Electromagnetic Contribution to the Decays $K_S \to l\bar{l}$ and $K_L \to l\bar{l}$*

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Using a model in which the decays $K_{S,L} \to l\bar{l}$ occur through a two-photon intermediate state, and considering only the absorptive part of the amplitudes, we obtain lower bounds on the ratios $\text{Rate}(K_{S,L} \to l\bar{l})/\text{Rate}(K_{S,L} \to \gamma\gamma)$.

1. INTRODUCTION

THE decays $K_S \to l\bar{l}$ and $K_L \to l\bar{l}$ are of interest for several reasons. Both are forbidden to first order in the weak coupling constant w, since couplings of neutral lepton currents to hadrons are believed absent. A search for these decays, therefore, places upper limits on the possible strength of such couplings.[1] At the same time, the decays are not strictly forbidden, and can occur in two distinct ways: by the combined effect of weak and electromagnetic interaction,[2,3] leading to an amplitude of order $w\alpha^2$ (α being the fine-structure constant), and by the weak interaction in second order,[4] with an amplitude of order w^2. The latter contribution is of special interest as it depends on the "weak-interaction cutoff" in those theories which give divergent results for higher-order weak processes.[4,5] A theory has also been proposed in which the second-order weak contribution to these decays is finite.[6] To derive information about the details of the weak interaction from the experimental rates or limits of $K^0 \to l\bar{l}$, it is essential to know the relative importance of the weak-electromagnetic mechanism. In general, the second-order weak and weak-electromagnetic amplitudes will interfere. If, however, the observed rates are considerably higher than the estimates based on the weak-electromagnetic mode, one may attribute the decays to second-order weak effects.

A complete experiment on the decays $K_S \to l\bar{l}$ and $K_L \to l\bar{l}$, would consist of not only a measurement of the decay rates, but also measurement of lepton polarization and $K_L - K_S$ interference. These additional observations, though remote from present possibilities, would yield tests of CP and CPT invariance in these decays.[7] Here again, interpretation of the observations requires some knowledge of the weak-electromagnetic amplitudes.[8]

This paper attempts to give estimates of the weak-electromagnetic decay rates of $K_S \to l\bar{l}$ and $K_L \to l\bar{l}$. We use as our model the decay of the K meson into a pair of virtual photons which then materialize into a pair of leptons. A calculation along these lines involves evaluation of the Feynman diagram shown in Fig. 1, and is beset with the problem of divergent integrals. We note, however, that the Feynman amplitude has an absorptive part which is finite and may be evaluated exactly. It is thus possible to obtain an approximate lower bound on the decay rates of $K_S \to l\bar{l}$ and $K_L \to l\bar{l}$, based on the absorptive part of the amplitude only, if one knows the decay rates of $K_S \to \gamma\gamma$ and $K_L \to \gamma\gamma$. The latter has recently been measured by two groups[9] with the result $\text{Rate}(K_L \to \gamma\gamma)/\text{Rate}(K_L \to \text{all}) = 5 \times 10^{-4}$, corresponding to $\text{Rate}(K_L \to \gamma\gamma) = 1.0 \times 10^4$ sec^{-1}. The decay $K_S \to \gamma\gamma$ has not been detected so far, but a reasonable estimate of its rate can be made, based on the model $K_S \to \pi^+\pi^-(\text{virtual}) \to \gamma\gamma$, which gives $\text{Rate}(K_S \to \gamma\gamma) \simeq 2 \times 10^4$ sec^{-1}.[10] The main results of this paper are embodied in Eqs. (14) and (15), which represent approximate lower bounds on the ratios $\text{Rate}(K_{S,L} \to l\bar{l})/\text{Rate}(K_{S,L} \to \gamma\gamma)$.

The decay $K_L \to l\bar{l}$ was calculated by this method in a previous paper.[3] We have included the results here along with those for the decay $K_S \to l\bar{l}$ to facilitate comparison of the two cases. Throughout the discussion, we treat K_S and K_L as CP eigenstates, and assume CP invariance for the decay amplitudes.

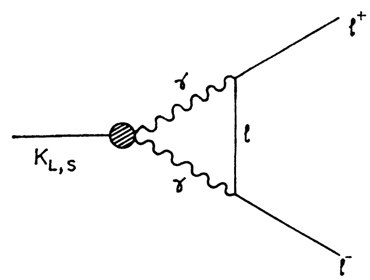

FIG. 1. Feynman diagram for the decays $K_{L,S} \to l^+l^-$.

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2. ELECTROMAGNETIC DECAY RATES

The absorptive part of the amplitude given by the Feynman graph of Fig. 1 is obtained most conveniently through the unitarity relation

$$\mathrm{Abs}\langle l^+l^-|T|K_{L,S}\rangle=\tfrac{1}{2}\sum_{k,k',\epsilon,\epsilon'}\langle l^+l^-|T|\gamma(k,\epsilon)\gamma'(k',\epsilon')\rangle^* \times\langle\gamma(k,\epsilon)\gamma'(k',\epsilon')|T|K_{L,S}\rangle(2\pi)^4\delta^4(Q-k-k'). \quad (1)$$

Here $|\gamma(k,\epsilon)\gamma'(k',\epsilon')\rangle$ is a real two-photon state (on the mass shell) with k, k' and ϵ, ϵ' specifying the 4-momenta and 4-polarizations, respectively. We denote by Q, p_+, and p_- the 4-momenta of the K meson, l^+, and l^-. The symbol $\sum_{k,k',\epsilon,\epsilon'}$ stands for an integration over the photon momenta and a sum over photon polarizations. If the matrix element $\langle l^+l^-|T|\gamma\gamma'\rangle$ is evaluated in the Born approximation, Eq. (1) gives exactly the absorptive part of the Feynman graph (Fig. 1).

Because of CP invariance, the decay of K_S produces an l^+l^- pair with quantum numbers $J=0$, $CP=+1$, corresponding to a ${}^3P_0$ state ($C=P=1$). Only the parity-violating weak interaction takes part here. Similarly, K_L decays into a l^+l^- pair with quantum numbers $J=0$, $CP=1$, which is a ${}^1S_0$ state ($C=1$, $P=-1$). Only the parity-conserving part of the weak interaction contributes in this case. Using CP invariance and gauge invariance, the matrix elements $\langle\gamma\gamma'|T|K_{L,S}\rangle$ may be parametrized as follows:

$$[(2\omega)(2\omega')]^{1/2}\langle\gamma(k,\epsilon)\gamma'(k',\epsilon')|T|K_L\rangle \equiv\langle\gamma(k,\epsilon)\gamma'(k',\epsilon')|\mathfrak{M}|K_L\rangle =2f(K_L\to\gamma\gamma)\epsilon_{\mu\nu\rho\sigma}\epsilon_\mu\epsilon_\nu'k_\rho k_\sigma'/M^2, \quad (2a)$$

$$[(2\omega)(2\omega')]^{1/2}\langle\gamma(k,\epsilon)\gamma'(k',\epsilon')|T|K_S\rangle \equiv\langle\gamma(k,\epsilon)\gamma'(k',\epsilon')|\mathfrak{M}|K_S\rangle =f(K_S\to\gamma\gamma)\epsilon\cdot\epsilon' \quad (2b)$$

(ω, ω' are the photon energies and M the K-meson mass). In the rest frame of the K meson, and using the gauge $\boldsymbol{\varepsilon}\cdot\mathbf{k}=\boldsymbol{\varepsilon}'\cdot\mathbf{k}'=0$, we get

$$\langle\gamma(k,\epsilon)\gamma'(k',\epsilon')|\mathfrak{M}|K_L\rangle=f(K_L\to\gamma\gamma)\boldsymbol{\varepsilon}\times\boldsymbol{\varepsilon}'\cdot\hat{k}, \quad (3a)$$

$$\langle\gamma(k,\epsilon)\gamma'(k',\epsilon')|\mathfrak{M}|K_S\rangle=f(K_S\to\gamma\gamma)\boldsymbol{\varepsilon}\cdot\boldsymbol{\varepsilon}'. \quad (3b)$$

In terms of $f(K_{L,S}\to\gamma\gamma)$, the decay rate of $K_{L,S}\to\gamma\gamma$ is given by

$$\mathrm{Rate}(K_{L,S}\to\gamma\gamma)=|f(K_{L,S}\to\gamma\gamma)|^2/16\pi M. \quad (4)$$

For the purpose of carrying out the polarization sum in Eq. (1), we choose as the two independent polarization states of the 2γ system the CP eigenstates given by

$$|k,k';CP=\pm1\rangle =(1/\sqrt{2})[|k(L)k'(L)\rangle\pm|k(R)k'(R)\rangle]. \quad (5)$$

Here L (R) denotes helicity -1 ($+1$). The decay amplitudes of K_L and K_S to these states are

$$\begin{aligned}&\langle k,k';CP=+1|\mathfrak{M}|K_L\rangle=0,\\ &\langle k,k';CP=-1|\mathfrak{M}|K_L\rangle=\sqrt{2}f(K_L\to\gamma\gamma),\\ &\langle k,k';CP=+1|\mathfrak{M}|K_S\rangle=\sqrt{2}f(K_S\to\gamma\gamma),\\ &\langle k,k';CP=-1|\mathfrak{M}|K_S\rangle=0.\end{aligned} \quad (6)$$

The unitarity equation [Eq. (1)] may now be written

$$\begin{aligned}\mathrm{Abs}\langle l^+l^-|\mathfrak{M}|K_{L,S}\rangle &=\tfrac{1}{2}\times\tfrac{1}{2}\int\frac{d^3k}{2\omega}\frac{d^3k'}{2\omega'}\frac{1}{(2\pi)^6}(2\pi)^4\delta^4(Q-k-k') \times\sum_{\epsilon,\epsilon'}\langle\gamma\gamma'|\mathfrak{M}|K_{L,S}\rangle\langle l^+l^-|\mathfrak{M}|\gamma\gamma'\rangle^*,\\ &=\frac{1}{64\pi}[\sqrt{2}f(K_{L,S}\to\gamma\gamma)] \times\int_{-1}^{+1}d\cos\theta\langle l^+l^-|\mathfrak{M}|k,k';CP=\mp1\rangle^*.\end{aligned} \quad (7)$$

The relation of the invariant amplitudes $\langle l^+l^-|\mathfrak{M}|K_{L,S}\rangle$ and $\langle l^+l^-|\mathfrak{M}|\gamma\gamma'\rangle$ to the T-matrix elements is

$$\begin{aligned}\langle l^+l^-|T|K_{L,S}\rangle&=(m^2/2ME^2)^{1/2}\langle l^+l^-|\mathfrak{M}|K_{L,S}\rangle,\\ \langle l^+l^-|T|\gamma\gamma'\rangle&=(m^2/4\omega^2E^2)^{1/2}\langle l^+l^-|\mathfrak{M}|\gamma\gamma'\rangle,\end{aligned} \quad (8)$$

where $E=\frac{1}{2}M$ is the energy of the lepton and m the lepton mass.

It remains now to obtain the matrix elements $\langle l^+l^-|\mathfrak{M}|k,k';CP=\pm1\rangle$ describing the amplitudes for pair creation by photons in a definite CP state. For photons of arbitrary polarization, the invariant amplitude is

$$\langle l^+l^-|\mathfrak{M}|\gamma(k,\epsilon)\gamma'(k',\epsilon')\rangle =e^2\bar{v}(p_+)\left[\frac{(\gamma\cdot\epsilon')(\gamma\cdot\epsilon)\gamma\cdot k}{2k\cdot p_+}+\frac{(\gamma\cdot\epsilon)(\gamma\cdot\epsilon')\gamma\cdot k'}{2k'\cdot p_-}\right]u(p_-). \quad (9)$$

Using $(\gamma\cdot\epsilon')(\gamma\cdot\epsilon)=\epsilon'\cdot\epsilon-i\sigma_{\mu\nu}\epsilon'^\mu\epsilon^\nu$,[11] this may be written

$$\begin{aligned}\langle l^+l^-|\mathfrak{M}|\gamma(k,\epsilon)\gamma'(k',\epsilon')\rangle &=\tfrac{1}{2}e^2\bar{v}(p_+)[i\boldsymbol{\varepsilon}\times\boldsymbol{\varepsilon}'\cdot(\boldsymbol{\sigma}\gamma\cdot k/k\cdot p_+-\boldsymbol{\sigma}\gamma\cdot k'/k'\cdot p_+)\\ &\quad-\boldsymbol{\varepsilon}\cdot\boldsymbol{\varepsilon}'(\gamma\cdot k'/k'\cdot p_++\gamma\cdot k/k\cdot p_-)]u(p_-).\end{aligned} \quad (10)$$

The first term is proportional to the amplitude for l^+l^- creation from a CP-odd state of two photons, the second from a CP-even state. Thus, we have

$$\langle l^+l^-|\mathfrak{M}|k,k';CP=-1\rangle =\sqrt{2}(\tfrac{1}{2}e^2)i\bar{v}(p_+)[\sigma_3\gamma\cdot k/k\cdot p_+-\sigma_3\gamma\cdot k'/k'\cdot p_+]u(p_-), \quad (11a)$$

$$\langle l^+l^-|\mathfrak{M}|k,k';CP=+1\rangle =-\sqrt{2}(\tfrac{1}{2}e^2)\bar{v}(p_+)[\gamma\cdot k'/k'\cdot p_++\gamma\cdot k/k\cdot p_-]u(p_-), \quad (11b)$$

[11] We follow the notation and conventions of J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill Book Co., New York, 1964).

where we have chosen the photon momenta to lie along the z axis: $\mathbf{k}=\hat{z}|\mathbf{k}|$. Since by angular momentum conservation the l^+l^- pair will be created in the helicity state $|l^+(L)l^-(L)\rangle$ or $|l^+(R)l^-(R)\rangle$, it is enough to know the decay amplitude to one of these states, as they contribute equally to the total decay rate. Evaluating the spinor products, we get from Eq. (7)

$$\mathrm{Abs}\langle l^+(R)l^-(R)|\mathfrak{M}|K_L\rangle = -\frac{e^2}{32\pi}f(K_L\to\gamma\gamma)\frac{2}{M}\int_{-1}^{+1} d\cos\theta \frac{1}{1-\beta^2\cos^2\theta}, \quad (12a)$$

$$\mathrm{Abs}\langle l^+(R)l^-(R)|\mathfrak{M}|K_S\rangle = i\frac{e^2}{32\pi}f(K_S\to\gamma\gamma)\frac{2}{M}\int_{-1}^{+1} d\cos\theta \frac{\beta\cos^2\theta}{1-\beta^2\cos^2\theta}, \quad (12b)$$

where $\beta=(1-4m^2/M^2)^{1/2}$. The lower bounds on the decay rates of $K_{S,L}\to l^+l^-$ are given by

$$\mathrm{Rate}(K_{S,L}\to l^+l^-)\geqslant(2\pi)^{-1}\beta(m/M)^2M \times|\mathrm{Abs}\langle l^+(R)l^-(R)|\mathfrak{M}|K_{S,L}\rangle|^2. \quad (13)$$

From Eqs. (4), (12), and (13), finally,

$$\frac{\mathrm{Rate}(K_L\to l^+l^-)}{\mathrm{Rate}(K_L\to\gamma\gamma)}\geqslant\tfrac{1}{2}\alpha^2\left(\frac{m}{M}\right)^2\frac{1}{\beta}\left(\ln\frac{1+\beta}{1-\beta}\right)^2, \quad (14a)$$

$$\frac{\mathrm{Rate}(K_S\to l^+l^-)}{\mathrm{Rate}(K_S\to\gamma\gamma)}\geqslant\tfrac{1}{2}\alpha^2\left(\frac{m}{M}\right)^2\frac{1}{\beta}\left[-2+\frac{1}{\beta}\ln\frac{1+\beta}{1-\beta}\right]^2. \quad (14b)$$

If we assume $\beta=(1-4m^2/M^2)^{1/2}\simeq1$, we get the approximate form of the above equations:

$$\frac{\mathrm{Rate}(K_L\to l^+l^-)}{\mathrm{Rate}(K_L\to\gamma\gamma)}\gtrsim2\alpha^2\left(\frac{m}{M}\right)^2\left(\ln\frac{M}{m}\right)^2, \quad (15a)$$

$$\frac{\mathrm{Rate}(K_S\to l^+l^-)}{\mathrm{Rate}(K_S\to\gamma\gamma)}\gtrsim2\alpha^2\left(\frac{m}{M}\right)^2\left[-1+\ln\frac{M}{m}\right]^2. \quad (15b)$$

Table I gives the numerical values for the ratios

$$\mathrm{Rate}(K_{L,S}\to l^+l^-)/\mathrm{Rate}(K_{L,S}\to\gamma\gamma),$$

computed from Eq. (15), as well as branching ratios for $\mathrm{Rate}(K_{L,S}\to l^+l^-)$ based on $\mathrm{Rate}(K_L\to\gamma\gamma)=1.0\times10^4$ sec^{-1} and $\mathrm{Rate}(K_S\to\gamma\gamma)=2.0\times10^4$ sec^{-1}.

3. DISCUSSION

We have calculated lower bounds on the electromagnetic rates of $K_L\to l^+l^-$ and $K_S\to l^+l^-$ by taking only the absorptive part of the decay amplitude in our model. A calculation of the dispersive part cannot be done reliably because of the uncertainty associated with the cutoff required for a convergent result. Still, one may hope that the dispersive part of the amplitude is roughly of the order of the absorptive part, so that the actual electromagnetic rates will be greater than the lower bounds of Table I by only a small factor. Two interesting situations can be envisaged. First, the decays $K_S\to l^+l^-$ and $K_L\to l^+l^-$ might be observed at rates considerably *higher* than our estimates. It would be natural to conclude then that the second-order weak amplitude dominates the weak-electromagnetic one. Second, the decays might be observed at rates much *lower* than the estimates we have made. This could be explained by assuming that the second-order weak and weak-electromagnetic amplitudes are comparable, but that they interfere destructively to produce a low decay rate. The possibility of interference certainly exists, because whereas the weak-electromagnetic amplitude has both an absorptive and a dispersive part, the second-order weak amplitude has no absorptive part at all, since no real intermediate states are possible.[8]

TABLE I. Electromagnetic decay rates of $K_L\to l^+l^-$ and $K_S\to l^+l^-$. The branching ratios

$$\mathrm{Rate}(K_{L,S}\to l^+l^-)/\mathrm{Rate}(K_{L,S}\to \mathrm{all})$$

are obtained on the assumption $\mathrm{Rate}(K_L\to\gamma\gamma)=1.0\times10^4$ sec^{-1} and $\mathrm{Rate}(K_S\to\gamma\gamma)=2.0\times10^4$ sec^{-1}.

Ratio	Lower bound	Experimental limit
$\mathrm{Rate}(K_L\to\mu^+\mu^-)/\mathrm{Rate}(K_L\to\gamma\gamma)$	$\geqslant1.1\times10^{-5}$	
$\mathrm{Rate}(K_L\to\mu^+\mu^-)/\mathrm{Rate}(K_L\to\mathrm{all})$	$\geqslant8.0\times10^{-9}$	$<1.6\times10^{-6}$ [a]
$\mathrm{Rate}(K_L\to e^+e^-)/\mathrm{Rate}(K_L\to\gamma\gamma)$	$\geqslant4.7\times10^{-8}$	
$\mathrm{Rate}(K_L\to e^+e^-)/\mathrm{Rate}(K_L\to\mathrm{all})$	$\geqslant3.6\times10^{-11}$	$<1.8\times10^{-5}$ [a]
$\mathrm{Rate}(K_S\to\mu^+\mu^-)/\mathrm{Rate}(K_S\to\gamma\gamma)$	$\geqslant1.6\times10^{-6}$	
$\mathrm{Rate}(K_S\to\mu^+\mu^-)/\mathrm{Rate}(K_S\to\mathrm{all})$	$\geqslant2.7\times10^{-12}$	$<7.3\times10^{-5}$ [a]
$\mathrm{Rate}(K_S\to e^+e^-)/\mathrm{Rate}(K_S\to\gamma\gamma)$	$\geqslant3.7\times10^{-9}$	
$\mathrm{Rate}(K_S\to e^+e^-)/\mathrm{Rate}(K_S\to\mathrm{all})$	$\geqslant6.0\times10^{-15}$	

[a] M. Bott-Bodenhausen *et al.*, Phys. Letters **24B**, 194 (1967).

We observe from Eq. (15) that in both K_L and K_S decays, the e mode is slower than the μ mode by roughly a factor of $(m_e/m_\mu)^2$. A similar strong suppression of the e mode relative to the μ mode is expected in the second order weak process.[7] If the $e:\mu$ ratio is very much different from $(m_e/m_\mu)^2$, one would be faced with a puzzle that might necessitate the introduction of neutral lepton currents[7,12] and/or a departure from μ-e symmetry.

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