PHYSICAL REVIEW VOLUME 183, NUMBER 5 25 JULY 1969

Breaking of Chiral Symmetry for Pseudoscalar Mesons*

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(Received 17 January 1969; revised manuscript received 23 April 1969)

The content of the generator-divergence commutators proposed by Gell-Mann, Oakes, and Renner is analyzed using the pole-dominance and smoothness approximations for the two- and three-point functions in the theory. This leads to very general sum rules which include both Hamiltonian and vacuum symmetry breaking. There are many different solutions to these sum rules. In particular, two interesting limiting cases, one with the Hamiltonian approximately $SU(2)\times SU(2)$-invariant and the other with the vacuum approximately $SU(2)\times SU(2)$-invariant, are found to be allowed. When η-η' mixing is included and the smoothness conditions imposed, a consistent solution exists which is in good agreement with present experimental results. We consider the applications of this model to the K_{l3} form factors, and compare our results with experiment as well as other theoretical work.

I. INTRODUCTION

IN a recent paper[1] Gell-Mann, Oakes, and Renner (GOR) considered the possibility of extending the $SU(3)\times SU(3)$ current algebra to include the commutators of the charges with the divergence of the currents. Such commutators are often referred to as σ terms in current-algebra calculations. Two simple ways to generate such commutators are either to use the quark model with unequal masses or to add $SU(3)\times SU(3)$ symmetry-breaking terms to the Hamiltonian of the system.[2] The former possibility corresponds to symmetry-breaking terms which transform like the $(3,3^*)\oplus(3^*,3)$ representation of the chiral group and is, therefore, a special case of the latter method. GOR consider the $(3,3^*)\oplus(3^*,3)$-type breaking only, and we shall also restrict ourselves to this case.

In this paper we analyze the content of these new commutators in the pole-saturation approximation. As we shall see, this leads to more general results than those in GOR because we allow for broken octet symmetry in the vacuum state as well as in the Hamiltonian. In the limit where the octet vacuum symmetry breaking is zero, we recover the GOR results. However, we also find a consistent solution in the opposite limit; namely, zero octet breaking in the Hamiltonian with large vacuum breaking. It is interesting to observe that even when both the vacuum and the Hamiltonian are octet-broken, one sum rule remains which is independent of our saturation scheme. However, in this case the equations obtained from saturation are too complex to analyze without further assumptions. If we assume that the three-point functions with poles removed are smooth functions of the momenta,[3] reasonable results are obtained. We include η-η' mixing in this general analysis and find that the smoothness assumption implies orthogonal field mixing with one angle only.[4] We apply this model to the K_{l3} form factors and obtain results which are consistent with experiment.

In Sec. II we define our model and introduce our basic notation. In Sec. III we obtain all of the algebraic consequences of the model, pointing out the formulas that are valid, independent of approximations. Except for one exact sum rule, the results are too general to yield useful results. In Sec. IV we discuss the two limiting cases: when the octet breaking is in the Hamiltonian only, and when it is in the vacuum only. The results in either case are not in disagreement with experiment. In Sec. V we consider the general problem, including η-η' mixing. Using the smoothness assumption, the ratios of renormalization constants are determined and a set of sum rules is obtained. A consistent solution of this set is found to be compatible with experiment. In Sec. VI we apply our model to K_{l3} form factors, and obtain results in agreement with experiment. A comparison is made of our work with other results in the literature.

II. THE MODEL

If use is made of the quark model with unequal masses to obtain divergences, we find

$$\partial_\mu V_i^\mu(x)=\omega_3 f_{3ik}\sigma_k(x)+\omega_8 f_{8ik}\sigma_k(x)\,, \tag{1}$$

$$\partial_\mu A_i^\mu(x)=\omega_0 d_{0ik}\phi_k(x)+\omega_3 d_{3ik}\phi_k(x)+\omega_8 d_{8ik}\phi_k(x) \quad (i=1,\cdots,8;\ k=0,1,\cdots,8)\,, \tag{2}$$

where ω_0, ω_3, and ω_8 are proportional to the average quark mass, isodoublet mass splitting, and the hypercharge mass splitting, respectively. The quantities $\{\sigma_k\}$ and $\{\phi_k\}$ are scalar and pseudoscalar quark densities that transform like the $(3^*,3)\oplus(3,3^*)$ representation of the $SU(3)\times SU(3)$ algebra. Their equal-time commuta-

* Research sponsored by the Air Force Office of Scientific Research, Office of Aerospace Research, United States Air Force, under Grant No. 69-1761.

[1] M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

[2] S. Weinberg and S. L. Glashow, Phys. Rev. Letters **20**, 224 (1968).

[3] For a discussion of smoothness, see H. J. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1967).

[4] A similar conclusion is drawn by R. Oakes, Phys. Rev. Letters **20**, 513 (1968).

tion rules are[5]

$$\begin{aligned}[F_i(t),\sigma_j(x)]&=if_{ijk}\sigma_k(x)\,,\\ [F_i^5(t),\sigma_j(x)]&=id_{ijk}\phi_k(x)\,,\\ [F_i(t),\phi_j(x)]&=if_{ijk}\phi_k(x)\,,\\ [F_i^5(t),\phi_j(x)]&=-id_{ijk}\sigma_k(x)\,,\end{aligned}\tag{3}$$

where $i=1,\cdots,8$, and $j,k=0,1,\cdots,8$.

Precisely the same equations result if one assumes that the Hamiltonian density has the form[6]

$$H=H^0-(\omega_0\sigma_0+\omega_3\sigma_3+\omega_8\sigma_8)\,.\tag{4}$$

Using Eqs. (1) and (2) with (3), it is now clear that we can write the commutators between charges and divergences in terms of σ's and ϕ's. In fact, using Eqs. (1) and (2), we can eliminate all σ's and ϕ's except σ_0, σ_3, and σ_8, and thus express the resultant commutators in terms of the divergences themselves and these three scalar fields. As we shall see, these fields play a special role in the theory.

Recently, some authors[7] have retained ω_3 in models which include weak interactions and have used it to determine the Cabibbo angle. As might be expected in this model, ω_3 is smaller than ω_0 and ω_8 by a factor of the order of 137, and we shall neglect it in the following.

In order to proceed with our saturation scheme, we need some simple matrix elements of the currents and fields. We repeat for completeness the following standard conventions[4]:

$$\begin{aligned}\langle 0|A_\mu^{1,2,3}|\pi\rangle&=iF_\pi\, p_\mu\,,\\ \langle 0|A_\mu^{4,5,6,7}|K\rangle&=iF_K\, p_\mu\,,\\ \langle 0|A_\mu^{8}|\eta\rangle&=iF_Y\cos\chi_Y\, p_\mu\,,\\ \langle 0|A_\mu^{8}|\eta'\rangle&=-iF_Y\sin\chi_Y\, p_\mu\,,\\ \langle 0|V_\mu^{4}|\kappa\rangle&=iF_\kappa\, p_\mu\,.\end{aligned}\tag{5}$$

In addition, we need to define

$$\begin{aligned}\langle 0|\phi_{1,2,3}|\pi\rangle&=(Z_\pi)^{1/2}\,,\\ \langle 0|\phi_{4,5,6,7}|K\rangle&=(Z_K)^{1/2}\,,\\ \langle 0|\phi_8|\eta\rangle&=(Z_\eta^{8})^{1/2}\,,\\ \langle 0|\phi_0|\eta\rangle&=(Z_\eta^{0})^{1/2}\,,\\ \langle 0|\phi_8|\eta'\rangle&=-(Z_{\eta'}^{8})^{1/2}\,,\\ \langle 0|\phi_0|\eta'\rangle&=(Z_{\eta'}^{0})^{1/2}\,,\\ \langle 0|\sigma_{4,5,6,7}|\kappa\rangle&=(Z_\kappa)^{1/2}\,.\end{aligned}\tag{6}$$

The notation used here is purely a definition. The ϕ's and σ's are probably not cannonical fields, so that the Z's are not constrained in any way. In fact, in a later section we shall use the smoothness condition to determine the ratios of the Z's, but we shall never restrict their absolute magnitude. Finally, because the symmetry can be broken by the vacuum as well as the Hamiltonian, the vacuum expectation value of the σ_0 and σ_8 fields are, in general, nonzero and are essential parameters of the theory. We introduce the following notation for simplicity:

$$\begin{aligned}\delta_0&=\langle 0|\sigma_0|0\rangle\,,\\ \delta_8&=\langle 0|\sigma_8|0\rangle\,.\end{aligned}\tag{7}$$

If we had retained ω_3, the vacuum expectation value of σ_3, i.e., δ_3, would also enter our equations. It is the vacuum breaking that selects the fields σ_0, σ_3, and σ_8 for a special role.

III. EQUATIONS OF THE MODEL

We shall consider the algebraic relations that emerge from three sets of equations: the partial conservation equations (1) and (2), the charge-field algebra equation (3), and the charge-divergence commutator equations.

By taking the vacuum to single-particle matrix elements of Eqs. (1) and (2) and using the definitions in Eqs. (5) and (6), we obtain

$$\begin{aligned}M_\pi^2F_\pi&=[(\sqrt{\tfrac{2}{3}})\omega_0+(\sqrt{\tfrac{1}{3}})\omega_8](Z_\pi)^{1/2}\,,\\ M_K^2F_K&=[(\sqrt{\tfrac{2}{3}})\omega_0-(\tfrac{1}{2}\sqrt{\tfrac{1}{3}})\omega_8](Z_K)^{1/2}\,,\\ M_\kappa^2F_\kappa&=\tfrac{1}{2}\sqrt{3}\,\omega_8(Z_\kappa)^{1/2}\,,\\ M_\eta^2F_Y\cos\chi_Y&=[(\sqrt{\tfrac{2}{3}})\omega_0-(\sqrt{\tfrac{1}{3}})\omega_8](Z_\eta^{8})^{1/2}\\ &\quad+[(\sqrt{\tfrac{2}{3}})\omega_8](Z_\eta^{0})^{1/2}\,,\\ M_{\eta'}^2F_Y\sin\chi_Y&=[(\sqrt{\tfrac{2}{3}})\omega_0-(\sqrt{\tfrac{1}{3}})\omega_8](Z_{\eta'}^{8})^{1/2}\\ &\quad-[(\sqrt{\tfrac{2}{3}})\omega_8](Z_{\eta'}^{0})^{1/2}\,.\end{aligned}\tag{8}$$

No saturation approximation has been made yet. These equations are exact within the context of our definitions.

To make use of Eq. (3), we now assume that the vacuum expectation values of the charge-field commutator algebra are saturated (dominated) by the one-meson contributions. This leads immediately to

$$\begin{aligned}F_\pi(Z_\pi)^{1/2}&=(\sqrt{\tfrac{2}{3}})\delta_0+(\sqrt{\tfrac{1}{3}})\delta_8\,,\\ F_K(Z_K)^{1/2}&=(\sqrt{\tfrac{2}{3}})\delta_0-(\tfrac{1}{2}\sqrt{\tfrac{1}{3}})\delta_8\,,\\ F_\kappa(Z_\kappa)^{1/2}&=\tfrac{1}{2}\sqrt{3}\,\delta_8\,,\\ F_Y\cos\chi_Y\,(Z_\eta^{8})^{1/2}+F_Y\sin\chi_Y\,(Z_{\eta'}^{8})^{1/2}&=(\sqrt{\tfrac{2}{3}})\delta_0-(\sqrt{\tfrac{1}{3}})\delta_8\,,\\ F_Y\cos\chi_Y\,(Z_\eta^{0})^{1/2}-F_Y\sin\chi_Y\,(Z_{\eta'}^{0})^{1/2}&=(\sqrt{\tfrac{2}{3}})\delta_8\,.\end{aligned}\tag{9}$$

We shall also use the charge-divergence commutators in the saturation approximation. However, before making this approximation, we can consider the exact results written in terms of the spectral functions defined as

$$\langle 0|[j_i^\mu(x),j_{j'}^\nu(0)]|10\rangle=\int dm^2\bigg[\bigg(g^{\mu\nu}-\frac{\partial^\mu\partial^\nu}{m^2}\bigg)\rho_{ij}^{1}(m^2)+\rho_{ij}^{0}(m^2)\partial^\mu\partial^\nu\bigg]\Delta(x,m^2)\,.\tag{10}$$

[5] Representations of $SU(3)\times SU(3)$ algebra are discussed by M. Gell-Mann, Physics 1, 63 (1964).

[6] Conditions on the Hamiltonian in order for PCAC-type equations to be valid are discussed by M. Gell-Mann, Phys. Rev. 125, 1067 (1962).

[7] R. Gatto, G. Sartori, and M. Tonnin, Phys. Letters 28B, 128 (1968); N. Cabibbo and Maiani, *ibid.* 28B, 131 (1968).

The notation is the usual one,[8] and ρ^1 and ρ^0 are the spin-1 and the spin-0 parts of the spectral functions. By taking the divergence of Eq. (10), and using Eqs. (1)–(3) to relate to commutators, we find that

$$\int [\rho_{ij}^{0V}(m^2)+\rho_{ij}^{0A}(m^2)]dm^2=A\delta_{ij}+Bd_{8ij} \quad (i,j=1,\cdots,8), \quad (11)$$

where the superscripts V and A refer to vector and axial-vector spectral functions, and the coefficients A and B are functions of ω_0, ω_8, δ_0, and δ_8. More explicitly, we can evaluate the left-hand side of Eq. (11) in pole saturation. This yields

$$\begin{aligned} M_\pi^2F_\pi^2&=[(\sqrt{\tfrac{2}{3}})\omega_0+(\sqrt{\tfrac{1}{3}})\omega_8][(\sqrt{\tfrac{2}{3}})\delta_0+(\sqrt{\tfrac{1}{3}})\delta_8],\\ M_K^2F_K^2&=[(\sqrt{\tfrac{2}{3}})\omega_0-(\tfrac{1}{2}\sqrt{\tfrac{1}{3}})\omega_8][(\sqrt{\tfrac{2}{3}})\delta_0-(\tfrac{1}{2}\sqrt{\tfrac{1}{3}})\delta_8],\\ M_\kappa^2F_\kappa^2&=\tfrac{3}{4}\omega_8\delta_8, \qquad (12)\\ F_Y^2(M_\eta^2\cos^2\chi_Y&+M_{\eta'}^2\sin^2\chi_Y)\\ &=[(\sqrt{\tfrac{2}{3}})\omega_0-(\sqrt{\tfrac{1}{3}})\omega_8][(\sqrt{\tfrac{2}{3}})\delta_0-(\sqrt{\tfrac{1}{3}})\delta_8]+\tfrac{2}{3}\omega_8\delta_8. \end{aligned}$$

Note that, although the right-hand side of Eq. (12) appears octet $\otimes$ octet broken, the one-octet-type sum rule still holds[9]:

$$\begin{aligned} 4(M_K^2F_K^2&+M_\kappa^2F_\kappa^2)\\ &=3F_Y^2(M_\eta^2\cos^2\chi_Y+M_{\eta'}^2\sin^2\chi_Y)+M_\pi^2F_\pi^2. \quad (13) \end{aligned}$$

It is clear from Eq. (11) that this sum rule holds among the integrals of exact spectral functions, too.

It is important to realize that the three sets of equations (8), (9), and (12) are not independent. Any two of the sets are sufficient to derive the third. Thus, we have a consistent saturation scheme. Since Eq. (8) is true by definition, *our basic approximation is to assume that the Weinberg-type*[8] *sum rule, Eq. (11), can be saturated by single-meson contributions.*

IV. TWO LIMITING CASES

We now wish to compare the solutions to our equations in two extreme cases; i.e., when $\delta_8=0$ *or* $\omega_8=0$. The first corresponds to a solution with an octet-broken Hamiltonian but a symmetric vacuum, and is the solution considered by GOR. The second relies on an asymmetric vacuum state to induce octet breaking, while the Hamiltonian is $SU(3)$-symmetric.

Referring to Eq. (12), we see that in both cases

$$F_\kappa^2M_\kappa^2=0, \quad (14)$$

and thus,

$$\begin{aligned} 4M_K^2F_K^2&\\ &=3F_Y^2(M_\eta^2\cos^2\chi_Y+M_{\eta'}^2\sin^2\chi_Y)+M_\pi^2F_\pi^2. \quad (15) \end{aligned}$$

In order to see the exact correspondence between our results and those of GOR, we neglect η-η' mixing in this section. We can then set $\chi_Y=0$, $Z_\eta{}^0=0$, $Z_{\eta'}{}^8=0$, and rewrite Eq. (15) as

$$M_i^2F_i^2=A+Bd_{8ii}, \quad i=1,\cdots,8. \quad (16)$$

When $\delta_8=0$,

$$A=\tfrac{2}{3}\omega_0\delta_0 \quad\text{and}\quad B=(\sqrt{\tfrac{2}{3}})\omega_8\delta_0; \quad (17)$$

and when $\omega_8=0$,

$$A=\tfrac{2}{3}\omega_0\delta_0 \quad\text{and}\quad B=(\sqrt{\tfrac{2}{3}})\omega_0\delta_8. \quad (18)$$

It follows from Eq. (8) that

$$M_i^2F_i=[(\sqrt{\tfrac{2}{3}})\omega_0+\omega_8d_{8ii}](Z_i)^{1/2}, \quad (19)$$

and Eq. (9) becomes

$$F_i(Z_i)^{1/2}=[(\sqrt{\tfrac{2}{3}})\delta_0+\delta_8d_{8ii}]. \quad (20)$$

When $\delta_8=0$, we see from Eq. (20), or from Eqs. (16) and (19), that

$$F_i(Z_i)^{1/2}=(\sqrt{\tfrac{2}{3}})\delta_0, \quad (21)$$

whereas, when $\omega_8=0$, we find instead that

$$M_i^2F_i=(\sqrt{\tfrac{2}{3}})\omega_0(Z_i)^{1/2}. \quad (22)$$

In both cases $M_i^2F_i^2$ is octet-broken [Eq. (16)]. For both cases, we can find a mass formula by using Eq. (16) and eliminating F_i by using Eq. (21) or (22). We thus have for $\delta_8=0$

$$M_i^2=(1/\delta_0)[\omega_0+(\sqrt{\tfrac{3}{2}})\omega_8d_{8ii}]Z_i, \quad (23)$$

and for $\omega_8=0$,

$$M_i^2=\omega_0Z_i/[\delta_0+(\sqrt{\tfrac{3}{2}})\delta_8d_{8ii}]. \quad (24)$$

If we now add the requirement that the Gell-Mann–Okubo (GMO) mass formula be part of the theory, then Eq. (23) implies that $Z_i=\langle 0|\phi_i|\pi_i\rangle^2=\text{const}$. Since the $\{\phi_\alpha\}$ form an octet, and since the vacuum is symmetric when $\delta_8=0$, this requirement is equivalent to assuming that the states $\{|\pi_i\rangle\}$ transform as an octet too. If we make this basic assumption, then Eq. (24) also makes sense, since we would expect $\langle 0|\phi_i|\pi_i\rangle$ to be octet-broken when the vacuum is octet-broken. In fact, for the two cases we would choose

$$\langle 0|\phi_i|\pi_i\rangle=\text{const} \quad\text{for}\quad \delta_8=0, \quad (25)$$

$$\langle 0|\phi_i|\pi_i\rangle=\text{const}\times[\delta_0+(\sqrt{\tfrac{3}{2}})\delta_8d_{8ii}] \quad\text{for}\quad \omega_8=0. \quad (26)$$

Conversely, assumptions (25) or (26) lead directly to the GMO mass formula, and in either case to

$$F_\pi=F_K=F_Y. \quad (27)$$

From the fact that M_π is small, GOR conclude from Eq. (23) that $\omega_8\sim-\sqrt{2}\omega_0$. Similarly, when $\omega_8=0$ one can see from Eq. (24) that $\delta_8\sim-\sqrt{2}\delta_0$. The former possibility corresponds to the Hamiltonian being $SU(2)\times SU(2)$-invariant, while the latter corresponds to the vacuum being $SU(2)\times SU(2)$-invariant.

At first glance the $\omega_8=0$ solution might seem to violate results such as the Goldberger–Treiman relation, since it leads (when one considers $\partial_\mu A_i{}^\mu$ between nu-

[8] S. Weinberg, Phys. Rev. Letters 18, 507 (1967).
[9] See footnote 8 of Ref. 2.

cleon states, $|N\rangle$) to

$$M_i^2F_i\langle N|\pi_i|N\rangle=(\sqrt{\tfrac{2}{3}})\omega_0\langle N|\phi_i|N\rangle. \quad (28)$$

Thus, if $\langle N|\phi_i|N\rangle$ is assumed symmetric, then $\langle N|\pi_i|N\rangle$ is badly broken because the M_i^2 are octet-broken. However, in the scheme with an asymmetric vacuum, there is no reason to suppose that $\langle N|\phi_\alpha|N\rangle$ does not contain octet-broken parts such as were allowed in one-particle-to-vacuum matrix elements in Eq. (26). This feature, however, makes the $\omega_8=0$ scheme somewhat complicated to use in practice, but careful consideration of vacuum breaking seems to lead to consistent results.

Finally, we comment on the mass of the κ meson in the two schemes. In the GOR scheme $Z_i=\text{const}$, and we may conclude from Eqs. (8) and (11) that, as $\delta_8\to 0$,

$$M_\kappa\to\infty\,,\quad F_\kappa\to 0. \quad (29)$$

In the scheme with $\omega_8=0$, on the other hand, we have

$$M_\kappa\to 0\,,\quad F_\kappa\to\sqrt{3}\delta_8/2(Z_\kappa)^{1/2}. \quad (30)$$

The massless κ is, of course, a consequence of the Goldstone theorem. Weinberg and Glashow[2] have found a general inequality for the κ mass. Using Weinberg's sum rules,[8] a low-lying κ mass is favored in that scheme. This would favor a solution with small ω_8 and large δ_8.

V. MORE GENERAL SOLUTION

The relations obtained in Sec. III are still too general to lead to any detailed results. In this section, we impose the "smoothness condition" on the three-point functions to determine the relations between different Z_i's. In this approximation, it is assumed that functions with pole singularities removed are as smooth functions of the momenta as possible. The approximation has been previously used and discussed by Schnitzer and Weinberg[3] and by Gertein and Schnitzer.[10] It is known that this approximation also corresponds to using tree diagrams in the effective-Langrangian approach. By its nature, it is an approximation which we expect to be good for small values of the momenta—which is where we use it. In the context of chiral breaking, Weinberg and Glashow[2] have used the approximation to determine the departure of the K_{l3} form factor from the symmetry limit. We shall consider here a general three-point function of two pseudoscalar fields and one scalar field, defined as

$$G_{ijk}(p^2,p'^2,q^2)\equiv\frac{(p^2-M_i^2)(p'^2-M_j^2)(q^2-M_k^2)}{(Z_iZ_jZ_k)^{1/2}}\times\int e^{ip\cdot x}e^{-ip'\cdot y}d^4xd^4y\langle 0|T\phi_i(x)\phi_j(y)\sigma_k(0)|0\rangle\,, \quad (31)$$

where $q=p-p'$, and for the moment we restrict i, j, $k=1,\ldots,7$ to avoid the η-η' mixing problem. On the mass shell, G is a coupling constant. By use of the partial conservation of vector current (PCVC) equation (1), we get

$$\partial_\mu V_l^\mu=\omega_8f_{8lk}\sigma_k.$$

For $k=4, 5, 6, 7$, we can relate the function G to another three-point function defined as

$$i(p+p')_\mu F_{ijl}^+(p^2,p'^2,q^2)+iq_\mu F_{ijl}^-(p^2,p'^2,q^2)\equiv\frac{-(p^2-M_i^2)(p'^2-M_j^2)}{(Z_iZ_j)^{1/2}}\int e^{ip\cdot x}e^{-ip'\cdot y}d^4xd^4y\times\langle 0|T\phi_i(x)\phi_j(g)V_{\mu,l}(0)|0\rangle. \quad (32)$$

The functions F^+ and F^- on the mass shell are the observed weak and electromagnetic form factors (e.g., $F_{345}{}^{\pm}$ are the K_{l3} form factors). Our G's and F's differ from Weinberg and Glashow's g's and f's by factors of $(Z_i)^{1/2}$ [see Eq. (10) of Ref. 2], and the smoothness approximation on these leads to quite different results, as we shall see. Using the PCVC equation and integrating by parts, we obtain the following identity between the G's and F's:

$$\begin{aligned}G_{ijk}(p^2,p'^2,q^2)&=[(q^2-M_k^2)/(Z_k)^{1/2}\omega_8f_{8lk}]\{(p^2-p'^2)F_{ijl}^+(p^2,p'^2,q^2)\\&\quad+q^2F_{ijl}^-(p^2,p'^2,q^2)-f_{ijl}[(Z_j/Z_i)^{1/2}(p^2-M_i^2)\\&\quad-(Z_i/Z_j)^{1/2}(p'^2-M_j^2)]\}. \quad (33)\end{aligned}$$

Here, we have approximated

$$i\int d^4x\,e^{ip\cdot x}\langle 0|T\phi_i(x)\phi_j(0)|0\rangle$$

by $(Z_i/p^2-M_i^2)\delta_{ij}$. Now, taking the limit $q^2\to 0$, and requiring that G and F^+ be constants independent of p^2 and p'^2 (smoothness), we obtain the following results:

$$F_{ijl}^+(p^2,p'^2,0)=f_{ijl}\,, \quad (34)$$

$$G_{ijk}(p^2,p'^2,0)=[M_k^2/(Z_k)^{1/2}\omega_8f_{8lk}]\times f_{ijl}(M_j^2-M_i^2)\,, \quad (35)$$

and

$$Z_i=Z_j. \quad (36)$$

It is of interest to note that smoothness leads to the symmetry value of F_{ijk}^+ and is consistent with the Ademollo-Gatto theorem,[11] which rules out first-order breaking. Since G_{ijk} is a first-order breaking effect, this result is also consistent. The value of G_{ijk} may be used to determine the width of the κ meson provided $|(Z_\kappa)^{1/2}\omega_8f_{8lk}/M_\kappa^2|=|F_\kappa|$ is known.[12] The last equation shows that in the smoothness approximation, the constants $(Z_i)^{1/2}\equiv\langle 0|\phi_i|\pi_i\rangle$ are symmetric. This possibility was discussed in Sec. IV, and an alternative derivation is given by GOR. Weinberg and Glashow, however,

[10] I. S. Gerstein and H. J. Schnitzer, Phys. Rev. **170**, 1638 (1968).

[11] M. Ademollo and R. Gatto, Phys. Rev. Letters **13**, 265 (1964).

[12] See, for example, C. H. Albright, P. R. Auvil, and N. G. Deshpande, Nuovo Cimento **52**, 301 (1967).

made the more general assumption that f^+ is a constant, while g is a constant plus a p^2-dependent term. Without this extra p^2 dependence, their choice of g and f would lead to the relation $Z_i=1$, instead of Eq. (36) [see Eq. (10) of Ref. (2)]. Such a result seems unacceptable, in general, since only the transformation properties of fields ϕ_i and σ_i were used, and these objects do not necessarily have the dimensions of fields. Even if they are assumed to be canonical fields, the result is unacceptable, as is pointed out in footnote 7 of Weinberg and Glashow's paper.[2] Our choice of the G and F functions is such that they correspond to physical observables on the mass shell, and seems more reasonable for this approximation. If we now use the partial conservation of axial-vector current (PCAC) equation (2), we can prove, in addition, that

$$Z_\kappa=Z_\pi. \quad (37)$$

The η-η' mixing problem can be treated if we replace $\phi_\pi/(Z_\pi)^{1/2}$ by ϕ_η or $\phi_{\eta'}$, defined as

$$\phi_\eta\equiv\frac{(Z_{\eta'}{}^8)^{1/2}\phi_0+(Z_{\eta'}{}^0)^{1/2}\phi_8}{(Z_\eta{}^8)^{1/2}(Z_{\eta'}{}^0)^{1/2}+(Z_{\eta'}{}^8)^{1/2}(Z_\eta{}^0)^{1/2}}, \quad (38)$$

$$\phi_{\eta'}\equiv\frac{(Z_\eta{}^8)^{1/2}\phi_0-(Z_\eta{}^0)^{1/2}\phi_8}{(Z_\eta{}^8)^{1/2}(Z_{\eta'}{}^0)^{1/2}+(Z_{\eta'}{}^8)^{1/2}(Z_\eta{}^0)^{1/2}}. \quad (39)$$

These fields are the proper interpolating fields for η and η', because they satisfy the conditions

$$\langle 0|\phi_\eta|\eta\rangle=\langle 0|\phi_{\eta'}|\eta'\rangle=1,$$
$$\langle 0|\phi_\eta|\eta'\rangle=\langle 0|\phi_{\eta'}|\eta\rangle=0. \quad (40)$$

Using the PCVC and PCAC conditions, we obtain the following results:

$$Z_{\eta'}{}^8+Z_{\eta'}{}^0=Z_\eta{}^8+Z_\eta{}^0=Z_\pi,$$
$$(Z_\eta{}^8)^{1/2}(Z_{\eta'}{}^8)^{1/2}-(Z_\eta{}^0)^{1/2}(Z_{\eta'}{}^0)^{1/2}=0. \quad (41)$$

The relations between the Z_i are thus,

$$Z_\pi=Z_K=Z_\kappa,$$
$$Z_\eta{}^8=Z_{\eta'}{}^0=Z_\pi\cos^2\theta, \quad (42)$$

and

$$Z_\eta{}^0=Z_{\eta'}{}^8=Z_\pi\sin^2\theta,$$

where θ is an arbitrary parameter.

These results, along with Eqs. (8) and (9) of Sec. III, lead to the following new sum rules:

$$F_\kappa+F_K=F_\pi,$$
$$M_\kappa{}^2F_\kappa+M_K{}^2F_K=M_\pi{}^2F_\pi,$$
$$4(F_K{}^2+F_\kappa{}^2)=3F_Y{}^2+F_\pi{}^2, \quad (43)$$
$$4(M_K{}^4F_K{}^2+M_\kappa{}^4F_\kappa{}^2)$$
$$=3F_Y{}^2(M_\eta{}^4\cos^2\chi_Y+M_{\eta'}{}^4\sin^2\chi_Y)+F_\pi{}^2M_\pi{}^4.$$

Combining these with the one exact sum rule obtained in Sec. III, i.e.,

$$4(M_K{}^2F_K{}^2+M_\kappa{}^2F_\kappa{}^2)$$
$$=3F_Y{}^2(M_\eta{}^2\cos^2\chi_Y+M_{\eta'}{}^2\sin^2\chi_Y)+F_\pi{}^2M_\pi{}^2, \quad (44)$$

and using the masses of known pseudoscalar mesons, we find that a consistent solution exists. The equations are extremely sensitive to the value of these masses, and, consequently, only a range of values of F_K/F_π and M_κ are predicted. Taking M_K to be 497±3 MeV and M_η =549±2 MeV, we find that F_K/F_π ranges from 1.28 to 1.41, while M_κ is between 1020 and 890 MeV. We should also point out that the sum rules used depend on the precise equality of the Z's, while we expect only approximate equality. However, the results seem quite favorable when a comparison is made of our predictions of the K_{l3} form factors with experiment. We do this in Sec. VI.

It is possible to take the limit $\delta_8\rightarrow 0$ of our solution. In this case the η-η' mixing angle becomes zero, and we recover the GOR solution that we obtained in Sec. IV, i.e.,

$$F_\pi=F_K=F_\eta,$$
$$4M_K{}^2=3M_\eta{}^3+M_\pi{}^2,$$

and

$$F_\kappa=0.$$

However, the masses of both κ and η' then become large for consistency. In the smoothness approximation one cannot reproduce the alternate solution $\omega_8\rightarrow 0$ with δ_8 large, because of the equality of the Z_i. The equality of the Z_i suggests that the smoothness approximation corresponds to the lowest-order perturbation theory, where the states are unaltered in the lowest approximation. To include large vacuum breaking it may be necessary to include more momentum dependence than we have assumed.

VI. APPLICATION TO K_{l3} FORM FACTORS

The assumption of smoothness leads us to a solution which is consistent with Ward identities and pole dominance, in which the Z's are approximately equal. Such a solution yields predictions on the K_{l3} form factors which are tests of the model. Alternative solutions have been proposed recently by Chang and Leung[13] and by Gerstein and Schnitzer.[14] In both of these papers the authors assume "quadratic" smoothness, i.e., the functions with poles removed are at most quadratic in the momenta and satisfy the Ward identities. Our principal difference from these authors is in the different hypothetical relations made between model parameters. While we rely on the equality of the Z's for guidance, Gerstein and Schnitzer use Weinberg's second sum rule for $SU(2)\times SU(2)$, and Chang and Leung assume that $Z_K/Z_\kappa=g_{KA}{}^2/g_{K^*}{}^2$, where the g's are the constants[8] relating currents to fields.

The K_{l3} form factors are defined as usual to be

$$\langle\pi^j(p')|V_\mu{}^k(0)|K^i(p)\rangle$$
$$\equiv f_{ijk}[(p+p')_\mu f^+(q^2)+(p-p')_\mu f^-(q^2)]. \quad (45)$$

[13] L. N. Chang and Y. L. Leung, Phys. Rev. Letters **21**, 122 (1968).

[14] I. S. Gerstein and H. J. Schnitzer, Phys. Rev. **175**, 1876 (1968).

We can express $f^+(q^2)$, using vector dominance and quadratic smoothness, as

$$f^+(q^2)=f^+(0)\frac{M_{K^*}{}^2+A^+q^2}{M_{K^*}{}^2-q^2}\,, \qquad (46)$$

where A^+ is arbitrary. (Note that pure pole dominance would imply that $A^+=0$.) An expression has been obtained for $f^+(0)$ by Weinberg and Glashow[2] using quadratic smoothness:

$$f^+(0)=(F_K{}^2+F_\pi{}^2-F_\kappa{}^2)/2F_KF_\pi\,. \qquad (47)$$

It is interesting to note that, by using our sum rules in Eq. (43), we obtain $f^+(0)=1$. This is consistent with our Eq. (34). We may now determine A^+ from K^* width, if we use vector dominance in the form

$$V_\mu{}^{(4)}=g_{K^*}K_\mu{}^{*(4)} \qquad (48)$$

at the K^* pole. The constant g_{K^*} is determined from Weinberg's first sum rule:

$$g_{K^*}{}^2/M_{K^*}{}^2=g_\rho{}^2/M_\rho{}^2-F_\kappa{}^2\cong 2F_\pi{}^2-F_\kappa{}^2\,. \qquad (49)$$

Present experimental information on $K\rightarrow\mu\nu$, $\pi\rightarrow\mu\nu$, and $K\rightarrow\pi e\nu$ gives a relation[14]

$$F_K/F_\pi f^+(0)=1.28\pm0.06\,. \qquad (50)$$

Noting that $f^+(0)$ is unity in our solution, the value of F_K/F_π is consistent with our solution. For purpose of comparison with other theoretical work, we shall use the value of $F_K/F_\pi=1.28$. From Eq. (43) we then obtain $F_\kappa/F_\pi=-0.28$. The K^* width is given by the expression[14]

$$\Gamma(K^*\rightarrow K\pi)=(1/8\pi)q^3(M_{K^*}{}^2/g_{K^*}{}^2)[f_+(0)(1+A^+)]^2\,. \qquad (51)$$

Using the experimental width of $\Gamma(K^*)=49.2$ MeV, we can determine A^+. This value can be tested in K_{l3} decay.

TABLE I. Comparison of different theories of K_{l3} form factors.

	Chang and Leung	Gerstein and Schnitzer	Present work
F_K/F_π	1.24	1.09	1.28
F_κ/F_π	−0.56	−0.58	−0.28
$g_{K^*}{}^2/g_\rho{}^2$	1.12	1.10	1.28
$f_+(0)$	0.975	0.85	1.00
$\xi=f^-/f^+$	−0.01	−0.106	−0.056
λ^+	0.018	0.0238	0.0216
λ^-	0.02	−0.016	0.047
M_κ	1050	635	1020

Using the experimental parametrization

$$f^\pm(q^2)=f^\pm(0)(1+\lambda^\pm q^2/M_\pi{}^2)\,, \qquad (52)$$

we have

$$\lambda^+=(1+A^+)M_\pi{}^2/M_{K^*}{}^2\,. \qquad (53)$$

Using the value obtained for A^+, we find that $\lambda_+=0.0216$. The experimental value for λ^+ ranges from 0.013 to 0.023.[15] We can also determine $f^-(q^2)$ from $f^+(q^2)$ by taking the divergence of Eq. (45) and using κ-pole dominance. We then find

$$f^-(q^2)=(M_\pi{}^2-M_K{}^2)f_+(0)\left(\frac{1+A^+}{M_{K^*}{}^2-q^2}-\frac{1}{M_\kappa{}^2-q^2}\right)\,. \qquad (54)$$

Using experimental parametrization, we have

$$\xi=f^-(0)/f^+(0)=(M_\pi{}^2-M_K{}^2)\left(\frac{1+A^+}{M_{K^*}{}^2}-\frac{1}{M_\kappa{}^2}\right) \qquad (55)$$

and

$$\lambda^-=M_\pi{}^2\left(\frac{1+A^+}{M_{K^*}{}^4}-\frac{1}{M_\kappa{}^4}\right)\frac{f^+(0)}{f^-(0)}(M_\pi{}^2-M_K{}^2)\,. \qquad (56)$$

We choose a κ mass of 1020 MeV which is consistent with $F_K/F_\pi=1.28$ as found in Sec. V. This yields $\xi=-0.056$ and $\lambda^-=0.047$. The experimental results on ξ are at present spread between $\xi=0$ and $\xi=-1.2$, with rate measurements favoring small negative ξ and polarization experiments[15] favoring $\xi\approx-1.00$.

We have compared our solution to those of Chang and Leung and Gerstein and Schnitzer in Table I. Our solution favors a large κ mass, and some experimental evidence exists for this.[16] References to earlier work and discussion may be found in Ref. 14. A better measurement of λ^+ and ξ can, in principle, distinguish between these solutions.

ACKNOWLEDGMENTS

We are indebted to Professor R. J. Oakes for many stimulating discussions of this work. One of the authors (P.R.A.) wishes to thank the Aspen Institute of Physics for hospitality during the past summer; and the other author (N.G.D.) wishes to thank the Boulder Summer Institute of Theoretical Physics.

[15] For the recent status of K_{l3} decays see the rapporteur talk by J. S. Cronin, in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, 1968* (CERN, Geneva, 1968), p. 281.

[16] D. J. Crennell, U. Karshon, K. W. Lai, J. S. O'Neall, and J. M. Scarr, Phys. Rev. Letters **22**, 487 (1969); T. G. Trippe, C. Y. Chien, E. Malamud, J. Mellema, P. E. Schlein, W. E. Slater, D. H. Stork, and H. K. Ticho, Phys. Letters **28B**, 203 (1968).