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Uniform Treatment of Pion Electro- and Photoproduction in Regge Theory*

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The problems associated with the conventional kinematical interpretation of the pion pole in Regge theory are studied. It is found that a formulation leading to a dynamical interpretation is possible in an approach which emphasizes the Lorentz-invariance interpretation of gauge invariance. This approach leads to a uniform treatment of pion production by real and virtual photons in the sense that the former is demanded to be the zero-mass limit of the latter in a manner consistent with Lorentz-invariance requirements on helicity amplitudes, as suggested by Weinberg. Our formula for the meson Regge pole has a singularity at $t=\mu^2$ which explicitly comes from a factor $1/\sin\pi\alpha(t)$, rather than the kinematical factor $(t-\mu^2)^{-1}$ used in recent phenomenological analyses. Comments are made on some features arising from gauge invariance which are analogous to perturbation theory.

I. INTRODUCTION

RECENT experiments[1] on high-energy pion photoproduction from proton targets have focused considerable theoretical attention[2–4] on the sharp forward peak in the reaction $\gamma+p\rightarrow\pi^++n$. Processes such as this have long been of interest[5] because of the effect that the additional requirement of gauge invariance will have upon Regge-pole theory. This effect can be expected to be nontrivial since in the low-energy limit (or in perturbation theory) gauge invariance is satisfied by a set of dynamical pole terms in different invariants s, t, and u, through universality of charge.

Generally, the high-energy behavior of amplitudes in the s channel is determined by studying the crossed (t, say) channel helicity amplitudes. For photoproduction, however, use of this standard method requires caution because of the absence of longitudinal polarization states for a physical photon. For example, the pion-pole term in pion photoproduction cannot appear in the t-channel helicity amplitudes in perturbation theory. This fact is inevitably reflected in the corresponding Reggeization process where the pion Regge pole has been introduced kinematically.[2–5] Such a kinematical interpretation is rather unsatisfactory and aesthetically unappealing: Suppose there exists a heavy boson (hb) with the quantum numbers of the pion, such that its mass $m_{\text{hb}}>m_\pi$, along with its heavier Regge recurrences (e.g., 2^-, $\cdots$) which define a relatively low-lying trajectory that might be expected to contribute to the photoproduction amplitudes. The usual prescription (e.g., Ref. 2) introduces the pion pole into the Reggeized amplitude by a kinematical factor $(t-\mu^2)^{-1}$ from $\sin\theta_t$. But this same kinematic factor would multiply the amplitude for hb exchange, and attaching dynamical significance to this factor suggests that hb exchange has the same range as pion exchange. Also, this kinematical pole considerably complicates electroproduction, since the dynamical pole enters in addition. Further, the universality of charge is not reflected in the practiced prescription.

This paper should help to clarify this pion-pole problem and allow for a simultaneous treatment of pion photoproduction and electroproduction.[6] The basic ideas were presented elsewhere[7] and illustrated for the simple example of boson photoproduction from bosons. Here we shall elaborate upon a Reggeization scheme for photoproduction from nucleons which includes a dynamical pion pole and produces a close correspondence with perturbation theory.[8] It is shown that the gauge-invariance requirement, which can be regarded as reflecting the Lorentz invariance of S-matrix elements involving massless particles,[9] can be used to derive a kind of sum rule ensuring the dynamical nature of the pion.

Our basic approach consists of expanding helicity amplitudes in terms of invariant amplitudes proved[10] to be kinematical-singularity-free, and vice versa, rather than using the crossing-matrix approach. The two are believed to be equivalent, although the former

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might be more fundamental. In the next section these relations between helicity and invariant amplitudes are presented for completeness and to introduce the notation. In Sec. III it is shown that by requiring those helicity amplitudes which involve longitudinal states of a massive ($m_V \neq 0$) photon to vanish smoothly as $m_V \to 0$, one recovers the usual gauge-invariance restrictions on photoproduction amplitudes. This approach was discussed for general helicity amplitudes in reactions involving massless particles by Weinberg[11] but does not seem to have been applied till now.[12] Also, kinematical singularities of helicity amplitudes are treated in this section. Explicit application of the results of the previous section to Regge-pole theory is made in Sec. IV, and an abbreviated discussion of the relation with perturbation theory is given in Sec. V. The final section includes a discussion of the possible experimental and theoretical implications of our results.

II. INVARIANT AMPLITUDES AND HELICITY AMPLITUDES

In this section we define various quantities needed in subsequent sections. The s channel describes the process

$$\pi^\alpha(q)+N(p_1) \to V^\beta(k,\epsilon)+N(p_2)\,, \tag{1}$$

where q, p_1, k, and p_2 represent the 4-momenta of the corresponding particle, and ϵ is the polarization of the vector meson V with mass m_V. The indices α and β refer to the charge of the bosons; in the following we are mainly concerned with the $(-)$ amplitudes of the standard notation,[10] which correspond to the amplitudes with $I=1$ states in the t channel. The t-channel amplitudes describe the process

$$N(p_1)+\bar{N}(p_2) \to V(k,\epsilon)+\pi(q)\,. \tag{2}$$

We write invariant amplitudes in the s channel as

$$T=\bar{N}(p_2)T_\mu\epsilon^\mu N(p_1) = \bar{N}(p_2)\sum_{i=1}^{6} U_i(s,t,u)u_i N(p_1)\,, \tag{3}$$

where $\bar{N}(p_2)$ and $N(p_1)$ are the nucleon spinors and the u_i's are defined as

$$\begin{aligned} u_1&=\gamma_5(\gamma\cdot k)(\gamma\cdot\epsilon), & u_4&=\gamma_5(\gamma\cdot\epsilon)\,,\\ u_2&=\gamma_5(P'\cdot\epsilon), & u_5&=-\gamma_5(\gamma\cdot k)(q\cdot\epsilon)\,,\\ u_3&=-\gamma_5(q\cdot\epsilon), & u_6&=\gamma_5(\gamma\cdot k)(P'\cdot\epsilon)\,, \end{aligned} \tag{4}$$

with $P'=p_1+p_2$. In defining Eqs. (3) and (4) we have assumed that the vector meson under consideration never has a scalar component. Without such an assumption, there would be two additional invariant amplitudes proportional to $(k\cdot\epsilon)$. These U_i's are shown by Ball[10] to be kinematical-singularity-free.

The helicity amplitudes in the t channel can be expressed in terms of these invariant amplitudes. In the center-of-mass coordinate system defined by

$$p_2=(E,0,0,p)\,,\quad k=(\omega,\kappa\sin\theta,0,\kappa\cos\theta)\,,$$

with $\omega^2-\kappa^2=m_V^2$, we obtain the following:

$$\begin{aligned} X_1&=E\kappa U_1\,,\\ X_2&=-p\omega U_1-2pEU_2-2pM\omega U_6\,,\\ Y_1&=-M\kappa U_1+2p^2\kappa U_6\,,\\ Y_2&=pU_4+2p^2\kappa\cos\theta\, U_6\,,\\ Z_1&=-m_Vp\cos\theta\, U_1-m_V^{-1}(2E\omega p\cos\theta\, U_2+2E^2\kappa U_3\\ &\qquad+M\kappa U_4+2E\omega\kappa MU_5+2\omega^2Mp\cos\theta\, U_6)\,,\\ Z_2&=m_V^{-1}(p\omega U_4+2Ep\kappa^2U_5+2\omega\kappa p^2\cos\theta\, U_6)\,, \end{aligned} \tag{5}$$

where

$$\begin{aligned} X_{1,2}&=(2\sqrt{2}\sin\theta)^{-1}(\langle+1|T|+\tfrac12+\tfrac12\rangle\pm\langle+1|T|-\tfrac12-\tfrac12\rangle)\,,\\ Y_{1,2}&=(2\sqrt{2})^{-1}[\langle+1|T|-\tfrac12+\tfrac12\rangle/(1-\cos\theta)\\ &\qquad\pm\langle+1|T|+\tfrac12-\tfrac12\rangle/(1+\cos\theta)]\,,\\ Z_1&=\tfrac12\langle0|T|+\tfrac12+\tfrac12\rangle\,,\\ Z_2&=\tfrac12(\sin\theta)^{-1}\langle0|T|-\tfrac12+\tfrac12\rangle\,. \end{aligned} \tag{6}$$

The quantities appearing in Eq. (5) can be expressed in terms of invariants s, t, and u:

$$\begin{aligned} E&=\tfrac12\sqrt{t}\,,\\ p&=\tfrac12(t-4M^2)^{1/2}\,,\\ \kappa&=\{[t-(m_V+\mu)^2][t-(m_V-\mu)^2]\}^{1/2}/(2\sqrt{t})\\ &\equiv\lambda/(2\sqrt{t})\,,\\ \omega&=(t+m_V^2-\mu^2)/(2\sqrt{t})\equiv\xi/(2\sqrt{t})\,,\\ q_0&=(t-m_V^2+\mu^2)/(2\sqrt{t})\,,\\ \cos\theta&=t^{1/2}(s-u)/(t-4M^2)^{1/2}\lambda\equiv z\,, \end{aligned} \tag{7}$$

where μ and M are the masses of the pion and nucleon, respectively.

The expressions in Eq. (5) can readily be inverted to give the invariant amplitudes U_i in terms of helicity amplitudes; we shall use the result of this inversion in Sec. IV. Also, it can be readily seen by considering the $N\bar{N}$ quantum numbers in the t channel that the "parity-conserving" helicity amplitudes of Eq. (6) contain the following Regge trajectories:

X_1: $1^-(\rho)$, $2^+(A_2)$; X_2: $0^-(\pi)$; Y_1: $1^-(\rho)$, $2^+(A_2)$; Y_2: $1^+(A_1)$; Z_1: $0^-(\pi)$; and Z_2: $1^+(A_1)$.

Here, the particle indicated in parentheses represents a typical particle on the trajectory. The amplitudes X_1, X_2, Y_1, Y_2, Z_1, and Z_2 are essentially $\bar{f}_{10,\frac12\frac12}{}^+$, $\bar{f}_{10,\frac12\frac12}{}^-$,

[11] S. Weinberg, Phys. Rev. **134**, B882 (1964).

[12] N. Dombey [Nuovo Cimento **32**, 1696 (1964)], however, considers the problem of defining longitudinally polarized spin-1 states in the limit $m_V \to 0$ and presents some related calculations. The Appendix of Ref. 8 also has a brief relevant discussion.

$\bar{f}_{10,\frac{1}{2}-\frac{1}{2}}{}^{+}$, $\bar{f}_{10,\frac{1}{2}-\frac{1}{2}}{}^{-}$, $\bar{f}_{00,\frac{1}{2}\frac{1}{2}}$, and $\bar{f}_{00,\frac{1}{2}-\frac{1}{2}}$, respectively, in the more conventional notation with subscripts specifying helicities.

III. GAUGE INVARIANCE: KINEMATICAL-SINGULARITY-FREE PARITY-CONSERVING HELICITY AMPLITUDES

The gauge invariance conditions on the invariant amplitudes for photoproduction are[10]

$$(P'\cdot k)U_2+(q\cdot k)U_3=0\,, \qquad (8)$$
$$U_4+(q\cdot k)U_5+(P'\cdot k)U_6=0\,.$$

If the U_i were completely independent, the longitudinal helicity amplitudes Z_1 and Z_2 of Eq. (5) would diverge as $m_V\to 0$. However, the conditions of Eq. (8) are such as to make the residue of the poles in photon mass (m_V^{-1}) in Z_1 and Z_2 vanish. Thus Eq. (8) follows from Lorentz invariance as emphasized by Weinberg.[11]

We now find the kinematical-singularity-free (KSF) parity-conserving (PC) helicity amplitudes (HA's) from Eqs. (5) and (6). The proven analyticity properties of U_1, U_2, U_4, and U_6 can be satisfied by choosing the kinematical factors for transverse helicity amplitudes as

$$X_1=\lambda\tilde{X}_1\xrightarrow[m_V\to 0]{}(t-\mu^2)\tilde{X}_1\,,$$
$$X_2=[(t-4M^2)/t]^{1/2}\tilde{X}_2\,,$$
$$Y_1=(\lambda/\sqrt{t})\tilde{Y}_1\xrightarrow[m_V\to 0]{}[(t-\mu^2)/\sqrt{t}]\tilde{Y}_1\,, \qquad (9)$$
$$Y_2=(t-4M^2)^{1/2}\tilde{Y}_2\,,$$

which are identical with those listed by Frautschi and Jones[2] in their Table I. This choice, however, leads to a kinematical singularity of λ^{-2} in the invariant amplitudes U_3 and U_5. A more detailed examination of Eqs. (5) or of the inverted form of (5) shows that the determination of the kinematical factors used in constructing the KSF PC HA's is ambiguous, since these factors depend on the way the limit $m_V\to 0$ is taken. Thus, for example, if the U_i's obtained by inverting Eq. (5) are evaluated in this limit under the assumption that

$$m_V Z_i\xrightarrow[m_V\to 0]{}0\,,$$

the resulting kinematical factors are those listed by Ball, Frazer, and Jacob.[3] A complication in this "$m_V\to 0$ method" for finding the KSF PC HA's is due to the fact [see Eq. (8)] that many kinematical quantities lead to factors of $t-\mu^2$ when $m_V\to 0$.

To help clarify this problem, we now try to construct KSF PC HA's in such a way that the $m_V\neq 0$ case is smoothly connected with that for $m_V=0$. This approach is suggested by the Lorentz-invariance requirements on helicity amplitudes involving massless particles,[11] and, since gauge-invariance restrictions (at $m_V=0$) are automatically incorporated, it implies a smooth transition between electro- and photoproduction amplitudes.

First we show that the kinematical factors of Eq. (9) are not compatible with gauge invariance, i.e., Lorentz invariance, in the sense that the longitudinal amplitudes Z_i vanish as $m_V\to 0$. From Eq. (5) or the inverted form we have

$$\xi(s-u)U_2+\lambda^2U_3=-4m_V^2[2(t-4M^2)^{-1}(s-u)$$
$$\times(t\tilde{X}_1+2M\tilde{Y}_1)-4M\tilde{Y}_2+\tilde{Z}_1/t+2M\xi\tilde{Z}_2/t]\,, \quad (10)$$
$$\lambda^2U_5=8m_V^2\tilde{Z}_2-4\xi\tilde{Y}_2\,. \qquad (11)$$

To obtain the above equations, in addition to Eq. (9) we have used

$$Z_1=m_V(\lambda\sqrt{t})^{-1}\tilde{Z}_1\,, \qquad (12)$$
$$Z_2=m_V[(t-4M^2)/t]^{1/2}\tilde{Z}_2\,.$$

Gauge invariance is realized through the explicit appearance of the kinematical factor m_V as shown in Ref. 7. At $t=(m_V\pm\mu)^2$ or $\lambda=0$, Eq. (11) yields

$$m_V(m_V\pm\mu)^{-1}\tilde{Z}_2=\tilde{Y}_2\,.$$

Thus, at least at these two values of t, since $\tilde{Y}_2$ is finite as $m_V\to 0$, the longitudinal amplitude $\tilde{Z}_2$ also remains finite, contradicting gauge (Lorentz) invariance. Similarly, Eq. (10) combined with

$$U_2=-2(t-4M^2)^{-1}\{\xi\tilde{X}_1+[(t-4M^2)/t]\tilde{X}_2$$
$$+[2M\xi/t]\tilde{Y}_1\} \quad (13)$$

[which follows by inverting Eq. (5)] shows that at $t=(m_V\pm\mu)^2$ the finiteness of $\tilde{X}_2$ in the limit $m_V\to 0$ implies a nonvanishing Z_1 as $m_V\to 0$. To correct these shortcomings we redefine the kinematical factors of Eq. (9) for $\tilde{X}_2$ and $\tilde{Y}_2$ (this is relatively unique because of the analyticity properties of U_2 and U_3, as elaborated below),

$$X_2=\xi[(t-4M^2)/t]^{1/2}\tilde{X}_2'\,, \qquad (14)$$
$$Y_2=\xi(t-4M^2)^{1/2}\tilde{Y}_2'\,.$$

We find that with Eq. (14), the gauge-invariance relations [Eq. (8)] can be generalized to

$$U_3=-[p\kappa\cos\theta/E\omega]U_2+4m_V^2\varphi_1\,, \qquad (15)$$
$$U_5=-(2E\omega)^{-1}[U_4+2\kappa p\cos\theta\, U_6]+8m_V^2\varphi_2\,,$$

where φ_1 and φ_2 are analytic functions. Rewriting (15) in terms of helicity amplitudes, we obtain the sum rules

$$2(s-u)\tilde{X}_2'=t^{-1}(\tilde{Z}_1+2M\xi\tilde{Z}_2)-4M\xi\tilde{Y}_2'+\lambda^2\varphi_1\,, \quad (16)$$
$$2t\tilde{Y}_2'=\tilde{Z}_2-\varphi_2\,. \qquad (17)$$

The amplitudes $\tilde{Y}_2'$ and $\tilde{Z}_2$ can be eliminated from Eq. (16) by use of Eq. (17) to form

$$2(s-u)\tilde{X}_2'=t^{-1}(\tilde{Z}_1+2M\xi\varphi_2)+\lambda^2\varphi_1\,. \qquad (16')$$

It is evident that the φ_i should not be zero in order to reconcile the different $z=\cos\theta_t$ dependence of the HA's occurring in these sum rules. This is clearly important in Reggeization (as shown explicitly in Sec. IV) since, e.g., $\tilde{Z}_1$ and $\tilde{X}_2$ both can contain the 2^- pole but have different forms for their z dependence. The requirements of Eqs. (16) ane (17) can be met through identities like

$$zP_\alpha'(z)=\alpha P_\alpha(z)+P_{\alpha-1}'(z). \quad (18)$$

To close this section we note that the kinematical factors of Eqs. (9), (12), and (14) defining $\tilde{X}_2'$, $\tilde{Y}_1$, $\tilde{Z}_1$, and $\tilde{Z}_2$ are chosen according to the conspiracy hypothesis which appears to be required by the sharp forward peak in the data.[1] From Eq. (16) with these factors

$$\tilde{X}_2'=-\tfrac{1}{4}(U_1+2MU_6)-\tfrac{1}{2}tU_2/\xi, \quad (19)$$
$$\tilde{Y}_1=-\tfrac{1}{2}M(U_1+2MU_6)-\tfrac{1}{4}tU_6.$$

Evaluated at $t=0$, in terms of invariant amplitudes the conspiracy condition reduces to the trivial identity

$$\tilde{Y}_1=2M\tilde{X}_2'=-\tfrac{1}{2}M(U_1+2MU_6), \quad (20)$$

where U_1+2MU_6 is A_1 of Ball.[10] A similar argument can be presented for Z_1 and Z_2. From Eq. (6)

$$\tilde{Z}_1=-t\{\tfrac{1}{2}(s-u)U_1+\tfrac{1}{4}[\xi(s-u)U_2+\lambda^2U_3]/m_V^2+M(s-u)U_6\}-\tfrac{1}{2}M\lambda^2[U_4+\tfrac{1}{2}\xi U_5+\tfrac{1}{2}(s-u)U_6]/m_V^2, \quad (21)$$
$$\tilde{Z}_2=\tfrac{1}{4}\{\xi[U_4+\tfrac{1}{2}\xi U_5+\tfrac{1}{2}(s-u)U_6]-4m_V^2tU_5\}/m_V^2.$$

and at $t=0$ yields

$$\tilde{Z}_1(t=0)=2M(m_V^2-\mu^2)\tilde{Z}_2(t=0), \quad (22)$$

which says that a linear combination of U_4, U_5, and U_6 equals itself. The conspiracy relations for HA's [Eqs. (20) and (22)] have recently been discussed in more detail by other authors.[13] We just note that when they are expressed in terms of invariant amplitudes, it becomes apparent that "conspiracy" must hold independently of Regge theory; any theory (e.g., one based on perturbation pole diagrams) must satisfy Eq. (20) since this equation amounts to $A_1=A_1$.

IV. REGGEIZATION

As noted in Sec. II, Eq. (5) can be inverted to give the U_i in terms of helicity amplitudes and by use of Eqs. (9), (12), and (14) in terms of KSF PC HA's:

$$U_1=4\tilde{X}_1,$$
$$U_2=-2\xi(t-4M^2)^{-1}\{\tilde{X}_1+[(t-4M^2)/t]\tilde{X}_2'+(2M/t)\tilde{Y}_1\},$$
$$U_3=2(s-u)(t-4M^2)^{-1}\{\tilde{X}_1+[\xi^2(t-4M^2)/t\lambda^2]\tilde{X}_2'+(2M/t)\tilde{Y}_1\}+4m_V^2\lambda^{-2}[4M\xi\tilde{Y}_2'-(\tilde{Z}_1+2M\xi\tilde{Z}_2)/t], \quad (23)$$
$$U_4=2(t-4M^2)^{-1}[-2M(s-u)\tilde{X}_1-(s-u)\tilde{Y}_1+(t-4M^2)\xi\tilde{Y}_2'],$$
$$U_5=4\lambda^{-2}[2m_V^2\tilde{Z}_2-\xi^2\tilde{Y}_2'],$$
$$U_6=4(t-4M^2)^{-1}[2M\tilde{X}_1+\tilde{Y}_1].$$

[13] See, e.g., M. B. Halpern, Phys. Rev. **160**, 1441 (1967); H. Högaasen and Ph. Salin, Nucl. Phys. **B2**, 657 (1967); S. Frautschi and L. Jones, Phys. Rev. **167**, 1335 (1968).

That the $\tilde{X}_1$, $\tilde{X}_2'$, $\tilde{Y}_1$, $\tilde{Y}_2'$, $\tilde{Z}_1$, and $\tilde{Z}_2$ are KSF PC HA's is consistent with the analyticity of the U_i's (including the point $\lambda^2=0$ in U_3 and U_5). Therefore, the explicit contributions from various Regge poles to each KSF PC HA can be written through use of the standard method of continuation as

$$\tilde{X}_1=\beta_V{}^X\frac{1-e^{-i\pi\alpha_V}}{\sin\pi\alpha_V}P_{\alpha-1}{}^{(1,1)}(z)+\beta_T{}^X\frac{1+e^{-i\pi\alpha_T}}{\sin\pi\alpha_T}P_{\alpha-1}{}^{(1,1)}(z)+\cdots,$$
$$\tilde{X}_2'=\beta_P{}^X\frac{1+e^{-i\pi\alpha_P}}{\sin\pi\alpha_P}P_{\alpha-1}{}^{(1,1)}(z)+\cdots,$$
$$\tilde{Y}_1=\beta_V{}^Y\frac{1+e^{-i\pi\alpha_V}}{\sin\pi\alpha_V}[P_{\alpha-1}{}^{(2,0)}(z)+a_VP_{\alpha-1}{}^{(0,2)}(z)]+\beta_T{}^Y\frac{1+e^{-i\pi\alpha_V}}{\sin\pi\alpha_T}[P_{\alpha-1}{}^{(2,0)}(z)+a_TP_{\alpha-1}{}^{(0,2)}(z)]+\cdots, \quad (24)$$
$$\tilde{Y}_2'=\beta_A{}^Y\frac{1-e^{-i\pi\alpha_A}}{\sin\pi\alpha_A}[P_{\alpha-1}{}^{(2,0)}(z)+a_AP_{\alpha-1}{}^{(0,2)}(z)]+\cdots,$$
$$\tilde{Z}_1=\beta_P{}^Z\frac{1+e^{-i\pi\alpha_P}}{\sin\pi\alpha_P}P_\alpha{}^{(0,0)}(z)+\cdots,$$
$$\tilde{Z}_2=\beta_A{}^Z\frac{1-e^{-i\pi\alpha_A}}{\sin\pi\alpha_A}P_{\alpha-1}{}^{(1,1)}(z)+\cdots.$$

Here V, T, P, and A stand for 1^-, 2^+, 0^-, and 1^+ Regge poles respectively. All possible other contributions are represented by $\cdots$.

To eliminate the t^{-1} singularity in Eq. (23) for U_2, we require conspiracy between at least one contribution to $\tilde{X}_2'$ and one part of $\tilde{Y}_1$ (the remaining parts of each KSF PC HA evade). If only the Regge-pole contributions explicitly written in Eq. (24) are nonzero, then the conspiracy occurs among these (the 0^+ pion conspirator in $\tilde{Y}_1$ should then be explicitly displayed).

Next, we examine what gauge invariance implies for the Reggeized contributions of Eq. (24). Since $\tilde{Z}_2$ and $\tilde{Y}_2'$ do not contain the pion pole, we expect that the relevant part of Eq. (16) can be written as

$$2(s-u)\beta_\pi{}^X\frac{1+e^{-i\pi\alpha_\pi}}{\sin\pi\alpha_\pi}P_\alpha'(X)=\beta_\pi{}^Z\frac{1+e^{-i\pi\alpha_\pi}}{\sin\pi\alpha_\pi}P_\alpha(Z)+\lambda^2\varphi_1{}^\pi. \quad (25)$$

To obtain Eq. (25) we have assumed that the pion Regge pole *in* $\tilde{Z}_1$ evades, i.e., we have written $\beta_P{}^Z$ in Eq. (24) as $\beta_P{}^Z=t\beta_\pi{}^Z$. Should we choose the case in which the pion pole *in* $\tilde{Z}_1$ conspires with a 1^+ trajectory in $\tilde{Z}_2$, we would *formally* introduce a t^{-2} pole in U_2 and very much complicate the form of Eq. (25).

To satisfy Eq. (25), we can choose

$$\beta_\pi{}^X=(\beta_\pi{}^Z/2\alpha\pi)t^{1/2}(t-4M^2)^{-1/2}\lambda^{-1} \equiv(\beta_\pi{}^Z/(2\alpha\pi)z/(s-u)\,, \quad (26)$$

$$\varphi_1{}^\pi=-\lambda^{-2}(\beta_\pi/\alpha\pi)[(1+e^{-i\pi\alpha_\pi})/\sin\pi\alpha_\pi]P_{\alpha-1}{}'(z)\,. \quad (27)$$

In Eq. (27), $\varphi_1{}^\pi$ will not have a pole at $\lambda^2=0$ since $\tilde{Z}_1{}^\pi$ is kinematical-singularity-free. From Eqs. (26) and (27) we obtain

$$U_2{}^\pi=\frac{-1}{t}\frac{\xi}{s-u}\frac{\beta_\pi{}^Z}{\alpha_\pi}\frac{1+e^{-i\pi\alpha_\pi}}{\sin\pi\alpha_\pi}zP_\alpha{}'(z)$$
$$=\frac{-1}{t}\frac{\xi}{s-u}\beta_\pi{}^Z\frac{1+e^{-i\pi\alpha_\pi}}{\sin\pi\alpha_\pi}\left(P_\alpha(z)+\frac{1}{\alpha_\pi}P_{\alpha-1}{}'(z)\right), \quad (28)$$

$$U_3=\frac{1}{t}\beta_\pi{}^Z\frac{1+e^{-i\pi\alpha_\pi}}{\sin\pi\alpha_\pi}\left(P_\alpha(z)+\frac{\xi^2}{\alpha_\pi\lambda^2}P_{\alpha-1}{}'(z)\right).$$

In the limit $m_V{}^2\to 0$, Eq. (28) is comparable with Eq. (8) of Ball *et al.*[3] since A_2 of Ref. 10 can be written from Eq. (28) as

$$A_2{}^\pi\equiv\frac{U_2{}^\pi}{t-\mu^2}=-\frac{1}{t}\frac{1}{s-u}\frac{\beta_\pi{}^Z}{\alpha_\pi}\frac{1+e^{-i\pi\alpha_\pi}}{\sin\pi\alpha_\pi}zP_\alpha{}'(z)\,, \quad (29)$$

which is different by $\alpha_\pi{}^{-1}$ from the form in Ref. 3. The above approach to the problem of making gauge invariance and Regge-pole theory compatible, however, does not lead to the same $(t-\mu^2)^{-1}$ pole for a possibly existing second heavy pion (hb of Sec. I) as for the true pion but predicts the pole position to be given by $m_{\rm hb}{}^2$. This is consistent with the intuitive and long accepted idea that the mass of a particle is inversely related to the interaction range. For large s, $P_\alpha{}'$ gives a factor α in the numerator which cancels the α in the denominator and leaves $1/\sin\pi\alpha(t)$ to determine the pole position.

It is clear that the above is independent of whether a meson conspires or evades. If the pion, e.g., evades, and the conspiring contributions which give rise to the forward peak result from some other mechanism, e.g., cuts (which lead to a good data fit),[14] then the only modification required in the preceding is to change $t^{-1/2}$ to $t^{1/2}$ in the kinematical factor which multiplies the pion Regge term in $\tilde{X}_2{}'=\tilde{f}_{10\frac{1}{2}\frac{1}{2}}{}^-$, Eq. (14). According to perturbation theory, the above residue function β^Z in Eq. (29) has a zero which for the pion trajectory exchange should be at $t=-\mu^2$. By relaxing such rather stringent perturbation-theory restrictions, Shih and Tung[15] found that the data on photoproduction of π^+ and K^+ mesons are consistent with meson conspiracy, i.e., exchange of parity doublets π, π_c and K, K_c, respectively. Unfortunately, the form of (29) is not different enough from formulas used in analyses of either type, so that changes would not occur in existing work.

[14] J. Frøyland and D. Gordon, MIT Report, 1967 (unpublished); Phys. Rev. **177**, 2500 (1969).

[15] C. C. Shih and W. K. Tung, Phys. Rev. **180**, 1446 (1969).

V. CORRESPONDENCE WITH PERTURBATION THEORY AND THE FORWARD PEAK

In Sec. III we derived sum rules (19) and (20). The fact that these sum rules do not involve any of the helicity amplitudes $\tilde{X}_1$ and $\tilde{Y}_1$, which contain such trajectories as those of the ρ and those of the A_2 mesons, can be understood in terms of a perturbation calculation. As is well known, vector-meson–pion–photon coupling and tensor (parity even) meson–pion–photon coupling can only be gauge-invariant through parity conservation. We also notice that if the evasion solution for these Regge poles is taken, the correspondence between Regge theory and perturbation is recovered completely.

On the other hand, consider the case of the A_1 meson. If we denote the A_1, photon, and pion fields by U_μ, V_μ, and π, respectively, we would have two independent coupling forms from parity conservation alone,

$$(U_\mu V_\mu)\pi \quad\text{and}\quad (U_\mu\partial_\nu V_\mu)\partial_\nu\pi.$$

Gauge invariance restricts these to the two forms

$$U_\mu(\partial_\mu V_\nu-\partial_\nu V_\mu)\partial_\nu\pi \quad\text{and}\quad U_\mu(\partial_\nu{}^2 V_\mu)\pi\,,$$

and it is easy to see that the above situation is in exact analogy with the treatment in Sec. III.

Now we turn to the more interesting case of the pion pole. It is well known that in perturbation theory gauge invariance is achieved by combining the pion pole with the nucleon pole. If we calculate these pole contributions to the U_i's, we get

$$U_1{}^B=ge\left(\frac{1}{s-M^2}-\frac{1}{u-M^2}\right),$$
$$U_2{}^B=-ge\left(\frac{1}{s-M^2}-\frac{1}{u-M^2}\right), \quad (30)$$
$$U_3{}^B=+ge\left(\frac{1}{s-M^2}+\frac{1}{u-M^2}+\frac{4}{t-\mu^2}\right),$$
$$U_4{}^B=U_5{}^B=U_6{}^B=0.$$

At $t=\mu^2$ (with $m_V{}^2=0$) we obtain

$$-U_1{}^B=U_2{}^B=ge\frac{s-u}{(s-M^2)(u-M^2)}=\frac{-2ge}{s-M^2},$$

$$(t-\mu^2)U_3{}^B=ge\left(\frac{-(t-\mu^2)^2}{(s-M^2)(u-M^2)}+4\right)=4ge.$$

On the other hand, Eqs. (28) give, at $t=\mu^2$,

$$U_2^\pi = -2\beta/[2(s-M^2)\pi\alpha'(\mu^2)\mu^2],$$
$$(t-\mu^2)U_3^\pi = 2\beta/[\pi\alpha'(\mu^2)\mu^2].$$

Thus, correspondence with the perturbation result is clearly achieved for U_2 and U_3 if we set

$$\beta/[\pi\alpha'(\mu^2)\mu^2] = 2ge. \tag{31}$$

The nucleon pole in U_1, however, cannot be reproduced without further postulation. We found that the nucleon pole in U_2 is represented by the pion Regge pole in Regge theory, and we expect that the nucleon pole in U_1 also has some correlation with the pion. Therefore, we can assume the pion pole in $\tilde{X}_2'$ to enter into a conspiracy as is discussed by several authors.[2,3] The conspiracy relation is given by Eq. (20). Since $(t-\mu^2)\tilde{X}_2'$ does not have singularities in the neighborhood of $t=\mu^2$, we can write for $t\approx\mu^2$

$$(t-\mu^2)\tilde{X}_2' = f(t)[(t-\mu^2)\tilde{X}_2']_{t=\mu^2}, \tag{32}$$

where $f(t)$ is a gently behaving "form factor," $f(\mu^2)=1$. From Eqs. (23), (24), (27), and (31) we obtain

$$A_1(t=0) \equiv [U_1+2MU_6]_{t=0} = -(2/M)\tilde{Y}_1(t=0) = f(0)[4ge/(s-M^2)]. \tag{33}$$

With $f(0)=\frac{1}{2}$, this equation reduces to the perturbation result.[8] Also, in order that Eq. (33) be compatible with the perturbation result, we require the contribution of the conspirators to $\tilde{X}_1$—say, $\tilde{X}_1^c$—to satisfy the relation

$$\tilde{X}_1^c \approx -\tilde{Y}_1(t=0)/2M.$$

Finally, a particular parametrization of the function introduced in Eq. (32),

$$f(t) = \tfrac{1}{2}(t+\mu^2)/\mu^2,$$

will give the (perturbation theory) result as given in Ref. 8.

VI. REMARKS

We have shown that if the "sum rules" (16) and (17) are accepted, our Regge-pole representation [Eqs. (23) and (24)] can be applied readily to the processes involving virtual photons. Production of pions with high energy electrons is an experimentally feasible application of our theory and also a test of it through comparison with photoproduction processes, or, stated differently, electroproduction will enable us to measure the electromagnetic form factor of the Regge pole.

The approach of the present paper, we feel, aids in understanding the similarity between the ρ meson and the photon. It is evident, regardless of the invariant mass, that the $J=1$ state of two pions, say, can be regarded as V in the process of Eq. (1). In other words, our results in Eqs. (23) and (24) with relations (16) and (17) should be applicable to the production of any of the $J=1$ states (like $\pi\pi$, $\mu^+\mu^-$, etc., if allowed by the conservation of other quantum numbers,) in pion-nucleon collisions at high energy. This interpretation helps us to see why a ρ meson (which is "established" as a Regge particle) can behave like a photon (which may or may not be a Regge pole).

In the previous section, we argued that the comparison of Regge theory with perturbation theory suggests the existence of a pion conspirator. It is easy to see that a linearly polarized photon with polarization normal to the reaction plane can be very useful for the study of this conspirator since the pion Regge pole cannot contribute to $\gamma+p\rightarrow\pi^++n$ with such incident photons.

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