PHYSICAL REVIEW VOLUME 183, NUMBER 5 25 JULY 1969

Phase of the Large-Angle Scattering Amplitude and Van Hove's Uncorrelated Jet Model

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(Received 11 March 1969)

By considering the s plane at fixed $A=[(1-\cos\theta)/(3+\cos\theta)]^2$, where θ is the c.m. scattering angle, it is shown that the phase of the crossing-symmetric amplitude that describes the large-angle elastic scattering of spinless particles is energy-dependent, for almost all possible phenomenologically based assumptions about the high-energy behavior of the differential cross section. The consequences for the uncorrelated jet model are discussed. The work is extended to particles with spin.

1. INTRODUCTION

THE purpose of this paper is to discover what effects crossing symmetry and the requirement of precipitous energy dependence have on the phase of the amplitude that describes the large-angle elastic scattering of spinless, self-conjugate particles. In Sec. 2, we show that a consideration of the s plane at fixed $A=[(1-\cos\theta)/(3+\cos\theta)]^2$ enables us to discuss at the same time the large-angle scattering amplitude in both the s channel and the u channel [s, t, and u are the usual kinematical invariants and θ is the (c.m.) scattering angle]. We take an ansatz for the energy dependence of the large-angle (fixed A) differential cross section given by $\exp[-as^\gamma \ln^\beta(s)]$ where $a>0$ and a, γ, and β are functions of A. By using crossing symmetry, we show that for all of the values of γ and β that are in gross agreement with the experimental results for p-p elastic scattering, except for the single case $\gamma=1$ and $\beta=0$, the phase of the large-angle amplitude depends heavily on s. In Sec. 3, we show that the uncorrelated jet model with neglect of phase cannot reproduce an amplitude whose imaginary part oscillates with s. In Sec. 4 the arguments are extended to processes involving particles with spin.

2. THE PHASE OF THE SCATTERING AMPLITUDE AT LARGE ANGLES

Consider the scattering of two equal-mass (m), self-conjugate bosons of spin zero. Figure 1 displays the Res-Ret plane with the s and u physical regions. Consider a hyperbola passing through[1] $s=u=2m^2$, $t=0$ and asymptotically parallel to $\theta_s=\theta_u=$const, θ being the c.m. scattering angle. Such a curve is shown by the line $ABCD$ and has the equation

$$t^2/(u-s)^2=(1-\cos\theta)^2/(3+\cos\theta)^2=A(\theta). \quad (2.1)$$

We now consider the s plane at constant $A(\theta)$, assuming Mandelstam analyticity. The real axis follows the hyperbola $ABCD$ with cuts at $s=4m^2(\sqrt{A})/(1+\sqrt{A})$ and $s=4m^2$ (see Fig. 2).

For large positive s this is effectively the s plane at fixed θ; for large positive u it is essentially the u plane at fixed θ; the advantage over the s plane or the u plane separately is that we can invoke crossing symmetry. Using the variable $w=(s-u)$ at fixed A and for an amplitude that is even under crossing, we have

$$f(w+i0, A)=f(-w-i0, A). \quad (2.2)$$

Invoking Hermitian analyticity yields

$$f(-w-i0, A)=f^*(-w+i0, A). \quad (2.3)$$

We are now able to proceed in a way analogous to the

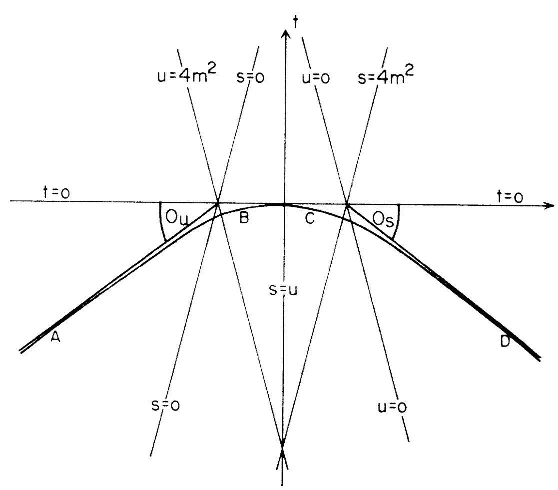

FIG. 1. The Res-Ret plane displaying the hyperbola $ABCD$ that is to serve as the real axis of the s plane at fixed A, A being defined in Eq. (2.1).

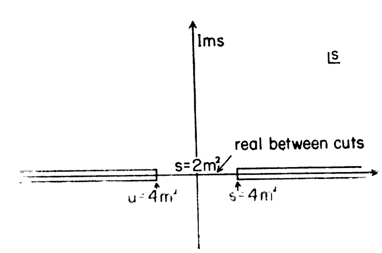

FIG. 2. The s plane at fixed A, showing cuts.

* Research sponsored by the Air Force Office of Scientific Research, Office of Aerospace Research, United States Air Force, under AFOSR Contract/Grant No. AF AFOSR 1268-67.
[1] This in an *ad hoc* choice. Anywhere inside the Euclidean triangle on the line $s-u=0$ will do.

method of Van Hove[2] when he discussed the phase of the scattering amplitude in the s plane at *fixed* t using the hypothesis of Regge asymptotic behavior. We first need an ansatz for the modulus of the large-angle scattering amplitude. This we look for in the fit of Orear[3] to the p-p elastic data at large angles,

$$d\sigma/d\Omega=(c/s)\exp(-ap_\perp)\,, \tag{2.4}$$

where c and a are constants. If β_L is the Lorentz factor v/c, v being the velocity of the proton in the c.m. system, there is an alternative fit due to Krisch,[4]

$$d\sigma/dt\propto\exp(-b\beta_L{}^2p_\perp{}^2)\,, \tag{2.5}$$

where b is another constant. Since $p_\perp\sim(\sqrt{s})\sin\theta$ as $s\to\infty$, we make the following assumption about high-energy behavior in the s plane at fixed A:

$$|f(s,A)|\propto\exp[-a'(A)|s|^{\gamma(A)}]. \tag{2.6}$$

We demand that $0<\gamma\leqslant1$ with both γ and a' functions of A (and $a'>0$), and that Eq. (2.6) be true for both $s\to+\infty$ and $u\to+\infty$. One simple way of doing this, and at the same time generalizing Eq. (2.6) somewhat, is to write, for $s\to+\infty$ and $s\to-\infty$ along the real axis,

$$f(s,A)=c(A)\exp[-as^\gamma\ln^\beta(s)-au^\gamma\ln^\beta(u)], \tag{2.7}$$

where β is a function of A. $C(A)$ is allowed to be complex but if it is in fact real we can tell by inspection that $f(s,A)$ defined in Eq. (2.7) already satisfies Eqs. (2.3) and (2.4).

We endeavor to prove that Eq. (2.7) defines a *unique* crossing-symmetric amplitude as follows. We define a function $\varphi(s,A)$ given by

$$\varphi(s,A)=C(A)\exp[-a(s+i)^\gamma\ln^\beta(s+i)-a(u-i)^\gamma\ln^\beta(u-i)] \tag{2.8}$$

and formulate our high-energy ansatz by requiring that

$$f/\varphi\to1\quad\text{as } s\to+\infty\,. \tag{2.9}$$

Using Eqs. (2.3) and (2.4), and arguing as in Ref. 2,

$$f/\varphi\to C^*(A)/C(A)\quad\text{as } s\to-\infty\,. \tag{2.10}$$

There are two possible ways of proceeding:

(i) f/φ is a function to which we may apply the Phragmén-Lindelöf theorems.[5,6] In order to do this, we have to assume that

$$|f/\varphi|<O(\exp[\delta|s|^\epsilon]) \tag{2.11}$$

as $|s|\to\infty$ in any direction in the upper half-plane, with $\delta>0$ and $O\leqslant\epsilon<1$. If this is the case the theorems apply[7] to f/φ to prove that $f/\varphi\to1$ in any direction in the upper half-plane and along the positive and negative real axes; so that $f(s,A)$ defined in Eq. (2.7) is a unique crossing-symmetric amplitude for the asymptotic large-angle elastic scattering process [provided that $C(A)$ is real].

(ii) If we relax assumption (2.11) we may no longer apply the Phragmén-Lindelöf theorems. For example, a function that satisfies Eqs. (2.9) and (2.10) but not Eq. (2.11) is

$$f/\varphi=1+d\exp(-a's^2)\,, \tag{2.12}$$

where d and a' are constants. Suppose

$$f/\varphi=1+g(s,A)\,, \tag{2.13}$$

the most general way in which we can write f/φ. Equation (2.9) demands that

$$g(s,A)\to0\quad\text{as } s\to+\infty \tag{2.14}$$

and Eq. (2.10) that

$$g(s,A)\to\Gamma e^{i\eta} \tag{2.15}$$

as $s\to-\infty$, with Γ and η constants such that

$$1+\Gamma^2+2\Gamma\cos\eta=1 \tag{2.16}$$

and

$$\frac{\Gamma\sin\eta}{1+\Gamma\cos\eta}=\tan2\delta. \tag{2.17}$$

δ is the argument of $C^*(A)/C(A)$ and is therefore a function of A only. It follows that the phase of the high-energy scattering amplitude at *fixed angle* is given by the phase of $f(s,A)$ defined in Eq. (2.7), *up to a constant.*

We may wish to accommodate some energy dependence of polynomial type outside the exponential in Eq. (2.7). This is easily done since an s^α dependence will contribute to the phase a term like that due to a crossing-symmetric Regge amplitude,[2] namely, $\pi(1-\frac{1}{2}\alpha)$. Likewise we can deal with terms like

$$s^\gamma\ln^\beta(s)[\ln\ln(s)]^\delta[\ln\ln\ln(s)]^\epsilon\cdots$$

in the exponent of Eq. (2.7); the treatment of these cases makes no important difference to our results. Equation (2.1) relates u and s. Since $s+t+u=4m^2$, and letting $\sqrt{A}$ be the positive square root of A, in the s channel

$$u=\frac{4m^2-s(1-\sqrt{A})}{1+\sqrt{A}}\,, \tag{2.18}$$

$$\sim-s(1-\sqrt{A})/(1+\sqrt{A})=-s\Delta\,, \tag{2.19}$$

as $s\to+\infty$.

$$\Delta=1\quad\text{at }\theta=0\,,$$
$$\Delta=0.5\quad\text{at }\theta=90°\,.$$

[2] L. Van Hove, Rev. Mod. Phys. **36**, 655 (1964).
[3] J. Orear, Phys. Rev. Letters **12**, 112 (1964); Phys. Letters **13**, 190 (1965).
[4] A. D. Krisch, Phys. Rev. Letters **11**, 217 (1963); Phys. Rev. **135**, B1456 (1964); Phys. Rev. Letters **19**, 1149 (1967).
[5] E. C. Titchmarsh, *The Theory of Functions* (Oxford University Press, London, 1939).
[6] N. N. Meiman, Zh. Eksperim. i Teor. Fiz. **43**, 2277 (1962) [English transl.: Soviet Phys.—JETP **16**, 1609 (1963)].
[7] The Phragmén-Lindelöf theorems are valid even when there are branch and points and other singular points on the boundary of the region in which they apply.

Therefore, an amplitude $f(s,A)$ given by Eq. (2.7) has a phase ϕ given, up to a number depending on A, and hence θ only, by

$$\phi=-a\Delta^\gamma s^\gamma \ln^\beta\left[-\sin(\pi\gamma)-\frac{\pi\beta\cos(\pi\gamma)}{\ln(\Delta s)}+\frac{\sin(\pi\gamma)\pi^2\beta(\beta-1)}{2\ln^2(\Delta s)}+O(\ln^{-3}(\Delta s))\right] \quad (2.20)$$

and a modulus L whose logarithm is given by

$$\ln L=\ln(C(A))-as^\gamma\Delta^\gamma\ln^\beta(\Delta s)\left[\cos(\pi\gamma)-\frac{\pi\beta\sin(\pi\gamma)}{\ln(\Delta s)}-\frac{\pi^2\beta(\beta-1)\cos(\pi\gamma)}{2\ln^2(\Delta s)}+O(\ln^{-3}(\Delta s))\right]-as^\gamma\ln^\beta(s) \quad (2.21)$$

as $s\to\infty$. We remember that u is in the $-i0$ limit. If the amplitude is odd under crossing, $\frac{1}{2}\pi$ should be added to Eq. (2.20). We discuss the following points.

(*a*) $\gamma\neq 1$, β has any value

$$\phi\sim a(\Delta s)^\gamma\ln^\beta(s)[\sin(\pi\gamma)+O(\ln^{-1}(\Delta s))] \quad (2.22)$$

as $s\to+\infty$. It follows that for fits of the Orear type, the phase is an increasing function of s.

(*b*) $\gamma=1$, $\beta\neq 0$

$$\phi\sim-\pi\beta a(s\Delta)^\gamma\ln^{\beta-1}(\Delta s)\quad \text{as } s\to+\infty. \quad (2.23)$$

Therefore, for fits of the Krisch type the phase may increase or decrease with s.

(*c*) $\gamma=1$, $\beta=0$

The phase of the amplitude does not depend on s and so there is the possibility that it may be purely real or purely imaginary, depending on what the A-dependent part of the amplitude, mentioned above, happens to be.

3. CONSEQUENCES FOR THE UNCORRELATED JET MODEL

The jet model assumes that high-energy elastic scattering is the shadow of inelastic processes.[2,8] The unitarity relationship is

$$-2\,\mathrm{Im}f(s,\theta)=F(s,\theta)+\int d^3k_a''d^3k_b''\delta^4(k_a'+k_b'-k_a-k_b)\times\langle\mathbf{k}_a'\mathbf{k}_b'|f^\dagger|\mathbf{k}_a''\mathbf{k}_b''\rangle\langle\mathbf{k}_a''\mathbf{k}_b''|f|\mathbf{k}_a\mathbf{k}_b\rangle, \quad (3.1)$$

where k_a', k_b', k_a, k_b are the initial- and final-particle four-momenta and the f on the right-hand side is a transition operator. $F(s,\theta)$ is the overlap function introduced by Van Hove[2,9] containing contributions from intermediate states containing more than two particles, $|n\rangle$;

$$F(s,\theta)=\langle f(\mathbf{k}_a'\mathbf{k}_b')|\delta^4(Q-k_a-k_b)|f(\mathbf{k}_a\mathbf{k}_b)\rangle \quad (3.2)$$

and

$$|f(\mathbf{k}_a,\mathbf{k}_b)\rangle=\textstyle\sum_n |n\rangle\langle n|f|\mathbf{k}_a\mathbf{k}_b\rangle, \quad (3.3)$$

where Q is the total c.m. four-momentum. The essence of the jet model is to consider the intermediate states with more than two particles as being most important in contributing to $\mathrm{Im}f(s,\theta)$ and that $\mathrm{Im}f(s,\theta)$ is the dominant part of $f(s,\theta)$. Many simplifications are necessary; for example, intermediate states of a certain mean multiplicity are considered, the wave functions of individual secondaries are uncorrelated, momentum conservation is neglected, a mean energy is taken for the secondaries, and multiparticle production amplitudes are supposed to be real.

Reference 8 shows that the uncorrelated jet model as described in the preceding page, with phenomenological fits to the multiparticle production data inserted as approximations to the single-particle uncorrelated wave functions, produces an $\mathrm{Im}f(s,\theta)$ (at large angles) whose magnitude does not oscillate with increasing energy. The work of Sec. 2 suggests that this is extremely unlikely to be the case, for the following reason. Two phenomenological fits have been widely used to explain the p-p data at large angles: that of Orear, $\gamma=\frac{1}{2}$, $\beta=0$ [eq. (2.7)], and that of Krisch with $\gamma=1$, $\beta=0$. Both fits reproduce the gross features of the data but the experimental results are insufficiently accurate to distinguish between them.[10] Therefore any value of γ between $\frac{1}{2}$ and 1, and many values of β other than zero, would presumably also provide a reasonable fit to the gross features of the large-angle data. As we have seen, there is only one of these fits that permits an $\mathrm{Im}f(s,\theta)$ that does not oscillate with s, $\gamma=1$, $\beta=0$. Since there are so many possibilities that both fit the data and demand an s-dependent phase for $f(s,\theta)$, we ought not to expect the uncorrelated jet model with neglect of the multiparticle production amplitude phase to work.

In order to strengthen our argument still further, by endeavoring to dispose of the one counterexample to our contentions, the Krisch fit itself, we refer to a recent paper[11] which discusses the Krisch three-exponential fit to the p-p data. We recall that each of these exponentials is supposed to correspond to a "region of interaction" within the proton and the authors of Ref. 11 ask the question, what if there are an infinite number of such "regions of interaction" with progressively smaller radii? (Data available at currently accessible regions

[8] J. A. McClure, Nuovo Cimento **53A**, 921 (1968); this paper gives references to other jet-model calculations.

[9] L. Van Hove, Nuovo Cimento **28**, 298 (1963).

[10] A. Diddens, in *Proceedings of the Topical Conference on High Energy Coll. Hadrons, Cern, Geneva, 1968* (Cern, Geneva, 1968), Vol. 1, p. 580.

[11] H. Fleming, A. Giovannini, and E. Predaggio, Nuovo Cimento **56A**, 1131 (1968).

are not inconsistent with the need for a fourth exponential in the Krisch fit.) The answer is that such a sum gives an asymptotic behavior for $f(s,\theta)$ with the γ of Eq. (2.7) equal to one-half.

The easiest way to dispose of the $\gamma=1$, $\beta=0$ case would be to state that it violates the Cerulus-Martin bound,[12,13] which is the reason why the authors of Ref. 11 wrote their paper; however, there are assumptions made in the derivation of the Cerulus-Martin bound which can be relaxed without violating any of the usually assumed analyticity properties of the scattering amplitude. In Ref. 12, for example, the amplitude is assumed to be bounded by a polynomial in s in a certain dumbbell-shaped domain in the $\cos\theta_t$ plane. The resultant lower bound is dependent on the shape of this domain and if a rather different, but equally reasonable, shape is taken $\gamma=1$ is found to be perfectly consistent with the resultant lower bound.[14,15] Likewise, in Ref. 13 Mandelstam analyticity is assumed in the t plane and it is also assumed (N.B.) that the scattering amplitude is bounded by $\exp(t^{1/2}\ln s)$ as $t\to\infty$. This last assumption may be relaxed and the case $\gamma=1$ is perfectly consistent with Mandelstam analyticity. We insert this paragraph not because the work in it is new but simply because what it says is not universally recognized.

What we assert, then, is that, from arguments based on crossing symmetry, it is almost certain that the uncorrelated jet model with neglect of phase will not work, since the phase of the intermediate multiparticle production amplitudes must vary subtly with energy to give the interferences, constructive and destructive, that cause $\mathrm{Im}f(s,\theta)$ to oscillate as the energy increases (at large angles). This is quite in agreement with the experience of jet-model calculations at *small angles* where the jet model may well apply since the forward scattering amplitude is known to be nearly pure imaginary. If a calculation is performed in which the multiparticle production amplitudes are given by the predictions of the multi-Regge exchange model, but with neglect of phases,[16] a slope A is found for the forward diffraction peak, where

$$d\sigma/dt\propto\exp(At)\,, \tag{3.4}$$

such that $A\sim1(\mathrm{GeV}/c)^{-2}$. It is well known that for the vast majority of two-body processes A should be more like 10 $(\mathrm{GeV}/c)^{-2}$. In Ref. 16 a second calculation is performed in which the phases are inserted into the multiparticle production amplitudes (albeit in an *ad hoc* manner) and it is found that A is increased by a factor of 10. In Ref. 17 an uncorrelated-jet-model calculation is performed in which data are taken from multiparticle production processes in the πp system at 8 GeV/c and again it transpires that $A\sim1$ $(\mathrm{GeV}/c)^{-2}$. The author of Ref. 17 attributes this explicitly to the neglect of the phases of the multiparticle production amplitudes.

4. PARTICLES WITH SPIN

A. Pion-Nucleon Scattering

Since pions are self-conjugate, we can cross elastic and charge-exchange events to give scattering processes in the same system. If p (p') and q (q') are the initial, (final) c.m. momenta of nucleons and pions, respectively, the πN scattering amplitude may be written in the form[18]

$$f(s,\theta)=\bar{u}(p')[-A(s,t,u)+i\tfrac{1}{2}\gamma^D\cdot(q+q')B(s,t,u)]u(p)\,, \tag{4.1}$$

where γ^D is a Dirac matrix and $u(p)$ is a Dirac spinor. The invariant amplitudes A and B are in fact 6×6 matrices. Let α, β run over the values 1, 2, 3. The most general forms for A and B are[18]

$$A_{\beta\alpha}=\delta_{\beta\alpha}A^{(+)}+\tfrac{1}{2}[\tau_\beta,\tau_\alpha]A^{(-)}\,, \tag{4.2}$$

$$B_{\beta\alpha}=\delta_{\beta\alpha}B^{(+)}+\tfrac{1}{2}[\tau_\beta,\tau_\alpha]B^{(-)}\,, \tag{4.3}$$

where τ_α are the analogs of the 2×2 Pauli matrices in isospin space. The invariant amplitudes $A^{(\pm)}$, $B^{(\pm)}$ have very simple crossing properties which, together with the assumptions of Hermitian analyticity, require that

$$A^{(\pm)}(u,t,s)^*=\pm A^{(\pm)}(s,t,u) \tag{4.4}$$

and

$$B^{(\pm)}(u,t,s)^*=\mp B^{(\pm)}(s,t,u) \tag{4.5}$$

in the limit $s+i0$, $t+i0$, $u-i0$. It is usual to assume Mandelstan analyticity for each of these amplitudes. There are dynamical poles in $B^{(+)}$ and $B^{(-)}$ at the positions of the nucleon mass, but if we wish to apply the Phragmén-Lindelöf theorems this does not prevent us from doing so.[5,6] All we need to show is that the high-energy ansatz (2.8) and (2.9) is appropriate for the amplitudes $A^{(\pm)}$, $B^{(\pm)}$. This we do by considering the two independent helicity amplitudes for π^+p elastic scattering.[19]

These are, in an obvious notation,

$$f(s,\theta\colon 0\tfrac{1}{2},0\tfrac{1}{2})=\cos\tfrac{1}{2}\theta\left[(A^{(+)}-A^{(-)})+(B^{(+)}-B^{(-)})\times\frac{s-m_\pi^2-m_N^2}{2}\right] \tag{4.6}$$

[12] F. Cerulus and A. Martin, Phys. Letters 8, 80 (1964).
[13] A. Martin, Nuovo Cimento 37, 671 (1965).
[14] C. B. Chiu, J. Harte, and C-I. Tan, Nuovo Cimento 53A, 174 (1968).
[15] R. J. Eden and C-I Tan, Phys. Rev. 172, 1583 (1968).
[16] L. Michejda, J. Turnau, and A. Białas, Nuovo Cimento 56A, 241 (1968).
[17] L. Michejda, Nucl. Phys. B4, 113 (1968).
[18] See, for example, S. Gasiorowicz, *Elementary Particle Physics* (Wiley-Interscience, Inc., New York, 1966).
[19] G. Cohen-Tannoudji, A. Morel, and H. Navelet, Ann Phys. (N. Y.) 46, 239 (1969).

and

$$f(s,\theta:0\tfrac{1}{2},0-\tfrac{1}{2})=\sin\tfrac{1}{2}\theta\left[(A^{(+)}-A^{(-)})\frac{s+m_\pi^2-m^2}{2m_\pi s^{1/2}}+(B^{(+)}-B^{(-)})\frac{s+m_N^2-m_\pi^2}{2s^{1/2}}\right]. \quad (4.7)$$

There is a good deal of structure in pion-proton elastic scattering angular distributions,[20] but there is still a precipitous fall-off with energy. There still remains the possibility that $A=(A^{(+)}-A^{(-)})$ and $B=(B^{(+)}-B^{(-)})$ fall off slowly with s but cancel subtly to allow the two helicity amplitudes to fall off rapidly (as they must do since there are no interference terms between helicity amplitudes in the expression for the differential cross section). An inspection of Eqs. (4.6) and (4.7) shows that if such a cancellation occurs for one helicity amplitude it cannot occur for the other so that both A and B decline precipitously with energy. In the same way the argument may be applied to π^-p elastic scattering and the amplitudes $A'=A^{(+)}+A^{(-)}$ and $B'=B^{(+)}+B^{(-)}$ to show that all of $A^{(\pm)}$, $B^{(\pm)}$ fall off quickly with energy. Therefore, the ansatz (2.8) and (2.9) is valid for each amplitude (although γ and β may differ) and the arguments of Sec. 2 may be carried through.

If one of $A^{(\pm)}$, $B^{(\pm)}$ eventually dominates over the other three, the phases of the helicity amplitudes in, say, π^+p scattering are clearly that of the dominant amplitude. If the rate of falloff of two or more is the same to within powers of s, there is the possibility that one of the helicity amplitudes will be zero, but then, by inspecting Eqs. (4.6) and (4.7), we can see that the other helicity amplitude will still have an s-dependent phase. Therefore, if we were to perform a very sophisticated jet-model calculation taking spins into account, we would still not expect to be able to calculate the helicity amplitudes correctly if we were to neglect the phases of the multiparticle production amplitudes.

B. Proton-Proton and Antiproton-Proton Scattering

The three processes

$$\begin{aligned} pp &\to pp \quad (\mathrm{I}),\\ p\bar{p} &\to p\bar{p} \quad (\mathrm{II}),\\ p\bar{p} &\to \bar{p}p \quad (\mathrm{III}), \end{aligned} \quad (4.8)$$

are related by crossing symmetry.

Consider process I. We use the five invariant amplitudes F_i, defined in Ref. 21. These amplitudes can be related to the helicity amplitudes and the same arguments as for pion-nucleon scattering can be used to show that each F_i falls off precipitously with s. The crossing relations between the F_i and the $\bar{F}_i$ in channels II and III may be used to show that each of the $\bar{F}_i$ also fall off rapidly with energy. Since channels II and III are the same process, the crossing relations between them are extremely simple and the arguments of Secs. 2 and 3 may also be applied to antiproton-proton elastic scattering and, by crossing back into channel I, to proton-proton elastic scattering.

C. Kaon-Proton Elastic Scattering

Since kaons are not self-conjugate particles the arguments of Sec. 4 A cannot be taken over immediately. If, however, $SU(3)$ invariance is invoked, then the crossing of the kaon-nucleon event leads to a scattering process in the same scattering system and the arguments of Secs. 2, 3, and 4 may be repeated.

D. Polarizations

The results of Sec. 4 A ought to have consequences for polarization measurements at large angles. Consider, say, elastic scattering. The polarization parameter is given by

$$P=\frac{\mathrm{Im}A^*B}{|A|^2+|B|^2}. \quad (4.9)$$

As we have seen, A and B fall off rapidly at large angles. There are two possibilities. If A and B have moduli that have the same asymptotic behavior, their phase must be the same and P will be a constant. (More precisely, the polarizations will be due to finer features of the scattering amplitude not included in our gross study.) On the other hand, if either A or B dominates at large angles, P will oscillate rapidly as a function of s but will also decrease like $\exp(-as^\gamma)$, where this is the behavior of the most rapidly decreasing of A or B. It is unfortunately true, therefore, that there are no useful experimental predictions about P to be derived from the approach of this paper.

ACKNOWLEDGMENTS

I am grateful to Dr. B. Kurşunoğlu for the hospitality of the Center for Theoretical Studies at the University of Miami and to Dr. T. Gudehus for a critical reading of the manuscript.

[20] M. L. Perl, in Ref. 10, Vol. 1, p. 252.

[21] M. L. Goldberger, M. T. Grisaru, S. W. MacDowell, and D. Y. Wong, Phys. Rev. 120, 2250 (1960).