$\hbar \to 0$) would we have

$$M+N=(n+\tfrac{1}{2}m-1)v+\tfrac{1}{2}(b_e+f_e)\,, \qquad \text{(A5)}$$

and then the contributions of the loop diagrams would vanish in the limit $\hbar \to 0$, in accordance with Refs. 1 and 2. But in this case, that is not a surprising result, because $e \to 0$.

The following formulas were used in the text:

$$2f_i+f_e=2nv\,, \qquad \text{(A6)}$$

$$2b_i+b_e=mv\,. \qquad \text{(A7)}$$

They connect f_e, f_i and b_e, b_i to v in theories whose interaction Lagrangian is given by Eq. (1), and can be easily proved.

PHYSICAL REVIEW VOLUME 183, NUMBER 5 25 JULY 1969

Remarks on the Kinematics of Multiperipheral Processes

Shau-Jin Chang* and R. Rajaraman†
Institute for Advanced Study, Princeton, New Jersey 08540

(Received 13 March 1969)

We study the implications of requiring that the "subenergies" s_i and the momentum transfers t_i of a multiple production process be in the Regge domain. We show that even for very high-energy accelerator or cosmic-ray processes, the s_i are too small compared to the t_i and the masses for one to apply a multi-Regge formalism for the two-particle $\to$ n-particle amplitude. Some alternatives are suggested.

THE problem of highly inelastic hadron scattering with the production of multiparticle final states has been one of increasing interest in recent years. The advent of newer and more energetic accelerators, as well as improvement of cosmic-ray data, is likely to render the study of such multiparticle processes even more interesting in the future. In particular, the "multi-Regge" models for describing these processes have been receiving much attention lately.[1–3] We wish to present here some remarks about the kinematics of such many-pronged processes (Fig. 1). These remarks are admittedly simple-minded, but are nevertheless very reasonable and relevant to the use of the multi-Regge formalisms.

The experimental situation on the asymptotic behavior of multiproduction amplitudes is, of course, not as good as that for elastic amplitudes. On the one hand, accelerator data,[4] while amenable to detailed analysis, are not really asymptotic (at $E_{\mathrm{lab}}\sim 10$–30 GeV) in all the subenergies of a multiparticle process. Cosmic-ray data,[5] on the other hand, are available up to lab energies in hundreds of TeV (1 TeV$=10^3$ GeV), where one can truly hope to be in the asymptotic region, but the analysis of these events is understandably not as detailed as in accelerator events.

Nevertheless, two gross features of multiparticle processes stand out fairly indisputably from both these sources of data:

(i) The transverse momenta of the outgoing particles do not increase appreciably with increasing incoming energy. This appears to be valid up to $E_{\mathrm{lab}}\sim 10^4$ GeV.[5] The average value of the $\langle p_\perp \rangle$ for pions is around 350 MeV.[6]

(ii) The average multiplicity of pions increases logarithmically[5] with $S=(W_{\mathrm{c.m.}})^2$, according to

$$n\simeq\frac{\log(S/\mu^2)}{\log 1.9}+C\,, \quad \text{or} \quad \frac{S}{\mu^2}=A(1.9)^n\,, \qquad (1)$$

where $W_{\mathrm{c.m.}}$ is the total energy in the center-of-mass system and μ is the pion mass.

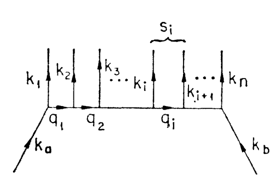

FIG. 1. A multiple production process with two particles $\to n$ particles.

* Supported in part by the National Science Foundation. Address after September 1, 1969: Department of Physics, University of Illinois, Urbana, Ill.

† Research supported by the Air Force Office of Scientific Research, Office of Aerospace Research, U. S. Air Force, under AFOSR Grant No. 68-1365.

[1] G. F. Chew and A. Pignotti, Phys. Rev. **176**, 2112 (1968); Phys. Rev. Letters **20**, 1078 (1968). A complete list of references can be found in these two papers.

[2] G. F. Chew, M. L. Goldberger, and F. E. Low, Phys. Rev. Letters **22**, 208 (1968).

[3] H.-M. Chan, J. Loskiewicz, and W. W. M. Allison, Nuovo Cimento **57A**, 93 (1968).

[4] For a discussion of the accelerator data, see O. Czyzewski, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 367.

[5] A recent survey of cosmic-ray data presented at the Tenth International Conference on Cosmic Rays, Calgary, 1967, is available in Can. J. Phys. **46** (1968). In particular, see C. A. B. McKusker *et al.*, *ibid.* **46**, S655 (1968); M. Akashi *et al.*, *ibid.* **46**, S660 (1968).

[6] See also G. Cocconi, Nuovo Cimento **57**, 837 (1968).

In this paper we will use only these two pieces of experimental information. Consider a two-particle $\rightarrow$ n-particle process as shown in Fig. 1. Assume for simplicity that all the particles are pions (although our conclusions will be valid for large n even if the incoming particles included a proton or a kaon). Define the usual subvariables $s_i=(k_i+k_{i+1})^2$ and $t_i=q_i^2$. Given that the total $S=(k_a+k_b)^2$ is large, let us first investigate the consequence of the requirement that $s_i\gg t_i$ and $s_i\gg 4\mu^2$, under which circumstances a multi-Regge form may be attempted without invoking any form of duality. Let $\mathbf{k}_i=(k_{iz},\mathbf{k}_{i\perp})$, where we are working in the c.m. frame and $\mathbf{k}_a=-\mathbf{k}_b$ is along the positive z axis. We will find it convenient to use the variable $\eta=k_0+k_z$ for each of the $n+2$ particles. The η's have some nice properties. First, they are always positive even as k_z varies from large positive to large negative values. Also,

$$\eta_a=k_{a0}+k_{az}\gg\mu\,,\quad \eta_b=k_{b0}+k_{bz}=k_{a0}-k_{az}=\mu^2/\eta_a\ll\mu\,,$$
$$k_{i0}-k_{iz}=(\mu^2+k_{i\perp}^2)/\eta_i\,,\quad i=1,\cdots,n \tag{2}$$
$$\eta_1+\eta_2+\cdots\eta_n=\eta_a+\eta_b\simeq\eta_a\,,$$

and η_i/η_j is invariant with respect to Lorentz transformations along the z axis, and is therefore the same in lab and c.m. frames. Further, the problem is symmetric with respect to the variables η_i and $1/\eta_i$, whose exchange simply reverses the z axis and exchanges the roles of particles a and b.

In terms of these η's, we have

$$s_1=(k_1+k_2)^2=(k_{10}+k_{20})^2-(k_{1z}+k_{2z})^2-(\mathbf{k}_{1\perp}+\mathbf{k}_{2\perp})^2$$
$$=(\eta_1+\eta_2)\left(\frac{\mu^2+k_{1\perp}^2}{\eta_1}+\frac{\mu^2+k_{2\perp}^2}{\eta_2}\right)-(\mathbf{k}_{1\perp}+\mathbf{k}_{2\perp})^2 \tag{3a}$$

and

$$t_1=(p_a-k_1)^2=(\eta_a-\eta_1)\left(\frac{\mu^2}{\eta_a}-\frac{\mu^2+k_{1\perp}^2}{\eta_1}\right)-k_{1\perp}^2\,. \tag{3b}$$

If $s_i\gg4\mu^2$, since the $k_\perp$'s are all roughly constant with increasing energy, we see from (3) that either η_1/η_2 or η_2/η_1 must be much greater than unity. But if $\eta_2/\eta_1\gg1$, rendering η_1 much smaller than η_a, then $|t_1|$ from Eq. (3b) would be large. Thus, requiring $s_1\gg4\mu^2$ and $s_1>|t_1|$ leads to $\eta_1\gg\eta_2$. Similarly, if

$$s_2=(\eta_2+\eta_3)\left(\frac{\mu^2+k_{2\perp}^2}{\eta_2}+\frac{\mu^2+k_{3\perp}^2}{\eta_3}\right)-(\mathbf{k}_{2\perp}+\mathbf{k}_{3\perp})^2\gg4\mu^2\,, \tag{4}$$

we have either η_2/η_3 or $\eta_3/\eta_2\gg1$ as before. But

$$t_2=(p_a-k_1-k_2)^2$$
$$=(\eta_a-\eta_1-\eta_2)\left(\frac{\mu^2}{\eta_a}-\frac{\mu^2+k_{1\perp}^2}{\eta_1}-\frac{\mu^2+k_{2\perp}^2}{\eta_2}\right)-(\mathbf{k}_{1\perp}+\mathbf{k}_{2\perp})^2$$
$$=(\eta_3+\eta_4+\cdots)\left(\frac{\mu^2}{\eta_a}-\frac{\mu^2+k_{1\perp}^2}{\eta_1}-\frac{\mu^2+k_{2\perp}^2}{\eta_2}\right)-(\mathbf{k}_{1\perp}+\mathbf{k}_{2\perp})^2\,. \tag{5}$$

If η_3/η_2 were $\gg1$, since all the η's are positive, $|t_2|$ would be large and comparable to s_2. Thus we are led to $\eta_2/\eta_3\gg1$. Proceeding thus, on requiring that each $s_i\gg4\mu^2$ and $s_i>|t_i|$, we obtain $\eta_i/\eta_{i+1}\gg1$, for all i.

Thus, to be assured of being in the Regge region for each adjacent pair, the particles must be arranged in decreasing order of η; furthermore, the ratio of the adjacent η's, namely, η_i/η_{i+1}, must be large. This ratio is invariant with respect to boosts along the z axis. In particular, in the c.m. frame, as η decreases from η_1 to $\eta_n\simeq0$, this corresponds to an algebraically decreasing sequence of k_z, varying from a large positive k_{1z} to a large negative k_{nz}.

We can obtain very easily an estimate of how well the observed events fulfill the criterion for being in the Regge region. We have

$$S=(p_a+p_b)^2=(\eta_a+\eta_b)\left(\frac{1}{\eta_a}+\frac{1}{\eta_b}\right)\mu^2\approx\mu^2\frac{\eta_a}{\eta_b}$$
$$=\mu^2\frac{\eta_a}{\eta_1}\left(\frac{\eta_1}{\eta_2}\frac{\eta_2}{\eta_3}\cdots\frac{\eta_{n-1}}{\eta_n}\right)\frac{\eta_n}{\eta_b}$$
$$\approx\mu^2\frac{\eta_a}{\eta_1}\frac{\eta_n}{\eta_b}\left(\frac{s_1}{\mu^2+k_{2\perp}^2}\right)\left(\frac{s_2}{\mu^2+k_{3\perp}^2}\right)\cdots\left(\frac{s_{n-1}}{\mu^2+k_{n\perp}^2}\right), \tag{6}$$

where use is made of Eqs. (3) and (4) for large s_i. For purposes of estimation, let us set all $k_{i\perp}$ equal to $k_\perp$, which is the experimental average of 350 MeV. Let all the s_i be equal, and consequently all η_i/η_{i+1} be equal. Note that since $\eta_a=\eta_1+\eta_2+\cdots+\eta_n$ and $\eta_1\gg\eta_2\gg\eta_3\cdots\gg\eta_n$, we have $\eta_a\approx\eta_1$. Similarly, $\eta_b\approx\eta_n$. Under these conditions Eq. (6) gives

$$S\approx\mu^2\left(\frac{\eta_i}{\eta_{i+1}}\right)^{n-1},\quad\text{or}\quad n-1\simeq\frac{\log(S/\mu^2)}{\log(\eta_i/\eta_{i+1})}\,. \tag{7}$$

Then, on assuming that pions are produced at a rate which maintains fairly constant s_i even as S increases, one gets the observed logarithmic law. Further, we know the coefficient of the $\log S$ dependence to be about $(\log1.9)^{-1}$ from experiment [Eq. (1)]. Thus

$\eta_i/\eta_{i+1}\approx 1.9$; and from Eqs. (3) and (4),

$$s_i \simeq (\mu^2+k_\perp^2)(1.9+1/1.9+2)-2k_\perp^2 \simeq 20\mu^2, \tag{8}$$

which can hardly be considered as being in the Regge region. The precise value in Eq. (8), of course, is not to be taken literally, since it was derived under the assumption that $s_i \gg 4\mu^2$. A more careful evaluation of the smaller s_i will depend on the relative orientations of the $\mathbf{k}_{i\perp}$ [see Eq. (4)], and can vary from $10\mu^2$ to $30\mu^2$. It can also be easily checked that the t_i are comparable to the s_i. Thus, regardless of the precise values of s_i and t_i, our derivation does show that the mean multiplicity observed in experiment as a function of S [given approximately by Eq. (1)] is not compatible with large s_i (i.e., large enough to be in the Regge domain). Thus, for the majority of two-particle $\rightarrow n$-particle events, even in ultra-high-energy cosmic-ray-type processes, a multi-Regge amplitude cannot be used for the differential cross sections.

We wish to emphasize that this is not a blanket criticism of the multi-Regge theory. We are aware that if one takes a *fixed n* and goes to increasing S, the s_i will eventually be in the Regge domain, and we would have no objection to attempting a multi-Regge fit to these events. Such events, of course, can be seen from our remarks to be a minority at that energy. In addition, even when the s_i are small, as Chew and Pignotti pointed out, a Regge amplitude may perhaps be used in *an average sense* for purposes of multiperipheral bootstrap, etc., if nature is kind enough to exhibit duality. Our remarks only refer to the inadvisibility of using multi-Regge theory to fit the general $2\rightarrow n$ scattering amplitude at high energies. However, it must be remembered that one of the ingredients in the execution of Chew and Pignotti's bootstrap model is the use of an asymptotic relation, which amounts (on translating from Toller variables) to

$$S \propto s_1 s_2 \cdots s_{n-1}. \tag{9}$$

This relation is what leads to an S^α dependence from the Regge amplitude of the form $\prod_i (s_i)^{\alpha_i}$. But relation (9) is valid only when the s_i are large, and our analysis shows that such is not the case for most of the events. Therefore, apart from the question of invoking duality and using a multi-Regge amplitude for small s_i, bootstrappers must take care not to use Eq. (9).

We conclude by noting that the reason behind the smallness of s_i is that the constant of multiplicity $(\log 1.9)^{-1}$ appearing in the multiplicity relation (1) is too large. In other words, too many particles are being created to give a large s_i. For this reason, let us momentarily assume Regge exchanges between clusters of r pions each (Fig. 2) instead of between individual pions. Then we should apply our considerations to the s_i of the neighboring clusters. For a given S, since the number of clusters is lowered by a factor r, the s_i for clusters will be increased exponentially by a factor r [see Eqs. (6) and (7)]. Thus,

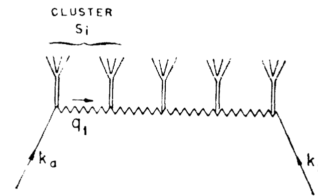

FIG. 2. Possible mechanism of Regge exchange between clusters or "fireballs."

$$s_i^{\text{clusters}} \approx 1.9^r s_i^{\text{pion}}.$$

On the other hand, indications are that the mass of the clusters will increase only linearly with r; therefore, for large enough r, one will be in the "Regge region" even as compared to the cluster masses. One can easily see that if we use ρ (or ω) mesons as candidates for the clusters, the gain due to the squaring (or cubing) of the factor 1.9 is lost as compared to the largeness of the ρ or ω masses squared. But if one uses clusters of four or more pions ("fireballs"), one will have $(s_i)^{\text{cluster}} \gg (m^2)^{\text{cluster}}$. Experimentally, there is strong evidence of such fireballs in cosmic-ray data,[5] and even in some accelerator data.[7] Whether or not one can extend the concept of Regge exchange to work between "fireballs" we cannot say. But if one could, and if one further assumed a spherically symmetric momentum distribution of pions within a fireball (for which there is some evidence[7]), then one could attempt to describe a majority of ultra-high-energy multiparticle production events.

Note that our statement that all the s_i^{pions} cannot be large does not require a detailed analysis of the multiproduction data, but only the gross information about the multiplicity rate, unlike the recent careful analysis of Lipes, Zweig, and Robertson.[8] Incidentally, their analysis does support our conclusion in that they get a substantial fraction of events only on letting one of the invariant masses (m_X in their language) be small, and exchanging Regge trajectories between the subsystem X and other particles. This is in the spirit of what we suggest in general. It should also be remembered that our simple analysis will work better and better as S increases.

[7] L. G. Ratner *et al.*, Phys. Rev. **166**, 1353 (1968).
[8] R. Lipes, G. Zweig, and W. Robertson, Phys. Rev. Letters **22**, 433 (1969).