and

$$a_0(s)-\frac{2}{4-s}\int_{(4-s)/2}^{(4-s)} dt\, a_0(t)$$

$$=\frac{2}{\pi(4-s)}\int_{(4-s)/2}^{4-s} dt\int_4^\infty ds' A_s(s',t)$$

$$\times\left[\frac{1}{s'-s}+\frac{1}{s'+s+t-4}-\frac{2}{4-t}\ln\left(\frac{s'}{s'-4+t}\right)\right],$$

$$0\leqslant s<4. \quad (32)$$

We can show that the integrand on the right-hand side of (31) is nonpositive for all values of t, s' in the range of integration if $c_2\leqslant s\leqslant c_3$, where c_2, c_3 are given by (16), and non-negative for all values of t, s', if $s=0$. Further, the integrand on the right-hand side of (32) is non-negative for all values of t, s' in the range of integration if $c_1\leqslant s<4$, where c_1 is given by (19). Thus, we obtain the right-hand side of inequality (15) and the left-hand sides of the inequalities (17) and (18). Finally, we notice that for the above values of s, the values of t in $A_s(s',t)$ are in the interval (0,2). Hence we combine (31) and (32) with (23) to obtain the remaining parts of the inequalities (15) and (18). In (17) an upper bound on $x(0)$ is not obtained because the integral of $f(t,0)$ diverges at $t=0$.

ACKNOWLEDGMENTS

I am grateful to Professor Virendra Singh for suggesting the derivation of the inequality (7). I wish to thank Dr. J. Pasupathy for helpful discussions and A. S. Anikhindi for help in the numerical computations.

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Predictions of a New Interference Model for Near-Forward Pion-Nucleon Scattering*

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Using parameters determined by outside means, we discuss the predictions for small-angle pion-nucleon scattering below 2 GeV/c of the interference model that equates the background to the Pomeranchon exchange amplitude. The quantitative predictions for elastic scattering are rather poor, but the moderate success of the direct-channel resonances alone in predicting the charge-exchange process leads to a suggestion for the determination of resonance parameters in phase-shift analysis, and to a reinterpretation of the model which has several interesting features.

I. INTRODUCTION

In this paper we examine some predictions of a new form of the interference model for small-angle pion-nucleon scattering at laboratory momenta between approximately 1 and 2 GeV/c.

The model is that suggested by Harari[1] on the basis of a conjectured link through finite-energy sum rules between low-energy nonresonant background scattering and high-energy Pomeranchon exchange.[2]

If it is supposed that an amplitude A at nonasymptotic energies can be divided into resonant and background parts:

$$A=A_{\rm res}+A_{\rm bgd}, \quad \nu\leqslant N, \quad (1)$$

then the suggestion is that

$$N^{-n-1}\int^N \nu^n\, {\rm Im}A_{\rm bgd}(\nu)d\nu\approx\tilde{P}(N), \quad (2)$$

and

$$N^{-n-1}\int^N \nu^n\, {\rm Im}A_{\rm res}(\nu)d\nu\approx\sum_{\text{poles } i,\text{ not } P}\tilde{R}_i(N). \quad (3)$$

In these equations, $\nu\propto s-u$, the integer n is even or odd as appropriate, N marks the onset of Regge asymptotic behavior, and $\tilde{P}(N)$ and $\tilde{R}_i(N)$ denote the Pomeranchon and "normal" Regge-pole terms, respectively.[3]

Harari's work[1] correlates a number of experimental facts concerning hadron scattering cross sections and embodies [in Eq. (3)] the apparent fact that perhaps only "normally" steep (slope ~ 1 GeV^{-2}) Regge trajectories can be built through sum rules from low-energy resonance saturation.[4] An important connection be-

* Work supported by the National Research Council of Canada.

[1] H. Harari, Phys. Rev. Letters **20**, 1395 (1968).

[2] Such a link was apparently first conjectured, for pion-pion scattering, by P. G. O. Freund, Phys. Rev. Letters **20**, 235 (1968).

[3] The formulation of finite-energy sum rules is reviewed by R. Dolen, D. Horn, and C. Schmid, Phys. Rev. **166**, 1768 (1968).

[4] D. J. Gross, Phys. Rev. Letters **19**, 1303 (1967); P. G. O. Freund, *ibid.* **20**, 235 (1968); C. Schmid, *ibid.* **20**, 628 (1968); C. Schmid and J. Yellin, Phys. Letters **27B**, 19 (1968) and Phys. Rev. (to be published); M. Ademollo, H. R. Rubinstein, G. Veneziano, and M. A. Virasoro, Phys. Rev. Letters **19**, 1402 (1967) and Phys. Letters **27B**, 99 (1968).

tween these ideas and the concept of duality has been pointed out,[5] and it has also been shown that some quark-model relations follow from them.[6] These relations are known to be fairly well satisfied.[7]

The old interference model,[8] which has a number of difficulties,[9,10] identifies A_{bgd} with the sum of all prominent exchanged Regge poles. The new interference model that we test here is constructed by equating A_{bgd} to the Pomeranchon-exchange amplitude. This local identification is equivalent to supposing that (2) is satisfied exactly for all values of n, however large. It must therefore be recognized as an assumption beyond Eqs. (2) and (3), which are not asserted to be completely accurate even for small n.[1]

This model suggests that the amplitudes for processes where vacuum exchange at high energy is forbidden should be described by resonances alone at lower energies. For example, pion-nucleon charge-exchange scattering should be well represented by N^* resonances alone. The resonance amplitudes should be supplemented by Pomeranchon Regge-pole exchange to describe the elastic pion-nucleon processes. Finite-energy sum rules connect the N^* states to the ρ, ρ', $\cdots$ and P', P'', $\cdots$ Regge exchanges.

We test the model for these processes in the near-forward direction, using the N^* resonances found in a phase-shift analysis below 2 GeV/c,[11,12] together with the new, more precise determinations of the Regge-pole parameters for small momentum transfer permitted by simultaneous fitting of generalized finite-energy sum rules and high-energy data.[13] We assume that a simple extrapolation of the P term of Barger and Phillips[13] gives the correct form of the Pomeranchon background below 2 GeV/c, but we do not expect it to be accurate right down to threshold. Details of resonance and background terms are given in Sec. II.

We choose simple forms for the N^* amplitudes that give a reasonable representation of the charge-exchange data. Adding the P background, we predict the elastic $\pi^{\pm}p$ data near the forward direction. These predictions are given in Sec. III.[14]

It appears that the Pomeranchon term gives a poor estimate of the elastic background, but the prediction of no background in the charge-exchange process is compatible with experiment. In the final section we discuss possible implications of this fact for the πN phase-shift analysis. A model is suggested for the effective P singularity which includes both a "normal" Regge pole and another vacuum singularity which is to be associated, through Eq. (2), with low-energy background scattering.

II. RESONANCES AND BACKGROUND

The N^* resonances have been extracted from a pion-nucleon phase-shift analysis extending up to about 2 GeV/c,[11,12] while the Regge poles, including the P, have been assumed to dominate above this momentum.[13] Because relatively little is known about N^* states above 2 GeV/c, we extrapolate the Pomeranchuk term to lower momenta to make predictions. The validity of this procedure is our first assumption.

Our second assumption is that the resonance parameters quoted by Lovelace[12] are approximately correct and refer to simple Breit-Wigner resonance amplitudes whose (probably unrealistic) long tails can be removed by a multiplicative cutoff function which goes quickly to zero a few full-widths away from the peak.

We try to gauge the effect of these two assumptions by repeating the calculations using some different formulas and parameters, as described in Sec. III.

For N^* resonances we parametrize the partial waves as[15]

$$f_{\text{res}}(W) = x\theta(m,\Gamma,W,n)\{2(m-W)/\Gamma - i\}^{-1}, \quad (4)$$

for a resonance of mass m, full width at half height Γ, and elasticity x. The variable W is the c.m. total energy $s=W^2$. The cutoff function θ is chosen to be

$$\theta(m,\Gamma,W,n) = 2\{1+e^{[(W-m)/n\Gamma]}\}^{-1}. \quad (5)$$

The length of the resonance tail is large or small according to the value of n. Possible improvements of this simple resonance expression include the use of energy-dependent widths.

Table I gives the resonances and their parameters, taken from Lovelace.[12] The numbers are slightly rounded, and the quoted[12] status of each state (established, probable, etc.) is indicated.

[5] R. C. Johnson, unpublished report, 1968; M. Kugler, Phys. Rev. (to be published); C. Schmid, CERN Report Th. 958, 1968 (unpublished).
[6] C. B. Chiu and J. Finkelstein, Phys. Letters **27B**, 516 (1968). Further predictions, concerning the effects of baryon-antibaryon channels, are given by J. L. Rosner, Phys. Rev. Letters **21**, 950 (1968) and D. P. Roy and M. Suzuki, CERN Report Th. 976, 1968 (unpublished).
[7] R. C. Johnson and R. K. Logan (unpublished).
[8] V. Barger and D. Cline, Phys. Rev. Letters **16**, 913 (1966); Phys. Rev. **155**, 1792 (1967); V. Barger and M. Olsson, *ibid.*, **151**, 1123 (1966).
[9] C. B. Chiu and A. V. Stirling, Nuovo Cimento **56A**, 805 (1968).
[10] R. Dolen, D. Horn, and C. Schmid, Phys. Rev. **166**, 1768 (1968).
[11] P. Bareyre, C. Bricman, and G. Villet, Phys. Rev. **165**, 1732 (1968), and Report CEA-R 3401 (unpublished); C. H. Johnson and H. Steiner, in Proceedings of the 1967 Irvine Conference on πN Scattering (to be published).
[12] C. Lovelace, in *Proceedings of the Heidelberg International Conference on High Energy Physics*, edited by H. Filthuth (North-Holland Publishing Co., Amsterdam, 1968).
[13] V. Barger and R. J. N. Phillips, Contribution to the Vienna International Conference on High Energy Physics, 1968 (unpublished).
[14] Some predictions for forward $\pi^{\pm}p$ elastic scattering data have been made independently by D. R. Dance and G. Shaw, Phys. Letters **28B**, 182 (1968). These authors do not consider charge-exchange or nonforward processes, and reach substantially different conclusions.
[15] Units and normalization are as defined by J. Hamilton and W. S. Woolcock, Rev. Mod. Phys. **35**, 737 (1963), and by Chiu and Stirling (Ref. 9).

TABLE I. Pion-nucleon resonances, taken from Ref. 12. Masses and widths are in MeV.

Partial wave	Mass, m	Width, Γ	Elasticity, X	Status[a]
P_{33}	1236	125	1	E
P_{11}	1470	210	0.7	E
D_{13}	1530	110	0.6	E
S_{31}	1630	180	0.3	E
D_{15}	1680	170	0.4	E
F_{15}	1690	130	0.7	E
S_{11}	1710	300	0.8	E
F_{37}	1950	220	0.4	E
G_{17}	2260	300	0.35	E
$H_{3,11}$	$\sim$2400	$\sim$340	$\sim$0.2	E
S_{11}	1550	120	0.3	P
P_{33}	1690	280	0.1	P
F_{35}	1910	350	0.2	P
P_{31}	1930	340	0.3	P
D_{13}	2060	290	0.3	P[b]
D_{33}	1690	270	0.1	D
P_{13}	$\sim$1860	$\sim$300	$\sim$0.2	D
D_{35}	$\sim$1950	$\sim$310	$\sim$0.2	D[b]
P_{11}	$\sim$1750	$\sim$330	0.3	U
F_{17}	1980	220	0.1	U[b]
H_{19}	$\sim$2450	$\sim$350	$\sim$0.4	U[b]

[a] According to Ref. 12, E≡established, P≡probable, D≡doubtful, U≡unconfirmed.
[b] Denotes that this wave was omitted in order to get agreement with Im$A'^{(-)}(\nu, t=0)$ just above 2 GeV/c.

The Pomeranchon amplitudes used in Ref. 13 are,[15,16]

$$A'^{(+)}(\nu,t)=\gamma(t)(\nu_0^2-\nu^2)^{1/2}, \tag{6a}$$

$$B^{(+)}(\nu,t)=\beta(t)\nu(\nu_0^2-\nu^2)^{-1/2}, \tag{6b}$$

where $\nu=(s-u)/4M$ and $\nu_0=\mu+t/4M$, and μ, M are pion and nucleon masses, respectively. A fit to sum rules and data for $0\geqslant t\geqslant -0.5$ (GeV/c)^2 was found for[13]

$$\gamma(t)=53.1e^{3.90t}\ \text{GeV}^{-1}, \tag{7a}$$

and

$$\beta(t)=42.2e^{2.16t}\ \text{GeV}^{-2}. \tag{7b}$$

The nonflip residue $\gamma(0)$ corresponds to an asymptotic total cross section of 20.7 mb, essentially indistinguishable from the preferred values obtained by Rogers and Schwarz,[17] who fitted high-energy total-cross-section data alone with both $(P+P')$ and $(P+\text{cut})$ models.

We use the amplitudes of Eqs. (6) and (7) as background in predicting $\pi^\pm p$ scattering for $0\geqslant t\geqslant -0.5$ (GeV/c)^2.

III. PREDICTIONS

In Fig. 1 we show the predictions of the model for the imaginary part of the forward charge-exchange amplitude, which is proportional to the difference of the π^+p and π^-p total cross sections. Indicated in the figure are the effects of the cutoff function on the resonance amplitudes. Evidently the model works quite well here below about 2 GeV/c. A small discrepancy above this momentum disappears if some of the more doubtful resonances are omitted. (See Table I.)

Figures 2 and 3 show the model predictions for the π^-p and π^+p total cross sections. It appears that the model fails, predicting much too large an imaginary part for the forward elastic amplitudes, particularly in the π^+p process at low energies. The disagreement can be reduced slightly but not significantly by cutting off the resonance tails more quickly.

Multiplying the background by small powers of p/E (p=lab momentum, E=lab energy), which re-

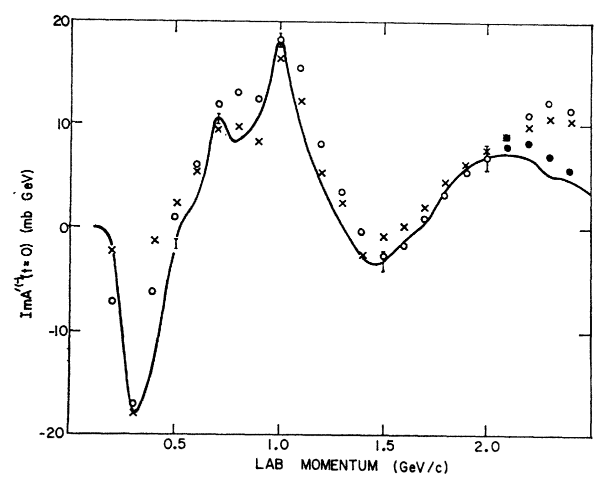

FIG. 1. Predictions for Im$A'^{(-)}$ at $t=0$. The solid line is from experiment, showing representative errors (Ref. 38), and the open circles and crosses are the predictions of the new interference model for "long-tailed" resonances [cutoff 20 widths away, $n=20$ in Eq. (5)], and "short-tailed" resonances, ($n=3$), respectively. The closed circles are the ("short-tailed") predictions above 2 GeV/c if some of the doubtful resonances of Table I (marked there by an asterisk) are omitted.

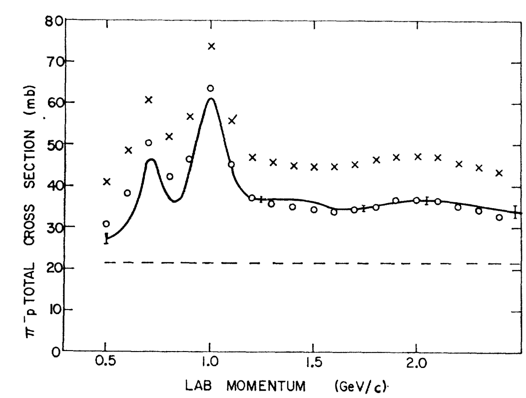

FIG. 2. Predictions for $\sigma_{\text{tot}}(\pi^-p)$. The solid line is from experiment, showing representative errors (Ref. 38), the broken line is the P-exchange background of Eqs. (6a) and (7a), and the crosses are the predictions of the model for "short-tailed" resonances. The circles are the predictions of the model with a background equal to one-half the P term of Ref. 13, as described in the text.

[16] The units and normalizations of Ref. 13 are identical to those of W. Rarita, R. J. Riddell, C. B. Chiu, and R. J. N. Phillips, Phys. Rev. **165**, 1615 (1968).

[17] T. W. Rogers and J. H. Schwarz, Phys. Rev. **172**, 1595 (1968).

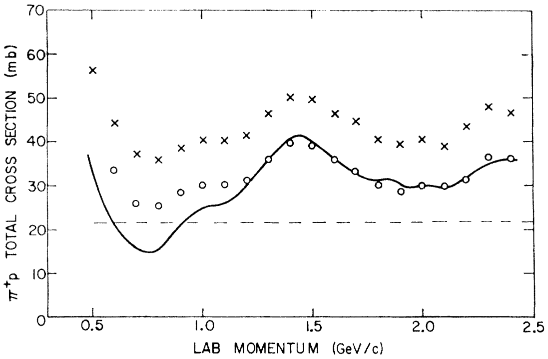

FIG. 3. Predictions for $\sigma_{tot}(\pi^+p)$. Notation is as for Fig. 2.

duces the P amplitude near threshold without changing its asymptotic behavior, has little effect above 1 GeV/c. This indicates that below this momentum the extrapolation of the Pomeranchuk term is more or less arbitrary, as expected, and we concentrate only on the region 1–2 GeV/c.

The results of Figs. 1–3 are reflected in the finite-energy sum rules[13] for $\mathrm{Im}A'^{(\pm)}(\nu, t=0)$, which read

$$\frac{1}{N^2}\int_0^N \nu\,\mathrm{Im}A'^{(+)}(\nu)d\nu=\sum_{P,P',P''}\gamma\frac{N^\alpha}{\alpha+2}, \qquad (8)$$

and

$$\frac{1}{N}\int_0^N \mathrm{Im}A'^{(-)}(\nu)d\nu=\sum_{\rho,\rho'}\gamma\frac{N^\alpha}{\alpha+1}, \qquad (9)$$

where N is fixed at a lab momentum of 2 GeV/c. The amplitudes are not well enough known for $\nu \lessgtr N$ to attach any significance to higher-moment sum rules.

The ρ' pole of Eq. (9) decouples at $t=0$ in the Barger-Phillips model,[13] and the ρ gives 2.58 mb GeV to the right-hand side. The resonances-plus-nucleon pole term give 3.03 mb GeV for the left-hand side. The discrepancy can probably be traced to the deficiency of our simple representation of the prominent $N^*(1238)$. If we parametrize it with an energy-dependent width,[18] we can get a better fit just above its peak (~0.4 GeV/c) and reduce the left-hand side by 0.35 mb GeV to give better than 95% agreement.

Cancelling the P contribution from both sides of Eq. (8),[19] we arrive at the appropriate version of Eq. (3). The resonances alone give for the left-hand side 14.5 mb GeV^2,[20] whereas the right-hand side is equal to[13]

$$8.3(P')-1.2(P'')\quad \text{mb GeV}^2.$$

[18] M. Gell-Mann and K. Watson, Ann. Rev. Nucl. Sci. 4, 219 (1954).

[19] The cancellation presumes that the Pomeranchon term is given by Eq. (6a) alone, and that powers of p/E, which mutilate its analytic properties, are not used.

[20] The nucleon pole term is negligible, being equal to 0.01 mb GeV^2. For the resonances here [and in Eq. (9) where in fact such details are numerically insignificant] we take "short-tailed" resonances, i.e., $n=3$ in Eq. (5).

It seems that not only does the P overestimate the background, but the resonances build more than just the rest of the vacuum trajectories. This may be regarded as evidence that the P is linked to some extent to low-energy resonances.

It is not inconceivable that the effective P singularity contains a normal trajectory, the first recurrence of which is the $F(1060)$, for which there is some evidence[21,22] of $J^P=2^+$.

This is consistent with the fact that if a Breit-Wigner form (4) is a realistic approximation to a resonance, then the sum of such contributions to $A'^{(+)}$ at $t=0$ (where all the states enter with the same sign) has a large imaginary part and a small real part. Such resonances would seem unsuitable to build just the P' (P'' etc., are presumably negligible) which has roughly equal real and imaginary parts, since[23] $\alpha_{P'}(0) \sim 0.5$. This argument should be regarded with some caution, however, because, for example, in $\pi\pi$ scattering, according to Harari's model, both P'-like and ρ-like exchanges should be built up from resonances which enter with the same sign. If the resonances build a trajectory above the P', they should also build one above the ρ. The difference in the two cases lies in the sign of the $I=2$ direct-channel contribution, usually assumed to be nonresonant. However, if the "normal" F trajectory is really significant (and not accompanied by an opposite-signature exchange-degenerate partner), there should be some resonant structure in $\pi\pi$ $I=2$ scattering, from Eq. (3). This could restore consistency. However, perhaps a more conservative viewpoint would be that the simple Breit-Wigner form is misleading.

In $A'^{(-)}$ at $t=0$ the resonances enter with opposite signs, so that both their real and imaginary parts tend to cancel, and the linking of the ρ to the N^*'s alone is plausible. As Fig. 4 shows, we find that the resonances alone represent $\mathrm{Re}A'^{(-)}$ less well than $\mathrm{Im}A'^{(-)}$ at $t=0$, giving, on the average, about 20% too low a value for the forward charge-exchange differential cross section.[24]

Such a disagreement persists away from $t=0$, where the large spin-flip amplitude $B^{(-)}$ begins to play a role. It turns out, however, that if energy-dependent resonance widths are used (in the way suggested by Gell-Mann and Watson[18]), most of the discrepancy can be removed. (See Fig. 5.)

Although here we are primarily concerned with

[21] C. Whitehead, J. G. McEwan, R. J. Ott, D. K. Aitken, G. Bennett, and R. E. Jennings, Nuovo Cimento 53A, 817 (1968); D. H. Miller, L. J. Gutay, P. B. Johnson, V. P. Kenney, and Z. G. T. Guiragossian, Phys. Rev. Letters 21, 1489 (1968); Phys. Letters 28B, 51 (1968).

[22] R. C. Johnson, Nucl. Phys. B5, 673 (1968).

[23] This point has been made also by Dance and Shaw (Ref. 14).

[24] A. S. Carroll, I. F. Corbett, C. J. S. Damerall, N. Middlemas, D. Newton, A. B. Clegg, and W. S. C. Williams, Phys. Rev. Letters 16, 288 (1966). As a representation of the forward differential cross section we use the dispersion relation fit given by G. Hohler, G. Ebel, and J. Giesecke, Z. Physik 180, 439 (1964). Fig. 3 of this paper [and Fig. 4 of Olsson's paper (Ref. 26)] show that this is in good agreement with experiment.

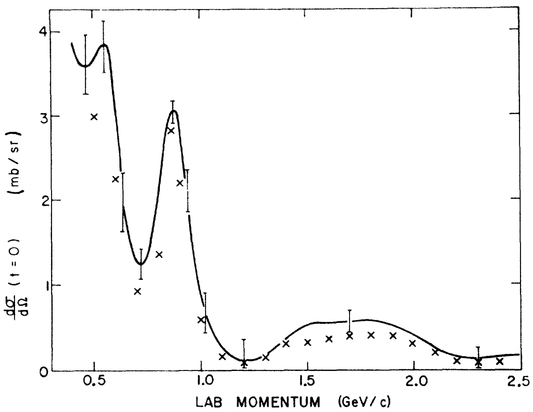

FIG. 4. Predictions of the N^* resonances alone (the crosses) for the forward charge-exchange differential cross section. The solid line is a dispersion relation prediction from Hohler *et al.* (Ref. 24). This is a good representation of the data. Some typical error bars are given.

near-forward scattering [since the background P amplitude[13] for the elastic processes is known only for $0\geqslant t\geqslant -0.5$ $(\mathrm{GeV}/c)^2$], we have found that there is enough uncertainty in the N^* states[25] to find reasonable

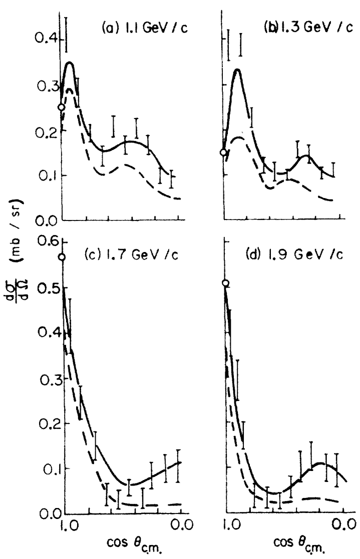

FIG. 5. Predictions of the N^* resonances alone for the charge-exchange differential cross section at (a) 1.1 GeV/c (data from Ref. 26), (b) 1.3 GeV/c (Ref. 26), (c) 1.7 GeV/c (Ref. 24), and (d) 1.9 GeV/c (Ref. 24). The broken line is the model prediction with "short-tailed" resonances [$n=3$ in Eq. (5)] with constant widths. The solid line is a prediction with energy-dependent widths given by the formula of Ref. 18, namely, $\Gamma(q)=\Gamma(q/q_R)^{2l+1}F(q)$, where q is the c.m. momentum (q_R=resonance momentum) and l is the relative angular momentum of the resonating pion and nucleon. The function $F(q)$ is $F=(1+q^2R^2)^{-1}$, where the "interaction range" R we take to be $0.9\mu^{-1}$, which is appropriate for the $N^*(1236)$ [see M. G. Olsson, Phys. Rev. Letters **14**, 118 (1965)]. The open circles at $t=0$ are the dispersion relation predictions of Hohler *et al.* (Ref. 24).

[25] Compare, for example, Refs. 11 and 12.

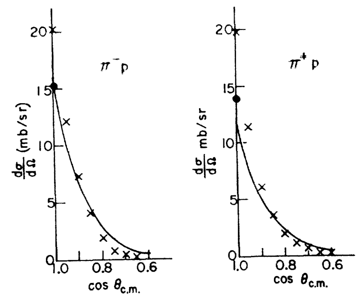

FIG. 6. Predictions for the $\pi^{\pm}p$ differential cross sections at 1.58 GeV/c. The data (the solid line) are from Duke *et al.* (Ref. 27). The errors are insignificant on the scale of the figure. The crosses are the predictions of the interference model using the background of Eqs. (6) and (7) with the resonance terms that give the best fits in Fig. 5. The closed circles are the predictions for the forward differential cross section if the P background there is halved, as described in the text.

fits to the available charge-exchange data[24,26] at all angles for momenta between threshold and 2 GeV/c. Besides the uncertainty in the resonance parameters, the freedom to parametrize energy-dependent widths and cutoff functions is sufficient to permit a number of acceptable solutions. We have no grounds for quoting any preferred fit.

Using the resonance forms corresponding to the better predictions given in Figs. 4 and 5, we can use the Pomeranchon background of Eqs. (6) and (7) to predict $\pi^{\pm}p$ elastic angular distributions,[27] and two examples are given in Fig. 6. These are chosen because they show features typical of the results for the momentum range 1–2 GeV/c. The model predictions overestimate the forward differential cross section, but they tend towards agreement or underestimate for $t\lesssim -0.2$ $(\mathrm{GeV}/c)^2$.

Therefore, it appears that, as it stands,[13] the P overestimates the background in a region very close to $t=0$, and falls slightly below it away from the forward direction, where its exponentially vanishing residues drop off quickly.

Above about 1.7 GeV/c, the model predicts correctly a crossover effect.[28] This is associated with a zero in the spin-non-flip amplitude $A'^{(-)}$ near $t=-0.2$ $(\mathrm{GeV}/c)^2$ through a finite-energy sum rule. Prominent resonances

[26] C. B. Chiu, R. D. Eandi, A. Carl Helmholz, R. W. Kenney, B. J. Moyer, J. A. Poirier, W. B. Richards, R. J. Cence, V. Z. Peterson, N. K. Sehgal, and V. J. Stenger, Phys. Rev. **156**, 1415 (1967); L. Guerriero, Proc. Roy. Soc. (London) **289**, 471 (1968). Further references are given by Bareyre *et al.* Ref. 11), by M. G. Olsson, Phys. Rev. **171**, 1681 (1968), and by L. D. Roper, R. M. Wright, and B. T. Feld, *ibid.* **138**, B190 (1965).

[27] P. J. Duke, D. P. Jones, M. A. R. Kemp, P. G. Murphy, J. D. Prentice, and J. J. Thresher, Phys. Rev. **149**, 1077 (1966); W. Busza, B. G. Duff, D. A. Garbutt, F. F. Heymann, C. C. Nimmon, K. M. Potter, T. P. Swetman, E. H. Bellamy, T. F. Buckley, R. W. Dobinson, P. V. March, J. A. Strong, and R. N. F. Walker, Rutherford High Energy Laboratory Report RPP/11/49, 1968 (unpublished). For a compilation of earlier data, see Roper *et al.* (Ref. 16).

[28] For a discussion see, e.g., R. J. N. Phillips and W. Rarita, Phys. Rev. **139**, B1336 (1965).

have their first zero together in this amplitude at such a momentum transfer.[10,29]

None of these qualitative features are changed by altering the parametrization of the resonance amplitudes.

The model predictions for the polarization in elastic $\pi^{\pm}p$ scattering are essentially random. The reason is that the largest part of it is given by interference between the wholly positive-imaginary Pomeranchon background and the real parts of the resonance amplitudes. The latter oscillate with energy and angle, and the precise nature of these fluctuations is very sensitive to the way that the resonance tails are eliminated, and to details of width parametrization. Polarization tests are therefore ambiguous.

IV. CONCLUSIONS

Although this new interference model in its present form fails to give good quantitative predictions of near-forward pion-nucleon elastic scattering, at least one of its qualitative aspects may be of some value. This is the suggestion that the background amplitude is isospin-independent.

We have found that the N^* resonances alone fail to describe the isospin-$\frac{1}{2}$ and isospin-$\frac{3}{2}$ pion-nucleon elastic amplitudes by approximately equal amounts, or, in other words, that the resonances give a quite good prediction for the charge-exchange process. This seems to suggest that N^* resonance parameters can be better determined from πN phase-shift analysis by eliminating the background between the partial waves of the same orbital and total angular momentum but different isospin.

In the present model the full amplitude is built up by the simple addition of resonant partial waves to a purely imaginary background. Examination of the Argand plots of the πN partial waves[12] shows that (a) It is rather likely that the nonresonant background has in general a real part, and that (b) it is also rather likely that there is a phase difference between resonance and background. Perhaps, therefore, a more realistic approach would be to try to fit (for example) the CERN set of pion-nucleon phases[12] using an isospin-independent background, allowing for a possible real part, and combining this with some resonance forms in the unitary manner of Dalitz and Michael.[30] This would have both parts of the partial wave separately unitary, and allow a phase difference between them. Quite possibly some of the less prominent members of the resultant set of N^* states would have substantially different properties from those usually suggested.[11,12,31]

We conclude by suggesting a possible modification of Harari's model. This is to suppose that the effective P singularity used in fitting the data[13,17] contains not only a fixed and possibly symmetry-independent diffractive singularity which is connected through sum rules to the low-energy background, but also a "normal" Regge trajectory which has the $F(1060)$ as its first recurrence.[22]

Such a trajectory would help the sum rules for the crossing-even pion-nucleon and pion-pion[32] amplitudes. Because it (presumably) has no partner of opposite signature and equal G parity, it would allow breaking of the rule against "exotic" particles described in Refs. 5–7, and permit resonances in, for example, KN,[33] NN,[34] and $\pi^+\pi^+$ [32] scattering. In an $SU(3)$ picture the $F(1060)$ would mix with the $f(1260)$ and $f'(1515)$. Then its possibly different couplings to $\pi\pi$ and $K\bar{K}$ would explain the presumed difference between the kaon-nucleon and pion-nucleon asymptotic total cross sections,[35] (while possibly the "true" diffractive P remains pure singlet).

The similarity of this model to that of Abarbanel, Drell, and Gilman[36] and of Chou and Yang[37] should be noted, but it must be stressed that we envisage not a universal current-current point interaction dominating at high energy, but rather a vacuum-exchange model with some J-plane singularity that does not contribute to charge-exchange and other inelastic processes.

If we assume that the $F_{2+}(1060)$ trajectory has unit intercept, (this may of course not be true) and observe that the P contributes[13] 13.8 mb GeV^2 to the right-hand side of Eq. (8), then we infer that about half the P is background. If that is so, then we predict that the $\pi^{\pm}p$ total cross sections will be as shown in Figs. 2 and 3, and the forward elastic differential cross sections as shown in Fig. 6. The agreement with experiment[27,38] is fairly satisfactory.

[29] This explanation is a good illustration of the powerful correlation between high- and low-energy scattering provided by finite-energy sum rules.

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[31] Such changes are likely to be confined to the most inelastic states, and therefore the fit to charge-exchange scatterings (which motivates the whole procedure) most probably would survive.

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