havior of the Bessel function $I_\nu(y)$ for large ν. From the representation (32), we have

$$F(z)=(\tfrac{1}{2}\pi)^{1/2}\int_0^\infty dy\, e^{-yz}y^{-1/2}\sum_{p=0}^{\infty}\zeta_p I_{p-\alpha-1/2}(y)\,. \quad \text{(A4)}$$

Comparing this with (25b) and (26), (A3) shows that $f(s,y)$ must be an entire function of y. Since $f(s,y)$ in (26) has branch points where

$$y/2a'q_s^2=\pm 2\pi i,\quad \pm 4\pi i,\ \ldots,$$

unless $\alpha(s)$ is an integer, it follows that $A(s,t)$ cannot have a convergent series of the form (A1) except for integer α. The points for which $\alpha(s)=1,\ 2,\ \cdots$ are singular points of $A(s,t)$. Hence $A(s,t)$ may only have a convergent series of the form (A1) when

$$\alpha(s)=-1,\ -2,\ \ldots. \quad \text{(A5)}$$

(It is interesting to note that these would be the so-called indeterminacy points of a potential theory.) It is trivial to show that $A(s,t)$ does indeed have a convergent series of the form (A1) for the points (A5).

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Chiral $SU(3)\otimes SU(3)$ as a Symmetry of the Strong Interactions

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Starting with the modern developments of current algebra and the hypothesis of partially conserved axial-vector current, it has gradually become apparent that the strong interactions are almost invariant under the group $SU(3)\otimes SU(3)$. In the limit that symmetry breaking is neglected, $SU(3)\otimes SU(3)$ does not appear as a symmetry of the particle states as $SU(3)$ does, but rather as a symmetry realized by eight Goldstone bosons, i.e., the pseudoscalar octet. Most papers on $SU(3)\otimes SU(3)$ symmetry have been concerned with soft-meson theorems and their connection with effective Lagrangians. This paper is devoted to other aspects of the symmetry. Part of the paper is frankly pedagogical. The physics behind a symmetry realized by way of Goldstone bosons is brought out through a study of the σ model. Then the general principles are stated abstractly and applied to the hadrons. One of the new results presented here is that there are two distinct ways in which $SU(3)\otimes SU(3)$ can be realized. In both cases there is an octet of massless pseudoscalar mesons. The two possibilities differ in the residual symmetry of the hadron spectrum: In one case, it is only $SU(3)$; in the other, it is $SU(3)$ times a discrete symmetry, which leads to parity doublets. It is conjectured that some of the observed parity doubling in nucleon resonances is a consequence of this new discrete symmetry. Symmetry breaking is discussed in detail and is found to be very complex. In particular, it is shown that, at least for the pseudoscalar-meson masses, octet enhancement can never occur for first-order perturbations around an $SU(3)\otimes SU(3)$-symmetrical limit. Since octet enhancement is an empirical fact, one is forced to conclude that lowest-order perturbation theory is not a good approximation. In connection with octet enhancement, we show how one can use a principle of pole dominance in the angular momentum plane to replace scalar "tadpole" mesons with Regge trajectories.

I. INTRODUCTION

FOR some time it has been apparent that the strong interactions are approximately $SU(3)$-symmetric. More recently, the joint successes of current algebra and partially conserved axial-vector current (PCAC)[1] have indicated that the strong interactions are nearly symmetrical under the bigger group $SU(3)\otimes SU(3)$. The larger symmetry does not, however, manifest itself in multiplets of particles as does $SU(3)$, but through the appearance of eight nearly zero-mass pseudoscalar mesons, i.e., Goldstone bosons.[2]

Historically, Nambu and his collaborators[3] were the first to suggest that both the small mass of the pion and PCAC might be consequences of an approximate symmetry of the strong interactions. The next major steps came out of Gell-Mann's suggestion[4] that the vector and axial-vector currents of the hadrons generate the algebra of $SU(3)\otimes SU(3)$. The combination of current algebra and PCAC lead to a large number of low-energy theorems[1] for processes involving soft pions and, occasionally, kaons. These low-energy theorems which are only approximate in the real world would become exact in a limit where the pseudoscalar-meson masses vanish and the axial-vector currents are conserved. Thus, the soft-meson theorems may be thought of as consequences of approximate symmetry. This

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[1] See, e.g., S. Adler and R. Dashen, *Current Algebras* (W. A. Benjamin, Inc., New York, 1968).

[2] J. Goldstone, Nuovo Cimento **19**, 154 (1961); J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. **127**, 965 (1962).

[3] Y. Nambu and D. Lurié, Phys. Rev. **125**, 1429 (1962), and references therein.

[4] M. Gell-Mann, Physics **1**, 74 (1964).

point of view has been especially stressed by Weinberg.[5]

Evidently, many of the applications of the symmetry mentioned above can be obtained simply from a consistent use of current algebra and pole dominance of the divergences of the axial-vector currents. The reader may wonder, then, why he should bother with the symmetry at all. There are several reasons. First, in the author's opinion, the idea that the strong interactions are almost $SU(3)\otimes SU(3)$-symmetrical is the only rational way in which one can understand the continued success of PCAC and current algebra. The second reason is an aesthetic one. There is good evidence that the weak and electromagnetic currents of the hadrons generate the algebra of $SU(3)\otimes SU(3)$. The hypothesis that the strong interactions are invariant under this algebra, except for some small piece, which may well be a separate "medium strong" interaction, clearly provides a beautiful connection between the symmetry of the hadrons and their weak and electromagnetic interactions. Finally, $SU(3)\otimes SU(3)$ symmetry does, in fact, have implications other than soft-meson theorems. In this paper we will be primarily concerned with these additional consequences of the symmetry.

Soft-meson processes as such are not discussed in this paper. In the following paper by Weinstein and the author, this aspect of the symmetry is worked out in detail. There we show how to obtain all the low-energy theorems implied by the symmetry and show explicitly how effective Lagrangians[6] arise in a natural way. The use of effective Lagrangians is, then, another facet of $SU(3)\otimes SU(3)$ symmetry which is not included in the present paper.

Since the idea that the strong interactions are nearly $SU(3)\otimes SU(3)$-symmetric does not appear to be widely understood or appreciated, Sec. II and part of Sec. III of this paper are frankly pedagogical. Through a study of the familiar σ model, the physics behind a symmetry realized by way of massless particles is brought out and then applied to the strong interactions. The reader who is familiar with the ideas behind $SU(3)\otimes SU(3)$ symmetry may wish to skip this background material. For other readers, it is hoped that there is enough material to make the paper fairly self-contained.

One of the most interesting new results presented here is that there are two distinct ways in which the strong interactions can realize $SU(3)\otimes SU(3)$ symmetry. Both realizations contain an octet of massless pseudoscalar bosons, but in one realization there must also be some parity doubling in the particle spectrum. Technically, the way the two realizations differ is in the subgroup of $SU(3)\otimes SU(3)$ which leaves the vacuum invariant; in one case the subgroup is only $SU(3)$, while in the other it is $SU(3)$ times the discrete operation $e^{i2\pi Y_5}$, where Y_5 is the axial hypercharge. In the latter case one predicts parity doublets which obey certain selection rules such as not being able to decay (except through symmetry breaking) into two undoubled states. We conjecture that some of the parity doubling observed experimentally in the nucleon resonances[7] is a reflection of this and make some tentative assignments.

A large part of the paper is devoted to symmetry breaking. A rather disconcerting conclusion is that for first-order perturbations around an $SU(3)\otimes SU(3)$-symmetrical limit, there can be no dynamical mechanism which enhances the octet part of the pseudoscalar-meson mass differences. Phenomenologically, however, octet enhancement seems to exist and work in essentially the same way for the meson masses as it does for the baryon masses. This is seen to be the root of the difficulties one encounters in trying to extend the current-algebra calculation of the pion electromagnetic mass differences[8] to the kaon mass difference. In nature, the octet electromagnetic mass difference $m_{K^+}^2 - m_{K^0}^2$ seems to have been enhanced relative to the 27-plet difference $m_{\pi^+}^2 - m_{\pi^0}^2$, but this enhancement cannot occur for perturbations around an $SU(3)\otimes SU(3)$ limit. It is argued that the only way out of this and some related difficulties is to assume that first-order perturbation theory around the $SU(3)\otimes SU(3)$ limit is never a good approximation. In particular, this means that second- and higher-order terms are important in the strong or "semistrong" breaking of $SU(3)\otimes SU(3)$. Another result is that in the breaking of $SU(3)\otimes SU(3)$, strong-interaction dynamics will *not* enhance any representation of $SU(3)\otimes SU(3)$ relative to other representations, in contrast to the way that **8** violations of $SU(3)$ appear to be enhanced relative to **27** violations. This fact, when folded in with the presence of second- and higher-order terms in the basic symmetry breaking interaction, suggests that the group-theoretic properties of deviations from $SU(3)\otimes SU(3)$ symmetry will turn out to be much more complicated than has been the case in violations of $SU(3)$.

The idea that scalar mesons may play some special role in the symmetry or its breaking is studied and found to be unattractive. As an alternative to the hypothesis that objects like divergences of vector currents are dominated by scalar-meson poles, we suggest a scheme of pole dominance in the angular momentum plane. This allows one to replace scalar mesons with Regge trajectories and, in this way, to accommodate the suggestions of various authors[9] that the tensor-meson

[5] S. Weinberg, Phys. Rev. Letters **16**, 163 (1966); Phys. Rev. **166**, 1568 (1968).

[6] See, e.g., the second paper in Ref. 5.

[7] A. Donnachie, R. Kirsopp, and C. Lovelace, Phys. Letters **26B**, 161 (1968).

[8] J. Das, G. Guralnik, V. Mathur, F. Low, and J. Young, Phys. Rev. Letters **18**, 759 (1967).

[9] M. Suzuki and F. Zachariasen, Phys. Rev. Letters **17**, 1033 (1967); S. Okubo, *ibid.* **18**, 257 (1967); H. Harari, *ibid.* **17**, 1303 (1967); D. Gross and H. Pagels, Phys. Rev. **172**, 1381 (1968); J. S. Ball and F. Zachariasen, *ibid.* **177**, 2264 (1969).

(A_2, f, etc.) trajectories play a special role in symmetry breaking.

The paper is organized as follows. Section II is, as mentioned above, devoted to background material and may be easily omitted by many readers. In Sec. III, we briefly outline the general principle of $SU(3)\otimes SU(3)$ symmetry and then turn to the possibility of parity doubling in the hadron spectrum. Symmetry breaking is discussed in Sec. IV, where we also develop the idea of pole dominance in the angular momentum plane. Finally, Sec. V contains a few additional comments.

II. BACKGROUND

Since for some readers the idea of realizing a symmetry through massless bosons may be unfamiliar, this section is devoted to some background material. First, we study the σ model of Gell-Mann and Lévy.[10] It turns out that this well-known Lagrangian model illustrates most of the physics behind Goldstone bosons. We then briefly discuss some generalizations of the σ model and finally summarize abstractly the essential features of Goldstone bosons.

A. σ Model

The Lagrangian

$$\mathcal{L}=\bar{N}i\gamma\cdot\partial N+g\bar{N}(\sigma'+i\boldsymbol{\tau}\cdot\boldsymbol{\pi}\gamma_5)N -\tfrac{1}{2}(\partial_\mu\sigma'^2+\partial_\mu\boldsymbol{\pi}^2)-B^2(\sigma'^2+\boldsymbol{\pi}^2-A)^2, \quad (1)$$

where N and $\boldsymbol{\pi}$ are fields with the quantum numbers of the nucleon and pion, and σ' is an isoscalar 0^+ meson field, is well known to be invariant under the group $SU(2)\otimes SU(2)$, with corresponding conserved vector and axial-vector currents $\mathcal{V}_i{}^\mu=\bar{N}\gamma^\mu\frac{1}{2}\tau_i N+(\boldsymbol{\pi}\times\partial_\mu\boldsymbol{\pi})_i$ and $\mathcal{A}_i{}^\mu=\bar{N}\gamma^\mu\gamma_5\frac{1}{2}\tau_i N+\sigma'\partial_\mu\pi_i-\partial_\mu\sigma'\pi_i$.

Let us ignore, for the moment, the first two terms in $\mathcal{L}$ and think of the remaining mesonic terms as a classical theory. In the ground state of this classical theory the fields σ' and $\boldsymbol{\pi}$ will be constants (independent of $\mathbf{x}$ and t) such that the potential energy $B^2(\sigma'^2+\boldsymbol{\pi}^2-A)^2$ is a minimum. For negative A, the state of lowest energy is clearly $\sigma'=0$, $\boldsymbol{\pi}=0$, but for positive A we have $\sigma'^2+\boldsymbol{\pi}^2=A$ at the minimum. Evidently, a straightforward perturbation expansion can make sense only for $A<0$. Classically, perturbation theory corresponds to expanding around $\sigma'=0$, $\boldsymbol{\pi}=0$, which is a stable minimum of potential energy for $A<0$, but is a local maximum for $A>0$. To do perturbation theory for $A>0$, we need to define new fields which vanish at the minimum of energy. To this end, we choose as the "physical" ground state $\boldsymbol{\pi}=0$, $\sigma'=-\sqrt{A}$, and introduce $\sigma=\sigma'+\sqrt{A}$. In terms of σ, the Lagrangian is

$$\mathcal{L}=\bar{N}i\gamma\cdot\partial N-(g\sqrt{A})\bar{N}N+g\bar{N}(\sigma+i\boldsymbol{\tau}\cdot\boldsymbol{\pi}\gamma_5)N -\tfrac{1}{2}[(\partial_\mu\sigma)^2+(\partial_\mu\boldsymbol{\pi})^2]-4B^2A\sigma^2 +(2\sqrt{A})B^2\sigma(\sigma^2+\boldsymbol{\pi}^2)-B^2(\sigma^2+\boldsymbol{\pi}^2)^2. \quad (2)$$

[10] M. Gell-Mann and M. Lévy, Nuovo Cimento **26**, 53 (1960).

We may now forget about our classical arguments and think of Eqs. (1) and (2) as defining a quantum theory which can be studied in perturbation theory. The rule is, of course, to use (1) for $A<0$ and (2) for $A>0$. Let us compare the two cases $A<0$ and $A>0$. If we neglect the interaction terms, Lagrangian (1) describes massless nucleons and π and σ mesons with a common mass of $(-4AB^2)^{1/2}$, while Lagrangian (2) describes nucleons with mass $g\sqrt{A}$, a σ meson with mass $(8AB^2)^{1/2}$ and massless pions. These qualitative features are not changed by the interaction terms. To all orders, Lagrangian (1) yields massless nucleons and a degenerate multiplet of π and σ, while Lagrangian (2) gives massive nucleons, a massive σ, and massless pions. In Appendix A, this is explicitly shown to be the case to first order the interaction. The argument that it must be true to all orders is standard. The matrix element $\langle N|\mathcal{A}_i{}^\mu|N\rangle$ with "physical" nucleon states $|N\rangle$ can be written as $\bar{u}[g(q^2)\gamma^\mu\gamma_5\tau_i+h(q^2)q^\mu\gamma_5\tau_i]u$, where q is the momentum transfer. Since $\mathcal{A}_i{}^\mu$ is conserved, we have $2M_Ng(q^2)+q^2h(q^2)=0$, which implies that either $M_N=0$ or that h has a pole at $q^2=0$ and therefore that the pion mass is zero.[11] Similar manipulations with $\langle\pi|\mathcal{A}_i{}^\mu|\sigma\rangle$ show that either $m_\sigma=m_\pi$ or $m_\pi=0$.

Evidently, the $SU(2)\otimes SU(2)$ symmetry of the model manifests itself in very different ways in the two cases $A<0$ and $A>0$. In the former case we have a "normal" symmetrical theory with π and σ forming a multiplet, while in the latter case there is no $SU(2)\otimes SU(2)$ multiplet structure, but we have instead a massless pion, i.e., a Goldstone boson. The transition between the two cases is interesting. If we approach $A=0$ from the negative side, the degenerate π and σ masses both approach zero. Then as A increases through positive values, σ picks up a mass, but the pion mass sticks at zero. Although this transition through $A=0$ is smooth and physically sensible, it is certainly not analytic. The origin of this nonanalyticity at $A=0$ is, of course, the factors of $\sqrt{A}$ in Eq. (2).

According to the Goldstone theorem,[2] a symmetry realized through massless bosons implies a degenerate vacuum (defined as the state of lowest energy). That this should be the case is easily seen in the classical theory where there are a threefold infinity of solutions to $\sigma'^2+\boldsymbol{\pi}^2=A$ and hence a threefold infinity of lowest-energy states. Physically, these extra vacua are obtained by adding zero-energy pions to the particular vacuum which we have chosen to be the physical one.

Let us now see what happens if we break the $SU(2)\otimes SU(2)$ symmetry of the model by adding a term such as $\epsilon\sigma'$ or $\epsilon\bar{N}N$ to the Lagrangian. For small ϵ, it is easy to see what such a term will do. If A is negative, the π-σ multiplet will split slightly and the nucleon will pick up a small mass. On the other hand, if A is positive, the main quantitative effect of the perturbation will be to give the pion a small mass. A perturbation

[11] We assume that $g(0)$ is nonvanishing.

will also remove the degeneracy of the vacuum. This is particularly easy to see when the breaking term is $\epsilon\sigma'$. The classical ground state is then unique and corresponds to $\pi=0$, $\sigma'=-\sqrt{A}+\epsilon/2B^2A+O(\epsilon^2)$.

If we break the symmetry with $\epsilon\sigma'$, the equations of motion give $\partial_\alpha \mathcal{A}_i{}^\alpha=\epsilon\pi_i$, which is the usual statement of PCAC in a Lagrangian model. On the other hand, breaking with $\epsilon\bar{N}N$ leads to $\partial_\alpha \mathcal{A}_i{}^\alpha=-\frac{1}{2}i\epsilon\bar{N}\gamma_5\tau_i N$, which does not exhibit manifest PCAC. It will be argued now, however, that for positive A and small ϵ the model satisfies a true PCAC for either breaking term, but does not satisfy PCAC for negative A, even though the formal equation $\partial_\alpha \mathcal{A}_i{}^\alpha=\epsilon\pi_i$ holds. Evidently, we first need a precise definition of PCAC. We say that a theory satisfies PCAC if, for sufficiently small momentum transfers q, all matrix elements of $\partial_\alpha \mathcal{A}_i{}^\alpha$ are completely dominated by the pion pole. Consider now the matrix element of $\partial_\alpha \mathcal{A}_i{}^\alpha$ between nucleons: it has the form $i\bar{u}\tau_i\gamma_5 u d(q^2)$. The function $d(q^2)$ can be written as

$$\frac{r}{q^2-m_\pi{}^2}+\int\frac{\rho(m^2)}{q^2-m^2}dm^2,$$

where the first term is the pion pole and the integral is the contribution of higher mass states. Both r and ρ are of order ϵ since $\partial_\alpha \mathcal{A}_i{}^\alpha$ is of that order. If $A>0$, then $m_\pi{}^2$ is also of order ϵ, and the pion pole gives a contribution of order $\epsilon/(q^2-\epsilon)$, which for small enough q^2 is of order unity. On the other hand, the spectral integral is of order ϵ, so that PCAC is indeed satisfied in the limit $\epsilon\to 0$. Note that this argument is equally valid for either symmetry breaking term $\epsilon\sigma'$ or $\epsilon\bar{N}N$. On the other hand, if $A<0$, the pion mass is nonvanishing in the limit $\epsilon\to 0$ and both terms in $d(q^2)$ are of order ϵ. In this case, then, there is no particular reason to expect the pion pole to dominate. We may summarize our conclusions with the following remarks. *PCAC is really a statement about the way in which the symmetry is realized in the limit* $\epsilon=0$. *When valid, PCAC is independent of the type of symmetry breaking and is correct only to zeroth order in the breaking parameter.*

B. Generalization of the σ Model

There are many possible generalizations of the σ model. Two of them are of particular interest to us.

First, as pointed out by Gell-Mann and Lévy[10] and emphasized by Weinberg,[6] one may impose the $SU(2)\otimes SU(2)$ symmetrical constraint $\sigma'^2+\pi^2=A$ and by expressing σ' as a function of π write a nonlinear Lagrangian which contains no σ field. Clearly, this only makes sense for $A>0$ which leads to a massless pion. The importance of this observation can be seen as follows. Suppose we have some random theory which is known to have an $SU(2)\otimes SU(2)$ symmetry realized by means of a massless pion. The original σ model would have suggested that the pion is accompanied by some massive state like the σ which, in a sense, may be thought of as being a partner of the pion in an $SU(2)\otimes SU(2)$ multiplet with a very large mass splitting. The nonlinear model which contains only pions shows that this need not be the case. Actually, this is a very general circumstance, not one restricted to the σ model. It has been shown that for any symmetry realized through massless bosons, one can construct a nonlinear Lagrangian model in which the Goldstone boson fields are not accompanied by any partners.[12] Thus, in general, the existence of a set of Goldstone bosons does not imply the existence of any related massive mesons.

Secondly, the σ model can be extended to have the symmetry of $SU(3)\otimes SU(3)$ rather than $SU(2)\otimes SU(2)$. This may be done, for example, by replacing the nucleon field in Eq. (1) by a quark field and the π and σ' fields by a set of mesons transforming as $(3,\bar{3})\oplus(\bar{3},3)$ under $SU(3)\otimes SU(3)$. This model is algebraically much more complicated than the usual σ model and does not contain any essentially different physics. For this reason and because this model is thoroughly discussed elsewhere,[13] we need not go into the details here. It suffices to say that for certain values of the coupling constants, this model produces an octet of massless pseudoscalar mesons which act as Goldstone bosons. The multiplet structure of particles in this model is then $SU(3)$, just as the particles in the σ model fall, for $A>0$, into degenerate $SU(2)$ multiplets [but not degenerate $SU(2)\otimes SU(2)$ multiplets].

Finally, before leaving models of Goldstone bosons, we should mention Nambu's original model[3] where there is no explicit pion field. In this model, Nambu argues that a massless pion can emerge as a nucleon-antinucleon bound state. It should be kept in mind, in what follows, that a Goldstone boson is not necessarily an "elementary particle." In particular, there is no obvious reason why a Goldstone boson cannot lie on a Regge trajectory like any other hadron.

C. Abstractions

We do not believe, of course, that the models discussed above really have anything to do with the actual hadrons. Nonetheless, we may abstract from these models certain general properties of Goldstone bosons. These more abstract statements can then be applied to the strong interactions.

To begin, let us imagine that we have a Hamiltonian which is symmetrical under a group $\mathcal{G}$. The vacuum will be invariant under some subgroup $\mathcal{G}'\subset\mathcal{G}$. As we have seen above, this subgroup $\mathcal{G}'$, which is the symmetry group of the particle spectrum, will not in general be all $\mathcal{G}$. For example, in the σ model $\mathcal{G}$ is $SU(2)\otimes SU(2)$ but $\mathcal{G}'$ can be only $SU(2)$, with the particles N, π, and σ falling into degenerate isospin [$SU(2)$] multiplets but not $SU(2)\otimes SU(2)$ multiplets.

[12] C. Callan, S. Coleman, J. Wess, and B. Zumino, Phys. Rev. **177**, 2249 (1969).

[13] W. Bardeen and B. Lee, Phys. Rev. **177**, 2389 (1969).

Let us denote those generators of $\mathcal{G}$ which are not in $\mathcal{G}'$ by E_i. Since the E_i are conserved and do not annhilate the vacuum, they must take the vacuum into other states with the same energy. The only possible such states are the vacuum plus some number of zero-energy, zero-momentum spinless bosons.[14] A zero-energy, zero-momentum boson must, of course, be massless. In general, then, there will be a set of zero-mass spinless bosons ξ_i which, since they can be created out of the vacuum by the E_i, must have the quantum numbers of the corresponding generators. Writing these generators as the space integrals of local conserved currents,

$$E_i=\int e_i^0 d^3x \quad \text{with} \quad \partial_\alpha e_i^\alpha=0,$$

the above conclusions are expressed mathematically by the existence of the nonzero matrix elements

$$\langle \xi_i(q)|e_i^\nu(0)|0\rangle = iq^\nu f_i^{-1}. \tag{3}$$

Multiplying by the momentum q^ν, one easily sees that current conservation is guaranteed by the fact that $q^2=m_\xi^2=0$. Evidently, the f_i's are fundamental parameters. They are not determined by group theory alone, but involve dynamics.

To illustrate these remarks, let us return to the σ model for $A>0$. Here the generators formed from space integrals of the time components of axial-vector currents do not leave the vacuum-invariant. The axial-vector currents themselves, written in terms of σ (not σ'), are

$$\mathcal{a}_i^\mu=\tfrac{1}{2}\bar{N}\gamma^\mu\gamma_5\tau_i N+\partial_\mu\sigma\pi_i-\sigma\partial_\mu\pi_i+(\sqrt{A})\partial_\mu\pi_i, \tag{4}$$

and clearly contain a term $(\sqrt{A})\partial_\mu\pi_i$ which creates a (massless) pion out of the vacuum with the parameter f^{-1} equal to $\sqrt{A}$ in lowest-order perturbation theory.

The above discussion contains, in condensed form, most of the basic ingredients of a symmetry which is realized by way of Goldstone bosons. Below we briefly discuss symmetry breaking. Before proceeding, however, a couple of remarks are in order.

(i) It is clear that some aspects of a theory with Goldstone bosons are very complex. For example, if we were to ask just how the degenerate vacuum transforms under the full group $\mathcal{G}$, the answer will probably be very complicated. Fortunately, however, the techniques of current algebra developed in the last few years allow one to extract most of the useful information contained in the symmetry without having to face complex questions like the one noted above. These techniques are illustrated in the following paper and to some extent in later sections of the present paper.

(ii) It should be stressed that the realization of a symmetry via Goldstone bosons is not a pathological or particularly unusual circumstance. In which way a system chooses to realize a symmetry is simply a question of dynamics. For example, in the σ model it is the sign of a coupling constant which determines the way in which the symmetry is realized. With this in mind, it should be clear that having a slightly broken symmetry with Goldstone bosons is no more or less aesthetically displeasing than having a slightly broken symmetry of the usual variety.

Let us now suppose that the symmetry group $\mathcal{G}$ is broken by a small term in the Hamiltonian. Qualitatively, such a perturbation will have two major effects. First, if the perturbation breaks the symmetry under $\mathcal{G}'$ as well as $\mathcal{G}$, the multiplets of states corresponding to representations of $\mathcal{G}'$ will be split. Second, the Goldstone bosons associated with the remaining generators will pick up a small mass. Also, now that the bosons have a mass, the theory will satisfy the usual sort of PCAC-like relation. That is, there will be a matrix element of $\partial_\alpha e_i^\alpha$ between the meson ξ_i and the vacuum, obtained by taking the divergence on both sides of Eqs. (3) which gives

$$\langle \xi_i|\partial_\alpha e_i^\alpha|0\rangle = m_{\xi_i}^2 f_i^{-1}. \tag{5}$$

Therefore, as a function of momentum transfer q, matrix elements $\langle\beta|\partial_\alpha e_i^\alpha|\alpha\rangle$ of the divergence of e_i^α will have a pole of the form

$$\frac{1}{q^2-m_\xi^2}(m_\xi^2 f^{-1})G_{\xi\beta\alpha},$$

where $G_{\xi\beta\alpha}$ is the ξ-β-α coupling constant. Since f^{-1} and $G_{\xi\beta\alpha}$ are finite in the symmetry limit, we see that for sufficiently small q^2 the contribution of this pole is of order $m_\xi^2\, m_\xi^{-2}$, or of order unity as the symmetry breaking is turned off. All other contributions to the matrix element are of order of the small symmetry-breaking interaction. Thus, the ξ pole dominates and a PCAC-like relation holds. This is, of course, simply a more general restatement of what we learned from the σ model. We repeat, for emphasis, that PCAC-like relations are independent of the particular type of symmetry breaking and hold, strictly speaking, only when terms of the same order as the symmetry breaking are neglected.

Finally, a word about our choice of language. In the literature, symmetries realized by Goldstone bosons are usually referred to as "spontaneously broken." In this paper we avoid the term "spontaneously broken" for two reasons. First, it gives the impression that

[14] We are implicitly assuming here that the E_i's take the vacuum into states which are physically different from the vacuum. This need not be the case when the theory has a local-gauge invariance; see P. Higgs, Phys. Rev. **145**, 1156 (1966). With regard to the general lack of mathematical rigor in this section, it should be understood that we are trying to describe a situation which appears to be experimentally present in the strong interactions; we are *not* attempting to discuss degenerate vacua in the most general case. A recent review of this subject is given by T. Kibble, in *Proceedings of the International Conference on Particles and Fields, Rochester, 1967* (Wiley-Interscience, Inc., New York, 1967).

symmetries realized by Goldstone bosons are not real symmetries when, in fact, they are. Second, in discussing the physical hadrons, we will be talking about symmetries that are really broken, presumably by some extra term in the Hamiltonian. We wish to avoid confusion between really broken and "spontaneously broken."

III. CHIRAL SYMMETRY OF THE STRONG INTERACTIONS

In this section we apply the basic ideas reviewed in Sec. II to the physical hadrons and their interactions.

A. Generalities

We assume, as appears to be the case experimentally, that the hadronic weak and electromagnetic currents generate the algebra[1,4] of $SU(3)\otimes SU(3)$. The subgroup $SU(3)$ generated by the vector currents is well known to be an approximate symmetry of the strong interactions leading to multiplets of particles. We do not, on the other hand, see larger multiplets corresponding to the full algebra $SU(3)\otimes SU(3)$. In Sec. II, however, we learned that producing particle multiplets is not the only way in which a symmetry can manifest itself. As an alternative, one can have nearly massless (massless in the symmetry limit) mesons which satisfy PCAC-like relations. Now PCAC for pions seems to be very accurate, which strongly suggests that the strong interactions almost have another symmetry, one in which the pion mass is zero and the axial-vector currents $\mathfrak{F}_i^{\nu 5}$, $i=1, 2, 3$, in Gell-Mann's notation,[4] are conserved. The analog of PCAC for kaons and the η meson has not been tested so well, but if there is one limit to the strong interactions in which $SU(3)$ is exact and another in which $\partial_\mu \mathfrak{F}_i^{\mu 5}=0$ for $i=1, 2, 3$, then there must be a joint limit in which $SU(3)$ is exact and all the axial-vector currents are conserved. Evidently, in this limit the kaon and η, as well as the pion, are massless.

We shall suppose, then that the Hamiltonian of the strong interactions can be written as $H=H_0+\epsilon H'$, where H_0 is invariant under $SU(3)\otimes SU(3)$, H' breaks the symmetry, and ϵ is small. We refer to $\epsilon=0$ as the symmetry limit. Our picture of the hadrons in this limit is one where all $SU(3)$ multiplets are exactly degenerate and there are eight massless pseudoscalar mesons. PCAC and its generalization to the kaon and η become, of course, exact in the symmetry limit.

We leave it to the reader to convince himself that the above picture of the world is consistent with the general principles discussed at the end of Sec. II. Also, for future reference we record the analogs of Eq. (5). With conventional normalization one writes

$$\langle P_i(q)|\mathfrak{F}_j^{\mu 5}(0)|0\rangle = iq^\mu(2f_i)^{-1}\delta_{ij}, \qquad (6)$$

where P_i is one of the eight pseudoscalar mesons and $\mathfrak{F}_i^{\mu 5}$ is an axial-vector current. In the symmetry limit we have $f_\pi=f_K=f_\eta=f$. The parameters f_π and f_K are, of course, measured in the decays $\pi\rightarrow\mu+\nu$ and $K\rightarrow\mu+\nu$.

The applications of $SU(3)\otimes SU(3)$ are rather different from the familiar applications of $SU(3)$. The main application of $SU(3)\otimes SU(3)$ is to scattering processes involving low-energy pseudoscalar mesons. As stated in the Introduction, however, we will not concern ourselves with this class of applications, except to remark that all the low-energy theorems for soft pions and kaons which have been derived in the last few years may be regarded as consequences of $SU(3)\otimes SU(3)$ symmetry, the reason being that these low-energy theorems are based[1] on PCAC and on the commutation relations

$$\left[\int d^3x\ \mathfrak{F}_i^{50}, \int d^3y\ \mathfrak{F}_j^{50}\right] = if_{ijk}\int d^3x\ \mathfrak{F}_k^{0},$$

both of which are implied by the symmetry.

Some different applications of $SU(3)\otimes SU(3)$ will be discussed in the remaining parts of this paper.

One is used to thinking of a sublimit in which $SU(3)$ is exact but $SU(3)\otimes SU(3)$ is broken. In fact, it is probably not generally useful to think in these terms. This way of thinking tends to suggest that the part of H' which conserves $SU(3)$ but breaks $SU(3)\otimes SU(3)$ should be large compared to the part which breaks $SU(3)$. That this is not the case can be seen by looking at the pseudoscalar-meson masses. Decomposing the masses into an $SU(3)$ octet piece m_8^2 and an $SU(3)$ singlet piece m_1^2, one finds that $(m_8/m_1)^2$ is of order unity, indicating that H' breaks $SU(3)$ and $SU(3)\otimes SU(3)$ by roughly equal amounts.

This identification of the $SU(3)$-symmetric limit with an $SU(3)\otimes SU(3)$-symmetric limit has some simple, but useful, consequences.[15] In the past there was always some question as to whether one should set $f_K^{-1}\approx f_\pi^{-1}$ or $m_K^2 f_K^{-1}\approx m_\pi^2 f_\pi^{-1}$ when comparing π and K decays. It should be clear, from the discussion around Eq. (6), that $SU(3)\otimes SU(3)$ tells us to use $f_K^{-1}\approx f_\pi^{-1}$. Another question involves the use of linear-versus-quadratic mass formulas. For all multiplets except the pseudoscalar mesons, it does not matter much since for heavy particles the mass differences are fairly small compared to the mean mass of a multiplet. For the pseudoscalars, however, it is essential to use a quadratic formula to obtain agreement with experiment. This is easily understood if the pseudoscalars are massless in the symmetric limit. Adding a perturbation to the symmetric Hamiltonian changes the energy of a pseudoscalar meson with momentum $\mathbf{q}$ from $(\mathbf{q}^2)^{1/2}$ to $(\mathbf{q}^2+m^2)^{1/2}=(\mathbf{q}^2)^{1/2}+m^2/2(\mathbf{q}^2)^{1/2}$ to first order, which clearly shows that one should use a quadratic mass

[15] This paragraph parallels, from a somewhat different point of view, some conclusions of M. Gell-Mann, R. Oakes, and B. Renner, Phys. Rev. **175**, 2195 (1968).

formula. In particular, we see that the meson masses squared, not the masses, are proportional to the symmetry-breaking parameter ϵ in lowest order.

B. Parity Doubling

Up to this point we have been tacitly supposing that out of the full symmetry group $SU(3)\otimes SU(3)$, only the subgroup $SU(3)$ leaves the vacuum invariant. Actually, there is a slightly different possibility that would lead to some parity doubling among hadron states. As will be discussed later, this may be the explanation of some of the observed parity doubling among nucleon resonances.

The point is that the subgroup that leaves the vacuum invariant can, in principle, be any subgroup of $SU(3)\otimes SU(3)$. As before, we call this subgroup $\mathcal{G}'$. The physical constraints on $\mathcal{G}'$ are that it contains $SU(3)$, since we see $SU(3)$ multiplets of particles, and that it is not the whole of $SU(3)\otimes SU(3)$, since the pseudoscalar mesons appear to be Goldstone bosons. An examination of $SU(3)\otimes SU(3)$ then shows that there are precisely two subgroups which satisfy these constraints[16]; they are (i) $SU(3)$ itself and (ii) the group formed from the direct product of $SU(3)$ times the discrete operation $Z\equiv e^{i2\pi Y_5}$, where Y_5 is the axial hypercharge $Y_5=(2/\sqrt{3})F_8{}^5$. In the symmetric limit, then, the vacuum must be invariant under $SU(3)$ alone or else invariant under $SU(3)\otimes Z$. If the former alternative holds, nothing new happens: For the rest of this section we will concentrate on the latter possibility.

One easily varifies that Z is an $SU(3)$-invariant operator satisfying $Z^3=1$ and $PZP^{-1}=Z^\dagger$, where P is the parity operator. It is convenient to add parity to our group: P certainly commutes with H_0 and leaves the vacuum invariant. We then obtain a group $SU(3)\otimes\mathfrak{Z}$ which leaves H_0 and the vacuum invariant, where $\mathfrak{Z}$ is the six-element group composed of 1, P, Z, $Z^\dagger$, PZ, and $PZ^\dagger$.

It should be clear, from the discussions of Sec. I, that the particle states will then fall into multiplets corresponding to irreducible representations of $SU(3)\otimes\mathfrak{Z}$. The irreducible representations of $SU(3)\otimes\mathfrak{Z}$ are, of course, just products of representations of $SU(3)$ and $\mathfrak{Z}$. It is easy to see that $\mathfrak{Z}$ has two kinds of representations. First, a "singlet" one-dimensional representation for which P can be either $+1$ or -1. Second, there is a two-dimensional "doublet" representation in which P must take both values $+1$ and -1. Thus, a $SU(3)\otimes\mathfrak{Z}$ multiplet would consist of any $SU(3)$ representation which will be parity-doubled[17,18] if the multiplet is a doublet under $\mathfrak{Z}$, but undoubled if it is a singlet with respect to $\mathfrak{Z}$. There is no *a priori* connection between the dimension of an $SU(3)$ representation and parity doubling.

In order to see if the hadrons actually do choose a $\mathfrak{Z}$-invariant vacuum in the symmetrical limit we must, obviously, see if we can classify hadrons into $\mathfrak{Z}$ singlets and doublets. There are a few simple properties of the group which are helpful in this respect. First, the currents $\mathfrak{F}_i{}^\mu$ and $\mathfrak{F}_i{}^{\mu 5}$ are invariant under $\mathfrak{Z}$. Therefore, any meson which is connected to the vacuum by one of these currents must be a singlet. This clearly implies that the pseudoscalar octet is a singlet. Also, the vector mesons must be singlets, since their obvious presence in form factors indicates that the vector currents connect them to the vacuum. Second, it is easy to see that $\mathfrak{Z}$ contains a selection rule which forbids the decay of a doublet to two singlets. This means, of course, that candidates for doublets must decay slowly into singlet states, with the decay proceeding through the symmetry breaking term $\epsilon H'$. Finally, all states on a given Regge trajectory must be either singlets or doublets; we leave it to the reader to convince himself of this point.

Experimentally, the spectrum of nucleon resonances contains several states which appear to be parity doubled, the most famous case being the $\frac{5}{2}^+$ and $\frac{5}{2}^-$ isodoublet resonances which are split only by a few tens of MeV. It is very tempting to suppose that the $\frac{5}{2}^+$-$\frac{5}{2}^-$ pair are a doublet. This cannot be the case, however. The strong decay of these resonances into πN plus the fact that the $\frac{5}{2}^+$ is believed to lie on the nucleon trajectory would then imply that the nucleon is also a doublet. Since the nucleon has no opposite-parity partner, we are forced to conclude that the $\frac{5}{2}^+$-$\frac{5}{2}^-$ pair is an accident.

On the other hand, the $I=\frac{1}{2}$, and $J^P=\frac{1}{2}^+$ and $\frac{1}{2}^-$ resonances[7,19] at 1466 and 1591 MeV could very well be a $\mathfrak{Z}$ doublet. These states are, in fact, coupled rather weakly to the πN system. To see this, we write the coupling between a $J^P=\frac{1}{2}^+$, $I=\frac{1}{2}$ resonance and the πN system as $f_\pi g\bar{N}^*\gamma_5\gamma^\mu\boldsymbol{\tau}N\cdot\partial_\mu\boldsymbol{\pi}$, and for a $\frac{1}{2}^-$ resonance as $f_\pi g\bar{N}^*\gamma^\mu\boldsymbol{\tau}N\cdot\partial_\mu\boldsymbol{\pi}$. The dimensionless parameters g are the analogs of G_A ($G_A=1.2$) for the nucleon. For a resonance of either parity, the width is then

$$\Gamma(N^*\to\pi N)=\tfrac{3}{4}g^2\frac{(2M_Nf_\pi)^2}{4\pi}\left(\frac{M_{N^*}{}^2-M_N{}^2}{2M_{N^*}M_N}\right)^3 M_N$$

$$\approx(8\text{ BeV})g^2\left(\frac{M_{N^*}{}^2-M_N{}^2}{2M_{N^*}M_N}\right)^3. \quad (7)$$

From the known widths, one then finds that g^2 is of

[16] This result is most easily obtained by going to a particular representation, say, $(3,1)\oplus(1,3)$ of the group.

[17] For mesons, the two members of a doublet also have opposite charge conjugation.

[18] A doublet of two fermions can either be MacDowell partners on a single Regge trajectory or could lie on two separate trajectories.

[19] We are assuming, for the purpose of making tentative assignments, that all the effects listed in Ref. 7 are true resonances. The pair at 1466 and 1591 MeV almost certainly do exist, although their masses tend to move around. The other pairs are less certain.

order 0.15 for both cases, which is to be compared to $G_A{}^2 \approx 1.4$ for the nucleon.

Another possible pair of states present in recent phase-shift analysis[7,19] are $I=\frac{3}{2}$, $J^P=\frac{3}{2}^+$ and $\frac{3}{2}^-$ states at 1688 and 1691 MeV. These states are also weakly coupled to the πN system. To obtain a measure of this coupling, we compare their decays with the $I=\frac{3}{2}$, $J^P=\frac{3}{2}^+$ resonance at 1235, which certainly is strongly coupled. We write $\Gamma(\Delta(1235)\rightarrow\pi N)=[g(1235)]^2q^3$, where q is the c.m. momentum of the πN system, and identical expressions for the 1688 and 1691 states. From the experimental widths we then find that both the ratios $[g(1688)/g(1235)]^2$ and $[g(1691)/g(1235)]^2$ are of order 1/50. Also, there are $I=\frac{3}{2}$, $J^P=\frac{5}{2}^+$ and $\frac{5}{2}^-$ states at 1913 and 1954 MeV, which again appear to couple rather weakly to the πN channel.

If, in fact, the above pairs of states are $\mathfrak{z}$ doublets, then we know that both members of each pair belong to the same $SU(3)$ representation. Experimentally, the $\frac{1}{2}^-$ state at 1591 MeV appears to belong to an octet.[20] We would then expect a $\frac{1}{2}^+$ octet containing the state at 1491 MeV. A characteristic property of these states would be that, since their decays into the pseudoscalar octet and baryon octet occur only through the symmetry-breaking interaction, these decays should not satisfy the usual $SU(3)$ intensity rules.[21]

We may summarize the above discussion of parity doublets as follows. In the symmetrical limit, the hadrons must choose either a vacuum which is invariant under $\mathfrak{z}\otimes SU(3)$ or one which is invariant only under $SU(3)$. In the latter case, the observed parity doubling is clearly not a consequence of $SU(3)\otimes SU(3)$ symmetry and must be explained by another presumably dynamical, means. In the former case, on the other hand, the existence of parity doublets is a direct prediction of the symmetry. Because of the selection rules implied by $\mathfrak{z}$, one should be able to distinguish between the two cases.

In Sec. IV we will discuss the breaking of $SU(3)\otimes SU(3)$ symmetry. To complete our present discussion, let us anticipate some results of that section and write a mass formula for $\mathfrak{z}$ doublets. For a parity-doubled $SU(3)$ multiplet, we write the masses as $m^{\pm}(I,Y)$, where I and Y are the isospin and hypercharge and the superscripts refer to the positive- and negative-parity states. Then, for the sum $m^+ + m^-$, one finds the usual result

$$m^+(I,Y)+m^-(I,Y)=m_0+aY+b[I(I+1)-\tfrac{1}{4}Y^2]. \quad (8)$$

For difference $m^+ - m^-$, the mass formula can only be conjectured. The simplest and perhaps most likely formula is

$$m^+(I,Y)-m^-(I,Y) = c+dY+e[I(I+1)-\tfrac{1}{4}Y^2-\Delta], \quad (9)$$

where Δ is $I(I+1)-\frac{1}{4}Y^2$ averaged over the multiplet and where c, d, and e are all of order of the symmetry breaking. Either c or d and e might always be zero. Clearly, if c or d and e vanish, the pattern of masses would be striking.

IV. SYMMETRY BREAKING

In this section we turn to the interaction $\epsilon H'$ that breaks $SU(3)\otimes SU(3)$. We will encounter some rather severe diffculties, the most likely explanation of which is that working to lowest order in $\epsilon H'$ is very misleading. In interpreting these difficulties it is important to take account of the observed pattern of "octet enhancement"[22] in the various breakings of $SU(3)$. Accordingly, we first review the phenomenology associated with octet enhancement. Then we turn to specific problems associated with the breaking of $SU(3)\otimes SU(3)$. The last part of this section deals with the possible relation of scalar mesons to the symmetry breaking.

A. Review of Octet Enhancement

Perhaps the most striking feature of $SU(3)$ violations is the observed phenomenon of octet enhancement. To introduce the idea of octet enhancement, let us consider the order e^2 electromagnetic breaking of $SU(3)$ neglecting the usual strong breaking. The effective Hamiltonian density for the electromagnetic breaking is

$$\mathcal{H}_{\rm em} = -e^2\int T(J_{\rm em}{}^\mu(x)J_{\rm em}{}^\nu(-x)\mathfrak{D}^{\mu\nu}(x))d^4x$$
$$=\mathcal{H}_{\rm em}{}^{(1)}+\mathcal{H}_{\rm em}{}^{(8)}+\mathcal{H}_{\rm em}{}^{(27)}, \quad (10)$$

where $\mathfrak{D}^{\mu\nu}$ is the photon propagator and we have indicated that $\mathcal{H}_{\rm em}$ can be broken up into pieces which are components of **1**, **8**, and **27** representations of $SU(3)$. The Clebsch-Gordan coefficients which go into the decomposition are all of order unity, but to a rather good approximation electromagnetic mass differences transform like an octet. Specifically, $\langle B|\mathcal{H}_{\rm em}{}^{(8)}|B\rangle$, where B means the usual $\frac{1}{2}^+$ baryon octet, is about a factor of 5 larger than $\langle B|\mathcal{H}_{\rm em}{}^{(27)}|B\rangle$. Similarly, in terms of mass squares the octet part of pseudoscalar-meson electromagnetic mass differences is large compared to the **27** part. This means that $\mathcal{H}^8$ is dynamically enhanced relative to $\mathcal{H}^{(27)}$.

[20] Along with s-wave enhancements at the Λ-η and Σ-η thresholds.

[21] If we accept the idea that the $\frac{1}{2}^-$ state at 1591 MeV belongs to an octet along with the Λ-η and Σ-η threshold bumps, then there is already evidence of large deviations from $SU(3)$ symmetry in the ratio of the coupling of the two bumps to Λ-η and Σ-η, respectively; see P. Dobson, Phys. Rev. **176**, 1757 (1968).

[22] R. Cutkosky and P. Tarjanne, Phys. Rev. **132**, 289 (1963); S. Coleman and S. Glashow, *ibid.* **134**, B671 (1964); R. Dashen, S. Frautschi, and Y. Hara, in *The Eightfold Way*, edited by M. Gell-Mann and Y. Ne'eman (W. A. Benjamin, Inc., New York, 1964); R. F. Dashen and S. Frautschi, Phys. Rev. **137**, B1331 (1965).

In nonleptonic weak interactions, the situation is similar. If we take an interaction of the form

$$\mathcal{H}_w = (G/\sqrt{2}) J_w^{\mu\dagger} J_{w\mu}\,, \qquad (11)$$

we again expect roughly equal amounts of **8** and **27**. Here $\langle B|\mathcal{H}_w|B\rangle$ cannot be measured directly, but these matrix elements can be obtained, using current algebra, from the known amplitudes for hyperon decay.[1] Again, one finds that the octet part of $\langle B|H_w|B\rangle$ is large compared to the 27-plet part.

It appears, therefore, that there is some dynamical mechanism that tends to enhance the octet part of any perturbation. There is a still more interesting piece of phenomenology which we now proceed to outline. Consider a perturbing Hamiltonian $\mathcal{H}_p$ which breaks $SU(3)$, and let $\delta M_{\alpha\beta}(\mathcal{H}_p)$, $\alpha,\beta=1,\cdots,8$ and $\delta m_{\alpha\beta}^2(\mathcal{H}_p)$, $\alpha,\beta=1,\cdots,8$ be the resulting changes in the baryon mass matrix and the pseudoscalar meson mass-squared matrix. As $\mathcal{H}_p$ runs over weak,[23] electromagnetic, and strong splitting, all experimental evidence is consistent with the simple formula[22]

$$\delta M_{\alpha\beta}(\mathcal{H}_p) = \sum_{\gamma=1}^{8} (c_1 f_{\alpha\beta\gamma} + c_2 d_{\alpha\beta\gamma}) D_\gamma(\mathcal{H}_p) + (\text{small } \mathbf{27} \text{ terms})\,, \qquad (12)$$

$$\delta m^2_{\alpha\beta}(\mathcal{H}_p) = \sum_{\gamma=1}^{8} c_3 d_{\alpha\beta\gamma} D_\gamma(\mathcal{H}_p) + (\text{small } \mathbf{27} \text{ terms})\,, \qquad (13)$$

where $d_{\alpha\beta\gamma}$ and $f_{\alpha\beta\gamma}$ are the usual symmetric and antisymmetric octet matrices and c_1, c_2, and c_3 are constants independent of the perturbation. Notice that the only dependence on the perturbation is in the $D_\gamma(\mathcal{H}_p)$ which specify a magnitude and the direction in $SU(3)$ space along which the effect points.

It is easy to see that Eq. (12) says that the D/F ratio of $M_{\alpha\beta}$ is c_2/c_1 independent of the perturbation. This is, in fact, experimentally true for all three cases, weak,[1] electromagnetic,[22] and strong,[22] with $c_2/c_1 = -0.3$ to -0.4. Together, Eqs. (12) and (13) also say that the ratio of $m^2_{\alpha\beta}$ to the symmetrical part of $M_{\alpha\beta}$ should also be a constant. This is again experimentally true for the ratio of electromagnetic mass differences to strong mass differences. To check the ratio for the case of weak breaking, we need to know the matrix element $\langle K|H_w|\pi\rangle$. The current-algebra determination of this matrix element is not on particularly sound footing, but if we accept this calculation[1] the ratio of weak mass matrices is also roughly equal to the ratio of strong mass splittings.

Thus, we see that not only are the octet pieces of perturbations enhanced but that the enhancement is such that all perturbations give essentially the same pattern of masses. This is presumably due to some dynamical property of the hadrons in the $SU(3)$-symmetrical limit.

[23] Here and in what follows, the term "weak" is understood to mean the parity-conserving part of the nonleptonic weak interaction.

B. Difficulties with the Breaking of $SU(3)\otimes SU(3)$

We now derive a formula for the mass matrix of the pseudoscalar mesons which is valid to lowest order in violations of $SU(3)\otimes SU(3)$. By looking at the electromagnetic violations, we will find that there are severe difficulties with a lowest-order formalism.

The desired formula is obtained as follows. We assume that the perturbing Hamiltonian can be written as

$$H_p = \int d^3x\, \mathcal{H}_p(\mathbf{x})$$

and has the property that

$$\left[\mathcal{F}_a^{05}(0), \int d^3X \mathcal{H}_p(\mathbf{x},0)\right] = [F_a^5, \mathcal{H}_p(0)]\,,$$

with

$$F_a^5 = \int d^3x\, \mathcal{F}_a^{05}(\mathbf{x},0)\,.$$

Then in the identity

$$q^\lambda q^\mu \int e^{iqx} \langle 0|T(\mathcal{F}_a^{5\lambda}(x), \mathcal{F}_b^{5\mu}(0))|0\rangle d^4x = \int e^{iqx}\langle 0|T(\partial_\lambda \mathcal{F}_a^{5\lambda}(x), \partial_\mu \mathcal{F}_b^{5\mu}(0))|0\rangle + i\langle 0|[F_a^5,[F_b^5,\mathcal{H}_p(0)]]|0\rangle\,, \qquad (14)$$

we observe that, except for a meson pole which has a denominator of order H_p, the first term on the right is of order $(H_p)^2$. Explicitly, removing this pole, we have

$$q^\lambda q^\mu \int e^{iqx}\langle 0|T(\mathcal{F}_a^{5\lambda}(x), \mathcal{F}_b^{5\mu}(0))|0\rangle d^4x = i\sum_{cd} \frac{m^2_{ac}}{2f}\left(\frac{1}{q^2-m^2}\right)_{cd} \frac{m^2_{db}}{2f} + i\langle 0|[F_a^5,[F_b^5,\mathcal{H}_p(0)]]|0\rangle + O(H_p^2)\,, \qquad (15)$$

where m^2_{ab} is the meson-mass-squared matrix. Finally, we note that for nonzero meson masses, the left-hand side of Eq. (15) vanishes as $q\to 0$, and taking this limit gives the desired formula

$$m^2_{ab} = 4f^2\langle 0|[F_a^5,[F_b^5,\mathcal{H}_p(0)]]|0\rangle + O(H_p^2)\,. \qquad (16)$$

The symmetry of m^2_{ab} is guaranteed by the fact that

$$m^2_{ab} - m^2_{ba} = i4f^2 f_{abc}\langle 0|[F_c,\mathcal{H}_p(0)]|0\rangle = 0\,, \qquad (17)$$

since the vector charge F_c annihilates the symmetrical vacuum.

Let us now insert $\mathcal{H}_{\rm em}$ from Eq. (10) into Eq. (16) and see what happens. First, since any electrically neutral F^5 commutes with $J_{\rm em}{}^{\mu}$, it is clear that the self-energies of π^0, η, K^0, and $\bar{K}^0$ all vanish in lowest order. Also there is no off-diagonal element between π^0 and η. This allows us to prove that

$$m_{K^+}{}^2-m_{K^0}{}^2=m_{\pi^+}{}^2-m_{\pi^0}{}^2 \tag{18}$$

to order e^2, neglecting strong violations of $SU(3)\otimes SU(3)$. The proof is very simple. Since K^+ and π^+ belong to the same U-spin multiplet, their electromagnetic self-energies must be equal. Equation (18) then follows from the fact that the self-energies of all neutral mesons are zero.

Experimentally, the two sides of Eq. (18) are of opposite sign and differ in magnitude by a factor of 5.

Besides being in violent disagreement with experiment, Eq. (18) has another theoretical trouble. The π^+-π^0 mass difference, being an $I=2$ object, comes only from the "small **27** terms" in Eq. (13), while the K^+-K^0 difference receives a contribution from the enhanced octet term. In fact, the standard explanation of why $m_{K^+}{}^2-m_{K^0}{}^2$ is large compared to $m_{\pi^+}{}^2-m_{\pi^0}{}^2$ is that the former has been "octet enhanced" while the latter has not. Equation (18) is, then, in direct contradiction with the general phenomenon of octet enhancement.

Before discussing the implications of this result let us further convince ourselves that the difficulty does exist. To this end, we consider the mass-squared matrix of the pseudoscalar mesons, $m^2{}_{ab}|_{\rm w}$, obtained from the weak interactions with neglect of the strong breaking of $SU(3)\otimes SU(3)$. From Eq. (16),

$$m^2{}_{ab}|_{\rm w}=4f^2\langle 0|[F_a{}^5,[F_b{}^5,\mathcal{H}_{\rm w}(0)]]|0\rangle. \tag{19}$$

Now if $\mathcal{H}_{\rm w}$ is a function only of $J_{\rm w}{}^{\mu}$ (not necessarily the local product $J_{\rm w}{}^{\mu\dagger}J_{{\rm w}\mu}$), then $[F_a+F_a{}^5,\mathcal{H}_{\rm w}]$, vanishes since $J_{\rm w}{}^{\mu}$ and $J_{\rm w}{}^{\mu\dagger}$ both commute with with $F_a+F_a{}^5$ for any index a. Then using $[F_a{}^5,\mathcal{H}_{\rm w}]=-[F_a,\mathcal{H}_{\rm w}]$, we may replace the F^5's in Eq. (19) by F's to obtain

$$m_{ab}{}^2|_{\rm w}=4f^2\langle 0|[F_a,[F_b,\mathcal{H}_{\rm w}(0)]]|0\rangle=0, \tag{20}$$

since the F's annihilate the symmetrical vacuum. This result is in direct contradiction with Eqs. (12) and (13). To see this we simply note that since c_3 is (experimentally) nonzero, the vanishing of $m^2{}_{ab}|_{\rm w}$ is consistent with Eqs. (12) and (13) only if the corresponding weak mass matrix for baryons vanishes in the same limit, i.e., when we neglect strong violations of $SU(3)\otimes SU(3)$. The baryon mass matrix cannot vanish, however, without destroying the successful current-algebra calculations of nonleptonic decays of hyperons.[1] Thus Eq. (20), like Eq. (19), contradicts the observed pattern of octet enhancement.

Actually, it is very easy to show in general that octet enhancement cannot occur, in first order, for perturbations around an $SU(3)\otimes SU(3)$ limit. Suppose that $\mathcal{H}_{\rm p}$ in Eq. (16) belongs to a single representation of $SU(3)\otimes SU(3)$. When this is the case, it can be shown that the ratio of **8** to **27** in the resulting mass-squared matrix is a purely group theoretical number which contains no dynamics. Therefore, in the $SU(3)\otimes SU(3)$-symmetrical limit there cannot be any dynamical mechanism which, in all cases, selectively enhances the octet part of $m^2{}_{ab}$. In Appendix B, this situation is illustrated in a particular Lagrangian model.

The physical implication of the above results is simply that the observed meson mass matrix is not, even approximately, given by Eq. (16) with $\mathcal{H}_{\rm p}=\epsilon\mathcal{H}'+\mathcal{H}_{\rm em}+\mathcal{H}_{\rm w}$. What is apparently happening in the meson electromagnetic mass differences, for example, is the terms of order $\epsilon\mathcal{H}'\mathcal{H}_{\rm em}$, $(\epsilon\mathcal{H}')^2\mathcal{H}_{\rm em}$, etc., are not small even though they contain the small symmetry-breaking parameter ϵ. The reason is, apparently, that these terms allow $\mathcal{H}_{\rm em}$ to couple into an octet-enhancement mechanism and therefore make a contribution which is as large or larger than the lowest-order term. The same thing must, of course, be happening in the case of weak interactions. It is important to realize that the same situation must also occur in the strong breaking of $SU(3)\otimes SU(3)$. That is, the term of order $\epsilon\mathcal{H}'$ cannot be octet enhanced, while the terms of order $(\epsilon\mathcal{H}')^2$, $(\epsilon\mathcal{H}')^3$, $\cdots$ are enhanced and presumably end up being comparable to or larger than the lowest-order terms.

So far, it has been shown only that higher-order terms are important for the meson masses. It will now be argued, however, that the observed pattern of octet enhancement in other mass differences, like those in the baryon octet, is also coming only from second- and higher-order terms in a perturbation expansion around the $SU(3)\otimes SU(3)$-symmetric limit. The argument runs as follows. Suppose, contrary to what was stated above, that other mass differences were octet enhanced in first order. The first-order term would then have a characteristic size $F\epsilon\mathcal{H}'$, where F is an octet-enhancement factor which experimentally appears to be on the order of 5. Second-order terms, if important, would be of order $F^2(\epsilon\mathcal{H}')^2$, since the enhancement mechanism can act twice. The meson mass differences, on the other hand, would be of order $\epsilon\mathcal{H}'$, $F(\epsilon\mathcal{H}')^2$, etc., since they cannot be enhanced in lowest order. We would then expect the pseudoscalar mass differences to be a factor of F^{-1} smaller than other mass differences when, in fact, they are not; witness the relation $m_K{}^2-m_\pi{}^2=m_{K^*}{}^2-m_\rho{}^2$ which holds almost exactly.

We have concluded, then, that *although the observed departures from $SU(3)\otimes SU(3)$ symmetry may be small, they are not first order in the symmetry-breaking interactions*. Hiding behind our inability to calculate the terms of order $\mathcal{H}_{\rm em}\epsilon\mathcal{H}'$, $\mathcal{H}_{\rm em}(\epsilon\mathcal{H}')^2$, etc., we no longer have to worry about the violent disagreement between Eq. (18) and experiment.

Given that the higher-order terms in $\epsilon\mathcal{H}'$ are important, we have now to understand why the Gell-Mann–Okubo mass formula works so well. There are two possible explanations for this. The $SU(3)\otimes SU(3)$ symmetry-breaking interaction can be decomposed into $\epsilon\mathcal{H}'=\epsilon\mathcal{H}_1'+\epsilon\mathcal{H}_8'$, where $\mathcal{H}_1'$ is an $SU(3)$ singlet and $\mathcal{H}_8'$ breaks $SU(3)$. The first possibility is that the important higher-order terms invoke only $\mathcal{H}_1'$ raised to higher powers. That is, we need keep only terms like $\epsilon\mathcal{H}_8$, $\epsilon\mathcal{H}_8\epsilon\mathcal{H}_1$, $\epsilon\mathcal{H}_8(\epsilon\mathcal{H}_1)^2$, etc., or $\mathcal{H}_{\rm em}$, $\mathcal{H}_{\rm em}\epsilon\mathcal{H}_1$, etc., and not terms like $(\epsilon\mathcal{H}_8)^2$ or $\epsilon\mathcal{H}_8\mathcal{H}_{\rm em}$. If this is the case, the $SU(3)$ properties of a perturbation [but not the $SU(3)\otimes SU(3)$ properties] are not affected by the higher-order terms. The other possibility is that terms like $(\epsilon\mathcal{H}_8')^2$ are important but that the octet parts of these terms are dynamically enhanced relative to the **27** and higher parts, so that the net effect is again an octet pattern of mass splittings. In connection with this latter possibility, it should be pointed out that one does not expect higher-order terms belonging to one representation of $SU(3)\otimes SU(3)$ to be enhanced relative to those belonging to other representations. That is, for perturbations around an $SU(3)$-symmetrical limit one can show that if there is a dynamical enhancement mechanism, the mechanism must be such that perturbations belonging to a definite representation of $SU(3)$ are enhanced.[22] This is not the case for $SU(3)\otimes SU(3)$ which is realized in a different way, i.e., via Goldstone bosons rather than multiplets of particles. In a model discussed in Appendix B, we show how the $SU(3)$ octet part of a perturbation belonging to any representation of $SU(3)\otimes SU(3)$ can be enhanced, without any selective enhancement of perturbations belonging to particular representations of $SU(3)\otimes SU(3)$. Actually, it is easy to see how this could come about. If there were an octet of scalar mesons with a very small mass, the resulting "tadpole diagrams" could enhance the octet part of any perturbation but would not, in general, enhance perturbations belonging to a specific $SU(3)\otimes SU(3)$ representation. We do not, therefore, expect the effects of higher-order terms in $\epsilon\mathcal{H}'$ to have any simple properties under $SU(3)\otimes SU(3)$.

To summarize, our conclusions about the breaking of $SU(3)\otimes SU(3)$ are the following. (i) Lowest-order perturbation theory around the $SU(3)\otimes SU(3)$-symmetrical limit is not valid. (ii) The net effect of the breaking can, as seems to be the case experimentally, still have simple $SU(3)$ properties. (iii) The deviations from symmetry are not, however, expected to have simple $SU(3)\otimes SU(3)$ transformation properties.

We now turn to some special features of symmetry breaking that come up if, as discussed in Sec. II, the symmetrical vacuum is invariant under the group $\mathfrak{z}$ and parity doublets exist. In this case, we break $\epsilon\mathcal{H}'$ into $\epsilon\mathcal{H}'=\epsilon\mathcal{H}_s'+\epsilon\mathcal{H}_d'$, where $\mathcal{H}_s'$ and $\mathcal{H}_d'$ belong to singlet and doublet representations of $\mathfrak{z}$.[24] Clearly, $\mathcal{H}_d'$ cannot be zero, otherwise the symmetry under $\mathfrak{z}$ would not be broken and parity doublets would remain degenerate and could not decay into two singlets. The singlet term may or may not be present. If $\mathcal{H}_s'$ is not present, the splitting of undoubled $SU(3)$ multiplets can occur only in second and higher orders which, according to the above discussion, does not mean that these splittings should be small.

There is now the possibility that perturbations transforming according to definite representations of $\mathfrak{z}\otimes SU(3)$, not just $SU(3)$, are dynamically enhanced. In writing the mass formulas in Eqs. (9) and (10), we have assumed that the net effect of symmetry breaking is to produce mass splittings that transform like (i) a $\mathfrak{z}$ singlet and $SU(3)$ octet, leading to Eq. (9); (ii) a $\mathfrak{z}$ doublet and $SU(3)$ singlet, whose strength is measured by the parameter c in Eq. (10); (iii) a $\mathfrak{z}$ doublet and $SU(3)$ octet, whose strength is measured by c and d in Eq. (10). The $\mathfrak{z}$ singlet–$SU(3)$-octet term is necessary to produce the usual octet pattern of splittings in undoubled $SU(3)$ multiplets. The other two terms are simply a guess.

C. Scalar Mesons and Symmetry Breaking

In the broken symmetry, the strange-vector currents as well as the axial-vector currents are not conserved. It is interesting to ask, then, if there is a strange meson κ which dominates the divergences of the strange-vector currents in the same way as the pseudoscalar mesons dominate the divergences of the axial-vector currents. There are two possible reasons why such a meson, if it exists, might dominate the strange-vector divergences. They are the following.

(i) In the symmetry limit, the hypothetical κ is a Goldstone boson and dominates for the same reason that the pseudoscalar octet dominates the axial-vector divergences.

(ii) The meson belongs to an octet of scalar mesons that have no particularly unusual properties in the symmetry limit, but play a special role in symmetry breaking.

Case (i) is unattractive because there is no good reason why one should have both (slightly broken) $SU(3)$ multiplets and a strange scalar Goldstone boson. We will not discuss the possibility further. Case (ii) is essentially the tadpole model of Coleman and Glashow.[22] It has two difficulties. First, one would not expect scalar mesons to play any special role in symmetry breaking unless they have an unusually low mass. Experimentally, the least massive candidates for

[24] A term in $\mathcal{H}'$ belonging to, say, $(3,\bar{3})\oplus(\bar{3},3)$, is a doublet under $\mathfrak{z}$, while a term like $(8,8)$ is a singlet. The distinction between the doublet and singlet terms is that in the former the individual right- and left-handed $SU(3)$ representations have nonzero triality.

a scalar octet seem to be in the region around 1 BeV. The other difficulty is that recent analyses of electromagnetic mass differences[9] have rather strongly suggested that it is not a real scalar meson but rather the A_2 Regge trajectory which is playing a special role in electromagnetic violations of $SU(3)$.

We may conclude, then, that it is not likely that there is a scalar meson which dominates the strange-vector divergences for either of the above reasons. We are left, however, with the interesting idea that the $SU(3)$ partners of the A_2 trajectory, which does seem to play a special role in symmetry breaking, might, in some sense, dominate these divergences. Assuming a slope of one unit of spin per BeV^2, the nonet of trajectories to which the A_2 belongs would make a scalar nonet with masses in the neighborhood of -300 MeV^2. This is indeed a small mass squared, but the trouble is that it is negative, so that the would-be scalar nonet is not really there. Since we cannot dominate a divergence with a state that does not exist, we need a new principle which, evidently, should amount to pole dominance in the angular momentum plane. The next few paragraphs will be devoted to a heuristic discussion of this idea, which is a mathematical formulation of the idea given in Appendix C.

Actually, rather than talk about pole dominance for the divergences of currents, we will work with matrix elements of $\mathcal{H}(x)$, where $H=\int d^3x\,\mathcal{H}(\mathbf{x})$ is the total Hamiltonian of the hadrons. This is essentially the the same thing, since if we assume that $[F_i(t),\mathcal{H}(\mathbf{x},t)] = -i\partial_\alpha \mathcal{F}_i^\alpha(\mathbf{x},t)$ holds,[4] then the divergences are $SU(3)$ rotations of the symmetry-breaking parts of $\mathcal{H}$. We will write the spin-averaged matrix element $\langle\alpha|\mathcal{H}(0)|\beta\rangle$ between members α and β of the same $SU(3)$ multiplet as

$$\langle\alpha|\mathcal{H}(0)|\beta\rangle = P_\alpha^0 P_\beta^0 h_{\alpha\beta}(q^2) + (\text{terms proportional to } q^0q^0 \text{ and } g^{00}), \quad (21)$$

where $q^\nu = P_\alpha^\nu - P_\beta^\nu$ is the momentum transfer and the states are assumed to be normalized invariantly to $\langle\mathbf{P}|\mathbf{P}'\rangle = 2P^0\delta^3(\mathbf{P}-\mathbf{P}')$. At $q^2=0$, $h_{\alpha\beta}$ is the mass-squared matrix of the multiplet, i.e.,

$$\mathfrak{M}^2{}_{\alpha\beta} = h_{\alpha\beta}(0). \quad (22)$$

With these preliminaries out of the way, we can return to our task of formulating a principle of pole dominance in the angular momentum plane. As a first step, let us recall the usual form of a pole term in a form factor like $h_{\alpha\beta}(q^2)$. One writes

$$h_{\alpha\beta}(q^2) = \sum_i \frac{G_i g_{i\alpha\beta}}{q^2-m_i^2} + (\text{other terms}), \quad (23)$$

where the sum runs over a multiplet of (hypothetical) scalar mesons. The essential ingredients of the pole term are as follows. It contains (i) a "momentum transfer denominator" $(q^2-m_i^2)^{-1}$ corresponding to the fact that we are a distance $q^2-m_i^2$ away from the pole, (ii) a coupling $g_{i\alpha\beta}$ of the meson to α and β, and (iii) a parameter G_i which is independent of α and β. A pole term in the angular momentum plane should contain the same factors. First, there will be an "angular momentum denominator" $-\alpha_i(q^2)^{-1}$, where $\alpha_i(q^2)$ is the spin of the ith trajectory at a mass squared equal to q^2. The reason why the denominator is simply $-\alpha_i^{-1}$ is that the spin of $\mathcal{H}$ is zero, so that $-\alpha_i$ is the distance "off the spin shell," just as $q^2-m_i^2$ is the "distance off the mass shell." Second, there will be a factor $\gamma_{i\alpha\beta}(q^2)$ which is the coupling of the trajectory to $\alpha\beta$ at a mass of q^2, and finally a factor of $\Gamma_i(q^2)$ which is the analog of G_i. Thus, we should be able to write

$$h_{\alpha\beta}(q^2) = -\sum_i \Gamma_i(q^2)\frac{1}{\alpha_i(q^2)}\gamma_{i\alpha\beta}(q^2) + (\text{terms from other trajectories, cuts, and background}), \quad (24)$$

where the sum is now understood to run over the nonet of trajectories of which the A_2 is a member. That a formula like Eq. (24) does, in fact, exist is shown in Appendix C.

If we now assume that for $q^2=0$, the $SU(3)$-violating part of the $\mathfrak{M}^2{}_{\alpha\beta}$ is dominated by the pole, we have

$$\mathfrak{M}^2{}_{\alpha\beta} \approx -\sum_i \Gamma_i(0)\frac{1}{\alpha_i(0)}\gamma_{i\alpha\beta}(0) + [SU(3)\text{-symmetrical terms}], \quad (25)$$

where, as before, the sum runs over the nonet of tensor-meson trajectories.

The reason for believing the formula in Eq. (25) is simply that it seems to agree with experiment. The basic conclusion of the papers listed in Ref. 9 is that the formula should work for electromagnetic mass differences. From the discussion around Eqs. (12) and (13), it should be clear that if Eq. (25) works for electromagnetic violations of $SU(3)$, it must also work for strong and weak violations of $SU(3)$. In fact, Eq. (25) is just Eqs. (12) and (13) with the D's replaced by $\Gamma\alpha^{-1}$ and the combinations of f and d symbols replaced by $\gamma_{i\alpha\beta}(0)$. As has been noted before,[9] the d-to-f ratio c_2/c_3 determined from mass differences is in agreement with the d-to-f ratio in the coupling $\gamma_{i\alpha\beta}(0)$ of the tensor mesons to the baryon octet, as determined from high-energy scattering data.[25]

Since the couplings $\gamma_{i\alpha\beta}(0)$ are directly measurable in high-energy forward scattering, it is very easy to devise further tests of Eq. (25). For example, one finds that at high energies

$$\frac{\sigma(\bar{p}p)+\sigma(pp)-\sigma(\bar{p}n)-\sigma(pn)}{\sigma(K^-p)+\sigma(K^+p)-\sigma(K^-n)-\sigma(K^+n)} \approx \frac{M_p^2-M_n^2}{m_{K^+}^2-m_{K^0}^2} \quad (26)$$

[25] V. Barger and M. Olsson, Phys. Rev. **164**, 1080 (1966).

should hold where, for example, $\sigma(\bar{p}p)$ is the total cross section for antiprotons on protons. The derivation of Eq. (26) is given in Appendix C. The cross sections [especially $\sigma(\bar{p}n)$] are not yet well enough known to make a meaningful comparison of the two sides of Eq. (26).

We have, then, a pole-dominance principle which for reasons listed above is very likely to be correct. The problem is now to find a theoretical reason why the pole should dominate. In thinking about this, one has to keep in mind that, according to our previous discussion, the pole can dominate only in second and higher orders when we consider perturbations around an $SU(3)\otimes SU(3)$-symmetrical limit. Evidently, this is anything but a simple problem and the author has nothing more to add except the following remark.

For the sake of argument, let us assume that Eq. (25) gives the correct description of the deviations from symmetry which appear in the real world. If this is so, and we imagine slowly turning off all symmetry-breaking interactions to approach an $SU(3)\otimes SU(3)$-symmetrical limit, then at some point the formula must fail, since it cannot be correct in lowest order. This must mean, then, that the tensor-meson trajectories do something very peculiar in the symmetry limit. An answer to our problem might, therefore, uncover some really new and fascinating features of the strong interactions.

V. DISCUSSION

We have outlined, in the previous sections, a highly symmetrical picture of the strong interactions which is particularly beautiful in the way that hadron symmetries are connected to their weak and electromagnetic interactions. There are, however, several unanswered questions.

If we were to build a Lagrangian model of $SU(3)\otimes SU(3)$ symmetry in analogy with the σ model, we would introduce fields transforming according to definite representations of $SU(3)\otimes SU(3)$. When such a model possesses a solution containing a massless 0^- octet, the particle states belonging to a given $SU(3)\otimes SU(3)$ field can split very far apart in mass but do, nevertheless, exist. It may be then that on a mass scale which is sufficiently large, we will be able to recognize supermultiplets of particles falling into representations of $SU(3)\otimes SU(3)$. This is by no means a definite prediction of the symmetry, however; the nonlinear models[12] provide a counterexample.

Then there is the whole question of symmetry breaking. In Sec. IV, we pointed out a number of problems associated with the breaking of $SU(3)\otimes SU(3)$. These particular aspects of the symmetry breaking, like octet enhancement, are probably more directly related to the dynamics of hadrons in the symmetry limit than to the specific character of the symmetry-breaking interaction $\epsilon H'$. One should also ask: Just what is $\epsilon H'$? It it simply an extra term in the Hamiltonian built out of whatever variables it is that describe hardons, or is $\epsilon H'$ the result of the coupling of hadrons to another kind of particle? In the latter case what we have in mind is something like Ne'eman's fifth interaction.[26] A perhaps, less fundamental, but more tractable question, is: What are the transformation properties of $\epsilon H'$ under $SU(3)\otimes SU(3)$? It has been suggested[15] that $\epsilon H'$ can be broken up into two pieces, one which is fairly large but conserves $SU(2)\otimes SU(2)$ and a considerably smaller piece which breaks $SU(2)\otimes SU(2)$ down to $SU(2)$. If this is the case, $SU(2)\otimes SU(2)$ should be a rather better symmetry than $SU(3)$. It is interesting that there is a direct test of this idea. The Callen-Treiman relation,[27] which relates a certain combination of form factors for $K\to\pi+e+\nu$ to the decay constant for $K\to\mu+\nu$, may be thought of as a consequence of $SU(2)\otimes SU(2)$. In the limit of $SU(3)$ symmetry this relation becomes an identity, but is nontrivial when $SU(3)$ is broken. If the various quantities which appear in the Callen-Treiman relation show deviations from $SU(3)$, which are large compared to the over-all error in the relation, then we will have good reason to believe that $SU(2)\otimes SU(2)$ is actually better conserved than $SU(3)$.

Finally, a remark about current algebra. If our view of the strong interactions as being almost $SU(3)\otimes SU(3)$-symmetric is correct, then all the tests of current algebra which employ PCAC are really consequences of symmetry. Now one reason that Gell-Mann suggested the current algebra in the first place was to give a rigorous algebraic definition of a *broken* symmetry. It would be very desirable, therefore, to have some good tests of current algebra that are sensitive to symmetry-breaking effects.

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APPENDIX A

In this Appendix, we verify that to second order in perturbation theory the pion mass, as calculated with Lagrangian (2), remains zero. Perturbation theory for the σ model is quite straightforward provided that one is careful to maintain the $SU(2)\otimes SU(2)$ invariance of the theory. This has mostly to do with the normal-ordering prescription. To begin with, since a γ_5 trans-

[26] Y. Ne'eman, Phys. Rev. 134, B1335 (1964). Actually, Ne'eman's particular interaction of a vector meson coupled to the strangeness current does not work. The reason is that the axial-vector current $\mathfrak{F}_8^{\alpha 5}$ would remain conserved so that the η mass would have to remain at zero.

[27] C. G. Callan and S. B. Treiman, Phys. Rev. Letters 16, 197 (1966).

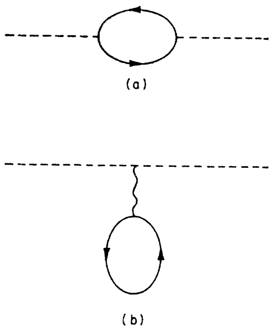

FIG. 1. Two diagrams for the pion self-energy in the σ model. The broken lines represent pions, the solid lines are nucleons, and the wiggly line is a σ.

formation mixes nucleon creation operators with antinucleon annihilation operators, the term $g\bar{N}(\sigma' + i\gamma_5 \boldsymbol{\tau}\cdot\boldsymbol{\pi})\, N$ in Eq. (1), cannot be normal ordered without destroying the $SU(2)\otimes SU(2)$ invariance. This has the consequence that diagrams where a σ line ends in a closed nucleon loop must be retained. Also, our argument for choosing the particular displacement $\sigma=\sigma'+\sqrt{A}$ depended critically on the fact that the last term in Lagrangian (1) is the square of an operator. This means that we should order the mesonic interaction term as $B^2(:\sigma'^2:+:\pi^2:-A)^2$ which leaves the trilinear and quadralinear terms in Eq. (2) only partially ordered. Consequently, we must keep diagrams where a σ line disappears into a σ closed loop. Also, one must keep diagrams where a single-pion line makes a closed loop and crosses itself. On the other hand, diagrams where, for example, a π^+ or σ leaves a π^0 line, makes a closed loop, and returns to the same point are not present because of the partial ordering of the quadratic term.

Of course, we also have to pick our expansion parameters in such a way that the $SU(2)\otimes SU(2)$ invariance is maintained in each order of perturbation theory. To do this and keep the bare masses fixed, one has to regard $g\sqrt{A}$ and $4B^2A^2$ as being fixed and equal to M_0 and $2m_{0\sigma}^2$. This means that only one of the variables, B^2 or A or g, can be considered as an independent expansion parameter. It is convenient to choose g. One then expresses A and B^2 in terms of $m_{0\sigma}$, M_0, and g and then expands in powers of g.

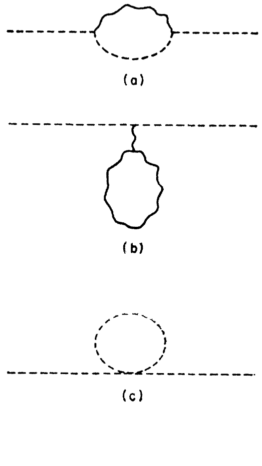

FIG. 2. (a) Three more diagrams for the pion self-energy in the σ model. (b) Only a single pion of the same charge as the external line is allowed to make the loop.

With these preliminaries, it is easy to calculate the pion mass to order g^2. The diagrams are shown in Figs. 1 and 2. Each of the diagrams is, by itself, quadratically divergent, but a straightforward calculation shows that Figs. 1(a) and 1(b) cancel each other exactly and that Figs. 2(a)–2(c) cancel among themselves.

APPENDIX B

In this Appendix we show how the general features of perturbations around an $SU(3)\otimes SU(3)$-symmetric limit discussed in Sec. IV manifest themselves in a particular Lagrangian model. We start with a set of meson fields φ_i transforming according to some representation of $SU(3)\otimes SU(3)$. Under infinitesmal transformation generated by the vector charges, the change in the fields is to be $\delta\varphi_i = R_{ij}{}^a\varphi_j$, where $a=1,\cdots,8$ runs over the octet of generators. Analogously, for infinitesmal transformations generated by axial-vector charges, we have $\delta\varphi_i = R_{ij}{}^{a5}\varphi_j$. Now consider a Lagrangian

$$\mathcal{L}_0 = -\tfrac{1}{2}\partial_\mu\varphi_i\partial^\mu\varphi_i + \mathcal{L}_I{}^0(\varphi)\,, \tag{B1}$$

where it is assumed that the fields are so chosen that $\varphi_i\varphi_i$ is invariant and $\mathcal{L}_I{}^0(\varphi)$ is an $SU(3)\otimes SU(3)$-invariant function of the fields. As in the σ model, we look for a minimum of the classical potential energy. We suppose that there is a minimum for $\varphi_i=\varphi_i{}^0$ and, defining new fields $\psi_i=\varphi_i-\varphi_i{}^0$, expand around $\psi_i=0$ to obtain

$$\mathcal{L}_0 = -\tfrac{1}{2}\partial_\mu\psi_i\partial^\mu\psi_i - \tfrac{1}{2}\mu^2{}_{ij}\psi_i\psi_j + g_{ijk}\psi_i\psi_j\psi_k + (\text{quartic and higher terms in } \psi)\,, \tag{B2}$$

where $\mu^2{}_{ij}$ is evidently the (bare) mass-squared matrix for the particles and g_{ijk} is a (bare) trilinear coupling. If we assume that the ground state $\varphi_i=\varphi_i{}^0$ is $SU(3)$-symmetric, $\mu^2{}_{ij}$ and g_{ijk} will be $SU(3)$-symmetric. For future reference, we need the conditions on $\mu^2{}_{ij}$ and g_{ijk} that follow from $SU(3)\otimes SU(3)$ symmetry. To do so, we write

$$\delta\mathcal{L}_I{}^0 = \frac{\partial\mathcal{L}_I{}^0}{\partial\varphi_i}\delta\varphi_i = \frac{\partial\mathcal{L}_I{}^0}{\partial\varphi_i}R_{ij}{}^{a5}\varphi_j = 0 \tag{B3}$$

for an infinitesmal transformation. Rewriting Eq. (B3) in terms of $\psi_i=\varphi_i-\varphi_i{}^0$ gives

$$\frac{\partial\mathcal{L}_I{}^0}{\partial\psi_i}R_{ij}{}^{a5}\psi_j = \frac{\partial\mathcal{L}_I{}^0}{\partial\psi_i}R_{ij}{}^{a5}\varphi_j{}^0 = \frac{\partial\mathcal{L}_I{}^0}{\partial\psi_i}\chi_i{}^a\,, \tag{B4}$$

where $\chi_i{}^a = R_{ij}{}^{a5}\varphi_j{}^0$ are eight constant vectors, Finally, expanding in powers of ψ and equating terms of a given order yields

$$\mu^2{}_{ij}\chi_j{}^a = 0\,, \tag{B5}$$

$$-\mu^2{}_{ik}R_{kj}{}^a = 3g_{ijk}\chi_k{}^a\,. \tag{B6}$$

Evidently, Eq. (B5) tells us that μ_{ik}^2 has eight eigenvectors with eigenvalue zero. Thus we have eight massless particles which are, of course, the octet of 0^- Goldstone bosons. Equation (B6) is a constraint on the trilinear couplings of these massless particles to themselves and other mesons.

Let us now add a perturbation $\epsilon\mathcal{L}'(\varphi)$ which breaks the $SU(3)\otimes SU(3)$ symmetry and find the new ground state. That is, we wish to find new values of the fields $\varphi_i^0+\delta\varphi_i^0$ for which

$$\partial\mathcal{L}_I^0/\partial\varphi_i+\epsilon\partial\mathcal{L}'/\partial\varphi_i=0. \quad \text{(B7)}$$

To order ϵ, Eq. (B7) can be replaced by

$$\mu^2{}_{ij}\delta\varphi_j{}^0=\epsilon\eta_i \quad \text{(B8)}$$

or

$$\delta\varphi_j{}^0=(\mu^2)^{-1}{}_{ji}\epsilon\eta_i\,, \quad \text{(B9)}$$

where $\eta_i=\partial\mathcal{L}'/\partial\varphi_i$ evaluated at $\varphi_j=\varphi_j{}^0$. To find the perturbed bare masses, we expand $\mathcal{L}_0+\epsilon\mathcal{L}'$ around the new ground state to obtain

$$\delta\mu^2{}_{ij}=\epsilon\lambda_{ij}-3g_{ijk}\delta\varphi_k{}^0+O(\epsilon^2)\,, \quad \text{(B10)}$$

where $\lambda_{ij}=-\partial^2\mathcal{L}'/\partial\varphi_i\partial\varphi_j$ evaluated at $\varphi_k=\varphi_k{}^0$.

To interpret Eq. (B10), let us choose a basis such that the unperturbed mass-squared matrix is diagonal, i.e., $\mu^2{}_{ij}=\delta_{ij}\mu^2{}_i$. Then inserting Eq. (B9) into Eq. (B10), one obtains

$$\delta\mu^2{}_{ij}=\epsilon\left(\lambda_{ij}-3\sum_k g_{ijk}\frac{1}{\mu^2{}_k}\eta_k\right). \quad \text{(B11)}$$

It is easy to see, now, that the factors of $(\mu^2{}_k)^{-1}$ on the right-hand side of Eq. (B11) provide a potential enhancement mechanism. That is, suppose the mesons belong to, say, (8,8) under $SU(3)\otimes SU(3)$ so that there is both an octet and 27-plet of scalar mesons; then if the mass of the octet is small, the **8** part of the second term on the right of Eq. (B11) will be enhanced, and similarly the **27** part will be enhanced if the 27-plet mass is small. This enhancement mechanism is, of course, independent of the perturbation as long as the relevant component of η_k is nonzero.

In the text it was pointed out that an enhancement mechanism like that found above cannot actually lead to enhancements in the pseudoscalar-meson masses. In the present model, this shows up when we take account of the constraint in Eq. (B6). Let us denote the pseudoscalar mass matrix by $\delta\mu^2{}_{ab}$, a, $b=1,\cdots,8$ and the trilinear coupling of pseudoscalars a and b to a scalar meson k by g_{abk}. Equation (B6) then implies

$$3g_{abk}=-\mu^2{}_k\sum_j\frac{R_{kj}{}^a\chi_j{}^b}{\chi^2}, \quad \text{(B12)}$$

where

$$\chi^2=\sum_k(\chi_k{}^a)^2=\sum_k(\chi_k{}^b)^2.$$

Evidently, the factor of $\mu^2{}_k$ in g_{abk} exactly cancels the enhancement factor in Eq. (B11), and for the pseudoscalar masses, we find

$$\delta\mu^2{}_{ab}=\epsilon(\lambda_{ab}+\chi^{-2}\sum_{kj}R_{kj}{}^a\chi_j{}^b\eta_k)$$

$$(a,\, b \text{ pseudoscalar mesons})\,, \quad \text{(B13)}$$

which is completely independent of the scalar-meson masses.

Mass splittings other than those in the pseudoscalar octet will, in this model, actually exhibit octet enhancement when the scalar octet has a small mass. This follows from the facts that Eq. (B6) is the only constraint on g_{ijk} and that this constraint effects only the pseudoscalar masses. We can use this result to prove a point stated in the text. The above considerations were completely independent of the $SU(3)\otimes SU(3)$ transformation properties of $\mathcal{L}'$. It follows then that in this model the octet part of a perturbation belonging to any $SU(3)\otimes SU(3)$ representation will be enhanced. Thus we have an enhancement of definite representations of $SU(3)$, but no enhancement of definite representations of $SU(3)\otimes SU(3)$.

APPENDIX C

In this Appendix we show how to derive Eq. (24). Consider an operator density $\mathcal{O}_J(0)$ which transforms under rotation like an object of spin J. We define form factors

$$f_J{}^i(t)=\langle 0|\mathcal{O}_J(0)|i,\, p^0=\sqrt{t},\, \mathbf{p}=0\rangle\,, \quad \text{(C1)}$$

where $|i,\, p^0=\sqrt{t},\, \mathbf{p}=0\rangle$ is a state at rest with energy $\sqrt{t}$ and internal quantum numbers i. In what follows, we treat the index i like a channel index in scattering theory. Thus we have a partial-wave scattering amplitude $T_{ij}(J,t)$ for $i\to j$ with angular momentum J, which we assume can be written in the matrix N/D form as $T_{ij}(J,t)=[N(J,t)D^{-1}(J,t)]_{ij}$.

The general solution of the Omnes equations for the $f_J{}^i(t)$ is then[28]

$$f_J{}^i(t)=\sum_j[D^{-1}(J,t)]_{ji}P_j(t)\,, \quad \text{(C2)}$$

where the P_j are polynomials in t. Our derivation of Eq. (24) from this formula.

If the scattering amplitude $T_{ij}(J,t)$ is an analytic function of J, then so is $D^{-1}(J,t)$. Thus we may write

$$[D^{-1}(J,t)]_{ji}=\frac{1}{2\pi i}\int_C\frac{1}{J'-J}[D^{-1}(J',t)]_{ji}dJ', \quad \text{(C3)}$$

where the contour C circles the pole at $J'=J$. Opening up the contour we can transform the integral in Eq. (C3) into an integral around the singularities of D^{-1}

[28] R. F. Dashen and S. Frautschi, Phys. Rev. **143**, 1171 (1966).

plus an integral around a large contour which we call the background term. The singularities of D^{-1} will come from poles at $J=\alpha_r(t)$, where $\alpha_r(t)$ is the position of the rth Regge pole at energy $\sqrt{t}$, and also from Regge cuts, etc. Near a pole $[D^{-1}(J,t)]_{ij}$ behaves like $\delta_i^r(t)\gamma_j^r(t)[J-\alpha_r(t)]^{-1}$, where γ_j^r is the coupling of the rth trajectory to channel j and the δ_i^r are a set of quantities which, in general, bear no simple relation to the γ's. Explicitly computing the contribution of the poles to the integral in Eq. (C3), we find

$$[D^{-1}(J,t)]_{ji}=\sum_r \frac{\delta_j^r(t)\gamma_i^r(t)}{\alpha_r(t)-J}$$

$$+(\text{contribution from cuts and background}). \quad \text{(C4)}$$

Finally, inserting Eq. (C4) into Eq. (C2), we obtain

$$f_J{}^i(t)=\sum_r \frac{\Gamma^r(t)\gamma_i^r(t)}{\alpha_r(t)-J}$$

$$+(\text{contribution from cuts and background})\,, \quad \text{(C5)}$$

where $\Gamma^r(t)=\sum_j P_j(t)\delta_j^r(t)$. Equation (25) is a special case of Eq. (C5) with $J=0$ and particle labels α and β replacing the channel index i. In the sum over poles one is, of course, only supposed to keep those which have the same internal quantum numbers, e.g., charge conjugation, as the operator $O_J(0)$. Also, in order to do the above analytic continuation of $D(J,t)$ properly, we should have kept track of signature. It turns out that only poles with even signature for even J and odd signature for odd J can appear in Eq. (C5).

The above derivation assumes that the representation in Eq. (C2) holds and that $D(J,t)$ has the requisite analyticity properties. Both of these assumptions are well known to be true in simple models based on two-particle unitarity. *Formally*, there is nothing wrong with Eq. (C2) or $D(J,t)$ when many-particle channels are included.[29] However, our derivation of Eq. (C5) should be understood as being based on assumptions that may not be justified.

Next is the derivation of Eq. (27), promised in the text. Actually, this is quite easy. The sums and differences of cross sections in Eq. (27) are so chosen that only Regge poles with isospin 1 and odd G parity can contribute. Since the highest trajectory with these quantum numbers is the A_2, the left-hand side of Eq. (27) is, for high enough energies, a direct measure of $\gamma_{A_2K}\gamma_{A_2N}/(\gamma_{A_2N})^2=\gamma_{A_2K}/\gamma_{A_2N}$, where γ_{A_2K} and γ_{A_2N} are the couplings (at zero momentum transfer) of the A_2 trajectory to kaons and nucleons. To obtain the right-hand side of Eq. (27), one simply uses Eq. (26) to express this ratio of couplings as a ratio of electromagnetic mass differences.

[29] S. Mandelstam, Phys. Rev. **140**, B375 (1965).