion into Eq. (4), yields

$$\frac{d}{dv}\left[\left(\frac{a^2}{3\nu_d}+\frac{\mu\nu_d KT}{m}\right)\frac{dY}{dv}+\left(\mu v\nu_d-\frac{2a^2}{3v\nu_d}-\frac{2\mu\nu_d KT}{mv}\right)Y\right]=\nu_a Y-\lambda Y, \tag{7}$$

and $$dT/dt = -\lambda T. \tag{8}$$

The combined solution to (7) and (8), $y(v,t) = Y(v)T(0)\exp - \lambda t$, is clearly exponential in t. The value of λ is found by integration of (7) with respect to v over all velocities.

$$\left[\left(\frac{a^2}{3\nu_d} + \frac{\mu\nu_d KT}{m}\right)\frac{dY}{dv} + \left(\mu v\nu_d - \frac{2a^2}{3v\nu_d} - \frac{2\mu\nu_d KT}{mv}\right) Y\right]_0^\infty = \int_0^\infty (\nu_a - \lambda) Y dv. \tag{9}$$

It is easy to show that the left-hand side of Eq. (9) is equal to zero. For potentials of the type used here, the momentum-transfer cross section tends to a constant value as the incident velocity tends to zero.[18] Furthermore, the probability density of positrons in velocity space $f(v,t)$ is finite at $v = 0$.[15] Thus $Y(v)$ approaches zero at least as rapidly as v^2. For high velocities, the momentum-transfer cross section is expected to remain finite. In addition, in order for the overall energy associated with the distribution $Y(v)$ to remain finite, Y must approach zero at high velocities more rapidly than v^{-3}.

With the left-hand side of Eq. (9) equal to zero,

$$\lambda = \int_0^\infty \nu_a(v) Y(v) dv \left[\int_0^\infty Y(v) dv\right]^{-1}. \tag{10}$$

This is the expression expected, of course, from Eq. (5). The quantity Z_{eff} is obtained by dividing Eq. (10) by $\pi r_0^2 cn_s$.

$$Z_{\text{eff}} = \int_0^\infty \zeta(v) Y(v) dv \left[\int_0^\infty Y(v) dv\right]^{-1}.$$

C. Numerical Solution of the Diffusion Equation

For a given set of velocity-dependent cross sections, the solutions to (7) and (10) can be readily obtained using a digital computer. Integrating (7) from $v' = 0$ to $v' = v$,

$$\left(\frac{a^2}{3\nu_d} + \frac{\mu\nu_d KT}{m}\right)\frac{dY}{dv} + \left(\mu v\nu_d - \frac{2a^2}{3v\nu_d} - \frac{2\mu\nu_d KT}{mv}\right) Y = \int_0^v (\nu_a - \lambda) Y dv. \tag{11}$$

The lower limit of the integral has been set equal to zero as dicussed in the previous section. If the velocity dependence of the annihilation rate is such that the right-hand side of Eq. (11) is small compared to the terms on the left-hand side [generally, this requires that $(\nu_a - \lambda) < \mu\nu_d$], the effect of the annihilation term on the velocity distribution is that of a perturbation. In argon, $\nu_a \sim 5\times 10^6$ $\text{sec}^{-1}\,\text{amagat}^{-1}$, whereas $\mu\nu_d \sim 10^8$ $\text{sec}^{-1}\,\text{amagat}^{-1}$ at thermal energies.

A zero-order solution to Y is obtained by solving (11) with the right-hand side set equal to zero. This yields

$$Y_0 = Cv^2 \exp\left[-\int_0^v \mu v\nu_d \left(\frac{a^2}{3\nu_d} + \frac{\mu\nu_d KT}{m}\right)^{-1} dv\right], \tag{12}$$

where C is determined by normalizing the integral of Y_0 over all velocities to unity. The zero-order value for λ is found using Eq. (10). Approximating (11) by the expression

$$\left(\frac{a^2}{3\nu_d} + \frac{\mu\nu_d KT}{m}\right)\frac{dY_j}{dv} + \left(\mu v\nu_d - \frac{2a^2}{3v\nu_d} - \frac{2\mu\nu_d KT}{mv}\right) Y_j = \int_0^v \left(\nu_a - \lambda_{j-1}\right) Y_{j-1} dv, \tag{13}$$

thus permits an iterative solution of Y in which the nth order of Y, Y_n is calculated in terms of the preceding Y_{n-1} and λ_{n-1}. This iterative procedure has been successfully used to solve Eq. (7) for the case of positrons in argon. In all cases, the procedure converged within five iterations for the cross sections used. The program was tested by comparing the numerical values to an analytic solution for the special case of a constant scattering rate.

IV. CROSS-SECTION CALCULATIONS FOR THE POSITRON-ARGON INTERACTION

In this paper, results presented earlier[19–21] in a preliminary way are discussed in detail, the analysis being performed in a similar way to that of Massey *et al.*[1] The analysis was carried out only for the special model due to Holtsmark[11] in which the interaction of the positron with an argon atom is represented in terms of an effective potential

$$V(r) = Z_p(r)/r + V_p(r). \tag{14}$$

Z_p/r is the Hartree-Fock potential of the unperturbed atom,[22] and $V_p(r)$ is an attractive polarization interaction which has the asymptotic behavior

$$V_p(r) \to -\alpha e^2/2r^4, \quad \text{as } r \to \infty, \tag{15}$$

where α is the polarizability of the argon atom ($11a_0^3$, in atomic units).

The problem thus reduces to solving the radial part of the relevant Schrödinger equation, which, in atomic units, is

$$\left(\frac{d^2}{dr^2} + k^2 - \frac{l(l+1)}{r^2} - 2V(r)\right)\chi_l(r) = 0, \tag{16}$$

where k^2 is the kinetic energy of the incident positron (in Rydberg units), and $V(r)$ is in atomic units (e^2/a_0).

(a) Calculation of Phase Shifts and Wave Functions

Equation (16) was solved numerically using the Runge-Kutta method. In order to save computing time, the integration was not carried out to values of r at which the solution attains its asymptotic sinusoidal character, but only to values for which the polarization potential $V_p(r)$ was negligible compared to k^2 and $l(l+1)/r^2$. In this region, the radial wave function $\chi_l(r)$ is given by[23]

$$\chi_l(r) = kr[A_l j_l(kr) - B_l n_l(kr)]. \qquad (17)$$

In the limit $kr \gg 1$, this solution approaches the standard form

$$\chi_l(r) = C_l \sin(kr - \tfrac{1}{2}l\pi + \delta_l). \qquad (18)$$

The A_l and B_l are in turn related to χ_l and $d\chi_l/dr$ as follows:

$$A_l = \chi_l[(l+1)n_l(kr) - krn_{l+1}(kr)] - r(d\chi_l/dr)n_l(kr) \qquad (19)$$

$$B_l = \chi_l[(l+1)j_l(kr) - krj_{l+1}(kr)] - r(d\chi_l/dr)j_l(kr).$$

Since the phase shift δ_l and normalization constant C_l are themselves functions of A_l and B_l, with

$$\tan\delta_l = B_l/A_l\,, \qquad (20)$$

and

$$C_l^2 = A_l^2 + B_l^2\,, \qquad (21)$$

they can both be expressed directly in terms of χ_l and $d\chi_l/dr$.

The above discussion applies for $k \neq 0$. For $k = 0$, a similar method was used, the details of which are given elsewhere.[24]

The total momentum-transfer cross sections σ_d (in units of πa_0^2) were determined from the phase shifts using[25]

$$\sigma_d = \frac{4}{k^2} \sum_{l=0}^{\infty} (l+1)\sin^2(\delta_l - \delta_{l+1}). \qquad (22)$$

In practice, all the phase shifts for partial waves up to and including $l = 5$ were calculated. The higher-order phase shifts were negligible in all cases.

(b) Determination of the Direct Annihilation Rate

The annihilation rate of a free positron in a gas is proportional to the atomic density of the gas and to the product of the electron and positron densities in the atom averaged over the positron position. This annihilation rate is given by[26]

$$\nu_a(k) = \pi r_0^2 c n_s \zeta(k), \qquad (23)$$

where r_0 is the classical electron radius, n_s the gas density, c the velocity of light, and

$$\zeta(k) = \int_{\substack{\text{all}\\ \text{space}}} d^3\vec{x}\, |\psi^-|^2 |\psi^+|^2 . \qquad (24)$$

In terms of the χ_l, ψ^+ is given by (for $k \neq 0$)

$$\psi^+ = \sum_{l=0}^{\infty} \frac{\chi_l}{kr} P_l(\cos\theta), \qquad (25)$$

where the P_l are Legendre polynomials. For the spherically symmetric electron distributions characterising the noble-gas atoms, integration over angle in (24) gives

$$\zeta(k) = \sum_{l=0}^{\infty} \zeta_l(k), \qquad (26)$$

where

$$\zeta_l(k) = \int_0^{\infty} |\psi^-|^2 (2l+1) \frac{\chi_l^2(kr)}{k^2 r^2}\, dr. \qquad (27)$$

The electron density $|\psi^-|^2$ in all the calculations presented here is that appropriate to the argon atom in its ground state as given by Hartree and Hartree.[22]

V. RESULTS

Preliminary results based on the procedure outlined above have been reported.[19–21] In one of them,[19] however, the results were obtained with two additional computational simplifications. One was that the total elastic scattering rather than momentum-transfer cross sections were used in the velocity-distribution calculations, and the other was that only the zero-order Y_0 were calculated.

Three slightly different forms of the positron-argon interaction were studied. All exhibited the required $1/R^4$ behavior at large distances, but differed in the detailed shape of the polarization term in the neighborhood of the atom. Specifically, the total interaction considered was the sum of the Hartree potential for argon[22] V_H and an empirical polarization potential V_p. Results were calculated for each of the following three estimates of V_p:

$$\text{(A)}\quad V_p^{\mathrm{A}} = -\tfrac{1}{2}\alpha(r^2 + r_0^2)^{-2};\quad r_0^2 = 2.5a_0^2;$$

$$\text{(B)}\quad V_p^{\mathrm{B}} = -\tfrac{1}{2}\alpha(r^2 + r_0^2)^{-2};\quad r_0^2 = 0.62a_0^2; \qquad (28)$$

$$\text{(C)}\quad V_p^{\mathrm{C}} = -\tfrac{1}{2}\alpha r^{-4}(1 - e^{(-r/r_0)^8});\quad r_0^8 = 14a_0^8.$$

In all three cases, a polarizability α of $11.0a_0^3$ was used.

In V_p^{A}, r_0^2 was chosen to be the value found to fit low-energy electron scattering in argon.[12] The same effective interaction was employed by Massey *et al.*[1] in their calculations of the momentum-transfer cross section and $\zeta(v)$ for positrons in argon.

Figures 1 and 2 show respectively the velocity dependence of ζ and the momentum-transfer cross section resulting from the solution of the relevant Schrödinger equation. The values obtained for V_p^{A} are in agreement with those reported by Massey *et al.*[1]

The results of the velocity-distribution calculations described in III are presented in Fig. 3. Comparison with the experimental results shows that the calculated annihilation rate (A) is far too small. Furthermore, the theoretical curve does not vary sufficiently rapidly with applied electric field.

In order to enhance the low-velocity positron annihilation rate, the cut-off parameter r_0^2 was reduced to the value $0.62a_0^2$ as indicated in (B). Reduction of the parameter r_0^2 by such a large amount, however, causes a correspondingly increased momentum-transfer cross section.

The potential described by V_p^{C} is also a one-parameter potential, but is a better representation of the short-range part of the polarization potential when incident electrons[27,28] are involved. The size of the parameter $r_0^8 = 14a_0^8$ was chosen so that the ζ at zero velocity matched that obtained using V_p^{B}. Again from Fig. 2, the momentum-transfer cross section is very large at $k=0$, and falls off rapidly to a fairly constant value. Although the parameters in both V_p^{B} and V_p^{C} were chosen to yield results approximating the experimental ones at zero electric field, neither potential adequately accounts for the observed rapid decrease in annihilation rate with increasing field. An important feature in both curves B and C of Fig. 3 is the "break" which occurs at about 70 V cm^{-1} amagat^{-1} and 120 V cm^{-1} amagat^{-1}, respectively. For smaller electric fields the average positron velocity changes slowly with increasing electric field, while for fields larger than that at the "break," the average velocity increases rapidly with increasing electric field. This behavior demonstrates the dependence of the velocity distribution on the relative sizes of the two terms in the coefficient of $\partial f/\partial v$ of Eq. (1). The *E/D* at the "break" for curve C is greater than the value for B because the low velocity momentum-transfer cross section is larger for C than for B.

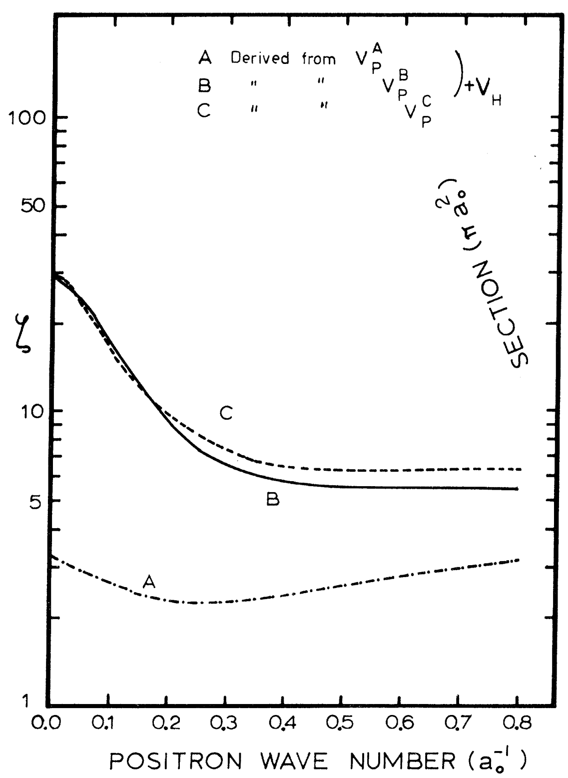

FIG. 1. Velocity dependence of the positron annihilation rate for three different representations of the positron–argon interaction. ζ is the effective atomic number of the atom, defined by Eq. (24).

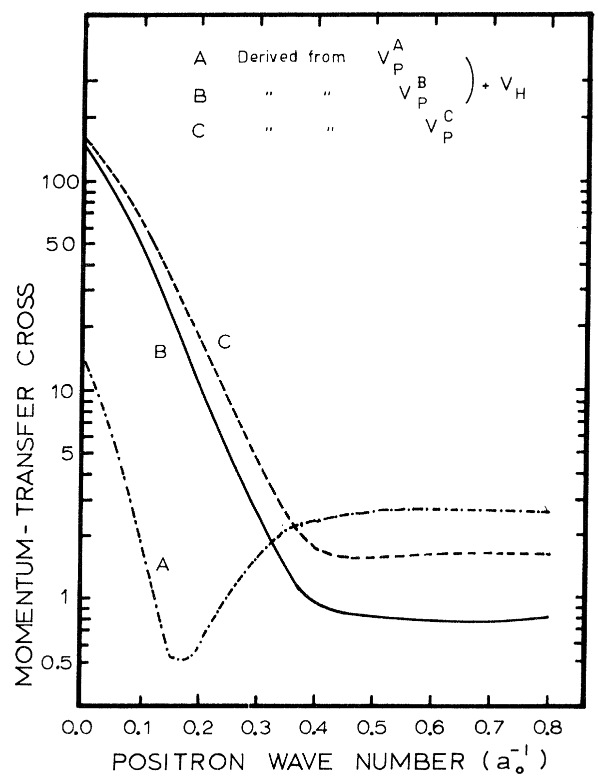

FIG. 2. Velocity dependence of the momentum-transfer cross section for three different representations of the positron–argon interaction.

VI. DISCUSSION

At low values of the electric field, the positron velocity distribution is a very weak function of field because the energy-gain term in Eq. (1) is much smaller than the elastic scattering term. That is,

$$a^2/3\nu_d \ll \mu\nu_d KT/m\,. \qquad (29)$$

Independent information concerning the velocity dependence of $\zeta(v)$ in this energy range can be obtained through measurements of the temperature dependence of the annihilation rate.

At room temperature, the results of such measurements for the case of positrons in argon,[29] can be summarized as follows:

$$d\lambda/\lambda \simeq -(0.30\pm0.015)dT/T\,. \qquad (30)$$

The change in the annihilation rate is interpreted as the result of the change in the positron velocity distribution brought about by the change in temperature, even though the shape of the distribution remains essentially Maxwell-Boltzmann. For an incremental change, ΔT (at zero electric field), the only change in Eq. (1) is a corresponding increment of the coefficient of the $\partial f/\partial v$ term by $\mu v^2\nu_d K\Delta T/m$. If, instead of a change of temperature, an electric field is applied, the corresponding increment in the coefficient of $\partial f/\partial v$ is $a^2v^2/3\nu_d$. If it is assumed that for small values of the electric field ["small" as defined by Eq. (29)], the shape of the velocity distribution remains essentially Maxwell-Boltzmann, then an experimental comparison of the relative effect of these two changes enables an estimate to be made of the value of the momentum-transfer scattering rate ν_d at thermal velocities. Clearly, this estimate would be exact if ν_d was velocity independent in this energy interval.

Fitting the low-field results of Fig. 3 to a quadratic yields

$$\Delta(\lambda/D)/\Delta(E/D)^2 \simeq -3.70\times10^3, \quad \text{at } E/D=0. \qquad (31)$$

Combining the results of (30) and (31) (at a density of 1 amagat and a temperature of 298 °K),

$$\Delta T/\Delta(E^2) \simeq 0.65\,\mathrm{K}^\circ\,(\mathrm{V/cm})^{-2}. \qquad (32)$$

From Eq. (1), a comparison of the relative magnitudes of the two terms in the coefficient of $\partial f/\partial v$ leads to

$$\Delta T/\Delta(E^2) = e^2/3\mu\nu_d^2 mK. \qquad (33)$$

Solving these two equations gives

$$\nu_d = 8.5\times10^{11}\ \mathrm{sec}^{-1}\ \text{(at 1 amagat)},$$

or $\sigma_d = 33\pi a_0^2$.

In view of the approximate nature of this calculation, a probable error of $\pm 5\pi a_0^2$ seems reasonable.

The only estimate for this cross section at thermal energies, that has been published previously, is that of Falk *et al.*[4] who obtained a value of $\sigma_d \simeq 20\pi a_0^2$. This result was deduced from fits of the detailed shape of annihilation lifetime spectra using a diffusion analysis in which a constant scattering rate was assumed. In the present calculation, no specific velocity dependence is assumed other than that, on application of small electric fields, no significant deviation from a Maxwell-Boltzmann shape occurs.

VII. CONCLUSIONS

The electric field dependence of the direct annihilation rate in argon has been used as a test of simple models of the interaction of positrons with multi-electron atoms. We have shown that, although incapable of providing explicit information regarding the separate velocity dependences of the momentum transfer and annihilation cross sections, the experimental measurement of the annihilation rate of free positrons in argon as a function of electric field strength does, together with an appropriate diffusion analysis, offer a stringent test of theoretical models of the positron-argon atom interaction.

It has been shown that the one-parameter approximations to the effective interaction between electrons and atoms, approximations which have been found to yield a reasonable fit to the low-energy, electron elastic scattering data, are inadequate to account in detail for the scattering of positrons by argon atoms. The rapid decrease in the direct annihilation rate observed at low values of applied electric field cannot be reproduced using the momentum-transfer cross sections and annihilation rates derived from such potentials.

Finally, an analysis based on the relative mag-

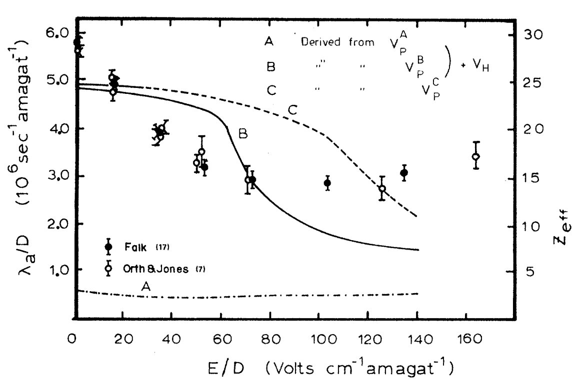

FIG. 3. Electric field dependence of the positron annihilation rate for three different representations of the positron-argon interaction. The circles are experimental values.

nitudes of the effect of temperature compared to that of electric field on the lifetime of positrons in argon gas yields a value of $(33 \pm 5)\pi a_0^2$ for the momentum-transfer cross section of positrons in argon at thermal energies.

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