The imaginary part of this equation is

$$\mathrm{Im}T(s,t)=-\frac{b}{\Lambda}\,\mathrm{Im}T\left(\frac{s+u}{2}+2a\Lambda,\,t\right)-\frac{2a^2\lambda}{[(s-u)/2]^2-4a^2\Lambda^2}\left\{A(s,t)-A(u,t)+\frac{s-u}{4a\Lambda}\left[A\left(\frac{s+u}{2}+2a\Lambda,\,t\right)-A\left(\frac{s+u}{2}-2a\Lambda,\,t\right)\right]\right\}. \quad (18)$$

Letting $\lambda\to 0$ in this gives us

$$2\,\mathrm{Im}T(s,t)=A(u,t)-A(s,t), \quad (19)$$

which is simply the (generalized) unitarity relation for $t\leqslant 0$.[7] Putting $\Lambda=-b$ in Eqs. (17) and (18) yields

$$T(s,t)=T^*(u,t), \quad (20)$$

the crossing relation. If we take Λ to ∞ in Eq. (17), we recover Eq. (14). If we leave Λ arbitrary in Eq. (17):

$$\Lambda=\nu_0/2a,$$

and change variables:

$$\nu\equiv(s-u)/2,\quad f(\nu,t)\equiv T(s,t),\quad F(\nu,t)\equiv A(s,t),$$

we get a *once-subtracted* dispersion relation

[7] cf. Gasiorowicz (Ref. 5), p. 340, Eq. (21.25).

$$\mathrm{Re}f(\nu,t)=\mathrm{Re}f(\nu_0,t)-\frac{\nu^2-\nu_0^2}{2\pi}P\int\frac{F(\nu',t)d\nu'^2}{(\nu'^2-\nu_0^2)(\nu'^2-\nu^2)}. \quad (21)$$

However, the full content of Eq. (16) is not apparent unless both Λ and M are left arbitrary:

$$\Lambda=\nu_0/2a,\quad \mathrm{M}=\nu_1/2a.$$

Then we get the fixed-t dispersion relation with *two arbitrary subtractions*:

$$\mathrm{Re}f(\nu,t)=\frac{\nu^2-\nu_0^2}{\nu_1^2-\nu_0^2}\,\mathrm{Re}f(\nu_1,t)+\frac{\nu^2-\nu_1^2}{\nu_0^2-\nu_1^2}\,\mathrm{Re}f(\nu_0,t)-\frac{(\nu^2-\nu_0^2)(\nu^2-\nu_1^2)}{2\pi}\times P\int\frac{F(\nu',t)d\nu'^2}{(\nu'^2-\nu_0^2)(\nu'^2-\nu_1^2)(\nu'^2-\nu^2)}. \quad (22)$$

Of course, there is no rigorous way of guaranteeing that even this prescription can be carried out. More subtractions may be required before the dispersion integral will exist. But any further subtractions will have to be introduced artificially, i.e., by actually writing Low equations for different values of s, subtracting one from another, and then taking γ to infinity. By way of contrast, the first two subtractions seem to come naturally out of the structure of the seagull terms and the two off-shell mass variables.

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Difficulties in Spin Treatments in the Quark Model for High-Energy Scattering*

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The nonrelativistic quark-model treatment of hadron scattering is shown to be inconsistent at relativistic momentum transfers. The validity of predictions is examined. Difficulties arise in the specification of the spin orientations and couplings of spectator quarks whose states are "unchanged" in the scattering process, but which undergo Wigner rotations in the transformation between the rest frames of the initial and final states. The small relativistic "tail" of the mainly nonrelativistic bound-state wave function is also shown to play an important role in relativistic scattering processes. A consistent formulation with spin form factors is developed and applied to determine which of the nonrelativistic predictions are still valid at relativistic velocities, and which break down.

I. INTRODUCTION. WIGNER ROTATIONS AND RELATIVISTIC TAILS

IN the application of the quark model to high-energy scattering[1–3] the following two assumptions are generally used: (a) The relative motion of the quarks inside a hadron is nonrelativistic, and (b) the scattering amplitude is the sum of contributions in which only

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[1] E. M. Levin and L. L. Frankfurt, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu 2, 105 (1965) [English transl.: Soviet Physics—JETP Letters 2, 65 (1965)]; V. V. Anisovich, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu 2, 439 (1965) [English transl.: Soviet Physics—JETP Letters 2, 272 (1965)].

[2] H. J. Lipkin and F. Scheck, Phys. Rev. Letters 16, 71 (1966).

[3] J. J. J. Kokkedee and L. Van Hove, Nuovo Cimento 42A, 711 (1966).

one active quark in each hadron interacts and the remaining quarks are spectators whose states are "unchanged" in the scattering process. The purpose of this paper is to point out an inconsistency in these two assumptions and to examine its effect on quark-model predictions. This inconsistency leads to ambiguities and possible errors when the model is taken literally and applied to spin couplings of quarks in scattering processes with high momentum transfer. All predictions which are based on spin couplings[4–7] must be reconsidered in view of this inconsistency. The validity of experimental tests of the model based upon these predictions is thus highly questionable.

The source of the difficulty is the failure of the two assumptions above to provide a unique prescription for the treatment of the spins of the spectator quarks. The statement that the spin orientation of the spectator quarks is unchanged in the scattering process is not Lorentz-invariant. Lorentz invariance is relevant even in the nonrelativistic quark model in all applications at high energies where the hadrons are moving at relativistic velocities. The rest frames for a given hadron before and after the scattering process differ by a relativistic velocity if the momentum transfer is appreciable. The coupling of the quark spins, to make the total hadron spin, must be done in the rest frame of each hadron. Thus, calculations of spin couplings in any given scattering process use at least four different Lorentz frames, all differing by relativistic velocities. These are the rest frames of the initial and final states of both particles. In addition, there is also the c.m. frame. The calculation of the scattering amplitude can be done blindly in one frame, following the prescription of keeping the spin orientations of the spectator quarks unchanged in that frame. The results, however, then depend upon the particular frame chosen for the calculation. As there is no objective criterion for choosing any given frame, the results are ambiguous.

One can attempt to avoid this ambiguity by using a consistent relativistic impulse-approximation formalism. The matrix elements $\langle XY|T|\alpha\beta\rangle$ for a transition from initial states α, β to final states X, Y are assumed to be given by the sum of matrix elements of two-body operators at the quark level

$$\langle XY|T|\alpha\beta\rangle=\sum_{ijkl} f_{ijkl}\langle X|t_{kl}|\beta\rangle\langle Y|t_{ij}|\alpha\rangle, \quad \text{(1a)}$$

where t_{ij} is a single-quark operator describing a transition of a quark between states i and j, f_{ijkl} is the two-body quark scattering amplitude for a transition from the states j, l to the states i, k, and the indices i, j, k, and l are summed over all possible quark and antiquark states. Equation (1a) can be rewritten

$$\langle XY|T|\alpha\beta\rangle$$
$$=\sum_{ijkl} f_{ijkl}\langle X_0|U_X^{\dagger}t_{kl}U_\beta|\beta_0\rangle\langle Y_0|U_Y^{\dagger}t_{ij}U_\alpha|\alpha_0\rangle, \quad \text{(1b)}$$

where α_0, β_0, X_0, and Y_0 represent each state in its own rest frame and U_H describes the Lorentz transformation from the rest frame of hadron H to the Lorentz frame in which the calculation is performed.

The wave function for each hadron state is assumed to be nonrelativistic only in its own rest frame. The ambiguity in the specification of the change in spin orientation of the spectator quarks is thus avoided. All spin effects are given directly by the form factors[3,4] which are obtained from the matrix elements on the right-hand side of Eq. (1b). These involve overlap integrals between the initial- and final-state wave functions including the spins. The overlap integrals can be written in a Lorentz-invariant manner to give the same results in all Lorentz frames. The wave functions for the individual hadron states are each given in the rest frame for that state and are then boosted by appropriate Lorentz transformations to the common frame in which the scattering calculation is carried out. The results should then be independent of the frame in which the scattering is calculated.

In the nonrelativistic approximation the results are the same as those given by the naive prescription that the spin of the spectator quark does not change. Spin is decoupled from the spatial variables and is unaffected by the Galilean transformations (Lorentz transformations in the nonrelativistic limit) from the rest frames to the frames where the scattering is calculated. The spin contribution to the form factor is therefore trivial. The overlap integral simply requires the spin-wave functions of the spectator quarks to be the same in the initial and final states.

In the relativistic case there are two new effects: (1) Wigner rotations[8] and (2) contributions from relativistic tails in the bound-state wave function. Wigner rotations appear, for example, in describing the spin of the spectator quarks in a Lorentz frame where the bound state is not at rest. In whichever Lorentz frame the overlap integral is calculated there must always be one bound state moving with relativistic velocity, so so that a description of the spin of the constituent quarks is complicated by Wigner rotations. This is seen formally in Eq. (1b), where the transformations U_H introduce Wigner rotations of the spectator quarks.

A more troublesome difficulty arises from the dependence of the overlap integral on the detailed structure of the wave function. The form factor is dominated by those Fourier components of the bound-state wave function which correspond to momenta of the order of the momentum transfer.[3,4] Even though the bound-

[4] H. J. Lipkin, F. Scheck, and H. Stern, Phys. Rev. **152**, 1375 (1967).
[5] C. Itzykson and M. Jacob, Nuovo Cimento **48A**, 909 (1967).
[6] J. L. Friar and J. Trefil, Nuovo Cimento **49A**, 642 (1967).
[7] A. Białas and K. Zalewski, Nucl. Phys. **B6**, 449 (1968); **B6**, 465 (1968); **B6**, 478 (1968).

[8] See, for example, S. Gasiorowicz, *Elementary Particle Physics* (John Wiley & Sons, Inc., New York, 1966), p. 74.

state wave function may be nonrelativistic to a good approximation, it must have a relativistic "tail" where the relative velocities and relative momenta are high. *For scattering processes of high momentum transfer*, the *form factor is dominated by this relativistic tail.* If the tail is small, the form factor and the scattering amplitude are small at high momentum transfer, but whatever amplitude exists must arise from the relativistic tail. If there is no relativistic tail, the scattering amplitude is zero. Thus, in order to describe the scattering at high momentum transfers, the nonrelativistic description breaks down and the relativistic tail of the wave function becomes relevant.

The role of the relativistic tail of the wave function can be seen formally by examination of Eq. (1b). Let α be a meson and expand the wave function in momentum space. The operator t_{ij} does not act on the spectator quark, but the operators U_α and U_Y do act on the spectator and change each momentum component in its wave function by an amount Δq corresponding to a Lorentz transformation between the initial and final rest frames. The overlap integral over the spectator quark momentum is just the overlap between two momentum distributions which are displaced from one another by an amount Δq. It therefore vanishes if there are no components in the distribution with momenta greater than $\frac{1}{2}\Delta q$. If only 1% of the wave function is in the region above $\frac{1}{2}\Delta q$, the overlap integral is proportional to this 1% tail and is very sensitive to the properties of the tail.

A pion at 100 MeV is already relativistic. Thus, the above relativistic effects must be considered even at momentum transfers normally considered to be low, where the neglect of double scattering in the quark model seems to be reasonable.

In the remainder of this paper we examine the effect of these relativistic considerations on predictions obtained from the nonrelativistic quark model to determine which are independent of these relativistic difficulties and are Lorentz-invariant and which are not valid and should not be used at high momentum transfer. We use the W-spin formalism[9] and take our axis of quantization as the x axis, chosen to be normal to the scattering plane. We describe all quark spins by using W spin rather than ordinary spin.

For the benefit of readers unfamiliar with the W-spin formalism, the following review of properties of W_x should be sufficient for an understanding of this paper. The definition of W_x for a single Dirac particle with momentum in the scattering plane is

$$W_x \equiv \tfrac{1}{2}\beta\sigma_x, \quad \text{for a Dirac particle with } p_x=0.$$

For a quark or antiquark at rest,

$$W_x \equiv \pm\tfrac{1}{2}\sigma_x \quad (+ \text{ for a quark at rest, } - \text{ for an antiquark at rest}).$$

For a hadron the total W_x is defined as the sum of the values of W_x for the constituent quarks

$$W_x \equiv \sum_i W_{xi} \text{ for a composite system.}$$

The operator W_x is defined to make it invariant under all Lorentz transformations in the scattering plane and also under reflections in the plane. Thus, we can define a basis of eigenfunctions of W_x which is the same in all relevant Lorentz frames for the scattering process, including the c.m. system and the rest frames for all initial and final states. If an arbitrary state is expanded in this basis, only the relative phases of the expansion coefficients change under Lorentz transformations in the scattering plane. The magnitudes are invariant. This property holds only for the component of W spin normal to the scattering plane, as other directions for W spin are not invariant under the relevant Lorentz transformation.

It is shown below that the invariance of the scattering amplitude and of W_x under reflections in the scattering plane leads to the result that the eigenvalue of W_x for a spectator quark does not change in the scattering process and the allowed changes in W_x for active quarks are considerably restricted.

The above properties of W_x explain why the W_x basis is the most convenient for treating the scattering of composite particles in a model where only a few particles are active and the remainder are spectators. The spin couplings for each hadron are defined in the rest system of the hadrons. These are easily expressed in the W_x basis in the hadron rest system by expanding the wave functions in a σ_x basis and reversing the sign when necessary for the antiquarks. No further transformations are necessary, and the spin changes in the scattering process are very simply described.

The W_x basis is clearly superior to the helicity basis where active and spectator quarks behave in different complicated ways in the scattering process and many transformations and couplings and recouplings of spins of active and spectator quarks would be necessary. In particular, the helicity states of the spectator quarks undergo complicated transformations from the initial to the final states which depend upon the Lorentz frame and on the scattering angle, while spectator quarks in the eigenstates of the W_x basis are not changed at all.

In our treatment we use a nonrelativistic wave function for a hadron in its own rest frame. We assume that when a hadron is in an eigenstate of W_x in the nonrelativistic approximation, the same eigenvalue holds for the relevant portion of the relativistic tail, namely, the components with high momentum in the scattering

[9] H. J. Lipkin and S. Meshkov, Phys. Rev. **143**, 1269 (1966); H. Harari, D. Horn, M. Kugler, H. J. Lipkin, and S. Meshkov, *ibid.* **146**, 1052 (1966); H. J. Lipkin, *ibid.* **159**, 1303 (1967).

plane. This seems reasonable, since such components can be transformed to rest by a Lorentz transformation which leaves W_x invariant.

II. RELATIVISTIC GENERALIZATION OF NONRELATIVISTIC SELECTION RULE

We first use the properties of W_x to place restrictions on the spin change of a spectator quark and to rederive a selection rule previously obtained nonrelativistically for hadron-hadron scattering in the quark model. The present derivation is valid even when the hadron is moving with relativistic velocities. The eigenvalues of W_x for a spectator quark cannot change in the scattering process. This can be verified by considering a transition in which the spectator quark has opposite eigenvalues of W_x in the initial and final states. The two states are orthogonal and remain orthogonal under any Lorentz transformation in the scattering plane. Thus, the overlap integral and form factor must vanish for such states, and we obtain the selection rule

$$\Delta W_x = 0 \quad \text{for all spectator quarks.} \tag{2a}$$

In quark-quark scattering, the change in W_x for each quark can only be 0 or ± 1. From angular momentum and parity conservation W_x is conserved[10] modulo 2. We thus find the following selection rule for the scattering of hadron A on hadron B in the quark model

$$|\Delta W_x{}^A| = |\Delta W_x{}^B| \leqq 1, \tag{2b}$$

where $\Delta W_x{}^A$ and $\Delta W_x{}^B$ are the changes in W_x for the active quarks in hadrons A and B, respectively. Since there is no change in W_x for the spectator quarks, $W_x{}^A$ and $W_x{}^B$ are also the *total* change in W_x for *hadrons* A and B.

The selection rule (2b) is conveniently expressed in terms of the ordinary spin **S** for each hadron state as measured in its rest frame. For a quark at rest $W_x = S_x$, while for an antiquark at rest $W_x = -S_x$. Thus, for either a quark or an antiquark transition

$$|\Delta S_x{}^A| = |\Delta W_x{}^A| = |\Delta W_x{}^B| = |\Delta S_x{}^B| \leqq 1, \tag{2c}$$

where ΔS_x is the change in S_x between the initial and final states measured in the rest frame of each state. This selection rule has also been obtained in the nonrelativistic description.[11] Many predictions which have been made with the nonrelativistic quark model can be obtained from the selection rule (2) without further assumptions. These predictions are therefore valid for relativistic velocities without any assumptions about the form factors of the spectator quarks.

For example, consider the class of reactions

$$P + B \rightarrow P + B^*, \tag{3}$$

where P denotes pseudoscalar meson, B denotes a spin-$\frac{1}{2}$ baryon, and B^* denotes a spin-$\frac{3}{2}$ baryon. Since $\Delta S_x = 0$ for the meson, the same is true for the baryon and therefore the final states having $S_x = \pm\frac{3}{2}$ are forbidden.[11]

In the nonrelativistic description the additional prediction is made that the states with $S_x = \pm\frac{1}{2}$ are equally populated in the final state if the initial baryon state is unpolarized.[5] This follows because the operator describing the baryon transition is a single-quark operator and connects states of spin $\frac{1}{2}$ and spin $\frac{3}{2}$. It must therefore transform like a vector in quark spin space. Thus, the transition matrix elements for all polarization states of the baryons are related by the Clebsch-Gordan coefficients for coupling spins of $\frac{1}{2}$ and 1 to make spin $\frac{3}{2}$. In the relativistic description this argument no longer holds. The transition operator is no longer a single-quark operator but depends also on the spins of the spectator quarks. The form factors could be different for different W_x eigenvalues of the spectator quarks. This could give rise to a component transforming like a tensor in quark spin space and predict a polarization of the final baryon state. Experimental tests of such polarizations would be of interest.

III. ALLOWED TWO-BODY QUARK AMPLITUDES AND SYMMETRY RELATIONS

We now consider in detail the calculation of predictions obtainable from the quark model with minimum assumptions regarding the spatial wave functions of the hadron states. We choose our basis for spin functions such that all hadron states considered are eigenfunctions of W_x. We then expand each hadron state in a basis where each individual quark is in an eigenstate of W_x. We use the quark model to describe the scattering in this basis and obtain the result for hadron-hadron scattering by summing all the contributing terms. Each term is the product of a two-body quark scattering amplitude and form factors for each hadron.

We now consider all the allowed quark-quark scattering amplitudes. Since there are four quark states, two initial and two final, and two spin states for each, there are a total of 16 possible spin configurations. The conservation of W_x modulo 2 reduces the number to eight allowed spin configurations. Using the notation of Białas,[7] we define these eight independent amplitudes as follows. There are four "nonflip" amplitudes, $\Delta W_x = 0$,

$$\begin{aligned} f_1 &= \langle ++|++\rangle, & f_2 &= \langle --|--\rangle, \\ f_3 &= \langle +-|+-\rangle, & f_4 &= \langle -+|-+\rangle, \end{aligned} \tag{4a}$$

and four "flip" amplitudes, $|\Delta W_x| = 1$,

$$\begin{aligned} f_5 &= \langle +-|-+\rangle, & f_6 &= \langle -+|+-\rangle, \\ f_7 &= \langle --|++\rangle, & f_8 &= \langle ++|--\rangle. \end{aligned} \tag{4b}$$

If time-reversal invariance is assumed, these eight

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[10] H. J. Lipkin, in *Proceedings of the Third Coral Gables Conference on Symmetry Principles at High Energy*, edited by Kunsunoglu, A. Perlmutter, and I. Sakmar (W. H. Freeman and Co., San Francisco, 1966), p. 97; D. Horn, Phys. Rev. **150**, 1218 (1966).

[11] H. J. Lipkin, Nucl. Phys. **B1**, 597 (1967).

amplitudes are not all independent. From the condition

$$\langle ab|cd\rangle = \langle cd|ab\rangle, \tag{5}$$

we obtain the relations

$$f_5 = f_6, \tag{6a}$$

$$f_7 = f_8. \tag{6b}$$

The number of independent amplitudes is now reduced to six, which is the correct number for the description of the scattering of two different spin-$\frac{1}{2}$ particles. For the case where the two particles are identical or where there exists a symmetry transformation which interchanges the internal degrees of freedom of the particle (e.g., isospin in nucleon-nucleon or nonstrange quark-quark scattering), time-reversal invariance also gives the relation

$$\langle ab|cd\rangle = \langle ba|dc\rangle. \tag{7}$$

This gives one additional relation between the amplitudes,

$$f_3 = f_4, \tag{8}$$

and reduces the number of independent amplitudes to five, as in the case of nucleon-nucleon scattering.[12] In the quark model the use of the additional relation (8) is allowed only for those quark states which are related by a symmetry transformation. If $SU(3)$ symmetry is assumed in addition to charge-conjugation invariance, any of the six quark and antiquark states can be transformed into one another and the condition (8) holds in general. Under the weaker and more realistic assumptions only of isospin symmetry and charge-conjugation invariance, the condition (8) holds for all amplitudes involving either only nonstrange quarks and antiquarks or only strange quarks and antiquarks but does not hold for amplitudes involving both strange and nonstrange objects.

IV. SPIN FORM FACTORS

In calculating the hadron-hadron scattering amplitude the relevant quark-quark scattering amplitude must be multiplied by a form factor which describes the change in state of the spectator quarks. For the relativistic case there is a spin form factor in addition to the spatial form factor. If all spectator quarks are eigenfunctions of W_x, the spin form factor depends upon the eigenvalue of W_x for the spectator quark and the relative velocity of the rest frames of the initial and final states. We must now consider possible changes in the form factor between different quark states having the same eigenvalue of W_x and the same given kinematic variables. It is reasonable to neglect this variation in the spin form factor between different meson states and between different baryon states for the following reasons.

(1) The quarks which are constituents *of the same hadron* should have the same form factor because of the separation of the spatial degrees of freedom from the spin and $SU(3)$ degrees of freedom implicitly assumed in the quark-model wave functions. Thus, each quark in a given hadron has the same spatial wave function and therefore the same form factor.

(2) The radial wave functions may very well be different in different baryon states, e.g., octet and decuplet, strange and nonstrange, and also in different meson states, e.g., vector and pseudoscalar, strange and nonstrange. However, these departures from $SU(6)$ symmetry in the wave functions also give rise to differences in the conventional form factors which take into account the momentum transfer but neglect spin effects. In the approximation where the differences in the conventional form factors are neglected, it is also reasonable to neglect these differences in the spin form factors.

The difference between the spin form factors of meson and baryon states cannot be neglected since there are two spectator quarks in a baryon and only one in a meson.

We now calculate explicitly the effect of the spin form factors on the expression for the scattering amplitude. The spin form factor for a given spectator quark s depends on the eigenvalue W_{xs} for the quark, on the kinematic parameters of the scattering process, and on the particular hadron in which the quark is found. We do not consider the dependence on the kinematic parameters, since the quark-model predictions generally relate different processes at the same values of energy and momentum transfer. We assume that the spin form factor for a quark depends upon whether it is bound in a meson or in a baryon, but is otherwise independent of the particular bound state. We denote the spin form factor for a spectator quark s bound in a hadron H by

$$\phi_s^H(W_{xs}).$$

The total spin form factor for hadron H is the product of the form factors for all the spectator quarks:

$$\phi^H(W_x^H - W_{xi}^H) = \prod_s \phi_s^H(W_{xs}), \tag{9a}$$

where W_x^H is the eigenvalue of W_x for the hadron, W_{xi}^H is the eigenvalue of W_x for the active quark, and the product with the index s is the product of spectator-quark form factors. The form factor depends only on the spins of the spectator quarks. Since these are determined by the difference $W_x^H - W_{xi}^H$, we write the spin form factor as a function of this difference. The expression (9a) can be rewritten

$$\begin{aligned}\phi^H(W_x^H - W_{xi}^H) &= \prod_q \phi_q^H(W_{xq})[\phi_i^H(W_{xi}^H)]^{-1}\\ &= g^H(W_x^H)[\phi_i^H(W_{xi}^H)]^{-1}, \end{aligned}\tag{9b}$$

where the product with the index q is a product of spin

[12] L. Wolfenstein and J. Ashkin, Phys. Rev. **85**, 947 (1952); R. H. Dalitz, Proc. Phys. Soc. (London) **A65**, 175 (1952).

form factors for all the quarks in the hadron and

$$g^H(W_x{}^H)=\prod_q \phi_q{}^H(W_{xq}). \quad (9c)$$

For a spin-flip transition, the eigenvalues of W_x and W_{xi} are different in the initial and final states. However, the expressions (9b) and (9c) are valid if the values for either the initial state or the final state are used, as long as the same state is chosen for both W_x and W_{xi}. For convenience we choose the eigenvalues of W_x and W_{xi} for the *initial* state. From Eq. (9b) the spin form factor can be written as the product of (a) a factor which depends only on the total W_x for the hadron and is independent of the individual quark spins and (b) a factor which depends only on the eigenvalue of W_x for the active quark and is independent of the W spin of the spectator quarks or of the hadron.

The scattering amplitude for a reaction from initial hadron states α and β is the sum of terms each involving one of the two-body quark scattering amplitudes f_i [Eq. (4)] multiplied by two spin form factors of the form (9), one for hadron α and one for hadron β. A typical term can then be written, using Eq. (9b),

$$f_i\phi^\alpha(W_x{}^\alpha-W_{xi}{}^\alpha)\phi^\beta(W_x{}^\beta-W_{xi}{}^\beta) = f_i{}'^{\alpha\beta}g^\alpha(W_x{}^\alpha)g^\beta(W_x{}^\beta), \quad (10a)$$

where

$$f_i{}'^{\alpha\beta}=f_i[\phi_i{}^\alpha(W_{xi}{}^\alpha)]^{-1}[\phi_i{}^\beta(W_{xi}{}^\beta)]^{-1}. \quad (10b)$$

For each of the scattering amplitudes f_i of Eq. (4), the eigenvalues W_{xi} for all active quark states are uniquely specified. Thus, the modified scattering amplitude $f_i{}'^{\alpha\beta}$ [Eq. (10b)] depends upon whether hadrons α and β are mesons or baryons, but is otherwise independent of hadron states.

V. RELATIONS BETWEEN RELATIVISTIC AND NONRELATIVISTIC PREDICTIONS

The scattering amplitude from the initial states α, β to final states X, Y which are all eigenfunctions of W_x is obtained by taking the corresponding nonrelativistic expression and replacing each two-body amplitude f_i by the corresponding expression (10a) which includes the spin form factors. Thus,

$$\langle XY|T|\alpha\beta\rangle=\langle XY|T_{NR}'|\alpha\beta\rangle g^\alpha(W_x{}^\alpha)g^\beta(W_x{}^\beta), \quad (11)$$

where T_{NR}' is related to the nonrelativistic expression for the scattering amplitude T_{NR} as follows: Each quark scattering amplitude f_i in T_{NR} is replaced by the corresponding modified amplitude $f_i{}'^{\alpha\beta}$ given by Eq. (10b). The relativistic correction thus consists of two parts: (1) the replacement of each f_i by the corresponding f_i', and (2) the additional factors g^α and g^β on the right-hand side of Eq. (11). We examine the consequences of each correction separately.

(1) In the nonrelativistic treatment of scattering in the quark model the eight amplitudes f_i are all unknown independent parameters, as long as the relations (6) and (8) which follow from time-reversal invariance are not used. Relations between observable scattering processes are obtained by eliminating these eight unknown parameters. The replacement in the relativistic treatment of these parameters f_i by a new set of eight independent unknown parameters f_i' does not change these predictions, as long as the dependence of $f_i{}'^{\alpha\beta}$ on α and β is not significant. We have assumed here that $f_i{}'^{\alpha\beta}$ has the same value for all meson-baryon scattering and for all baryon-baryon scattering but may be different in the two cases.

The relations (6) and (8) which follow from time-reversal invariance can no longer be used since they relate the amplitudes f_i and not the modified amplitudes f_i'. Unless the spin form factors on the right-hand side of Eq. (10b) are known, the relations (6) and (8) do not help in obtaining new predictions.

(2) The second type of relativistic correction involves the multiplication by factors which depend upon the eigenvalues $W_x{}^\alpha$ and $W_x{}^\beta$ for the two hadrons in the initial state. This will not effect predictions relating processes which all have the same eigenvalues of W_x for the particles in the initial state. Nonrelativistic predictions for baryon-baryon processes remain valid if they relate reactions which all have the same single initial polarization state with respect to an axis transverse to the scattering plane. For meson-baryon scattering the correction is more complicated because a meson state is not in an eigenstate of W_x. This can be seen explicitly as follows.

For a system of quarks and antiquarks

$$W_x=S_{qx}-S_{\bar{q}x}, \quad (12)$$

where S_q and $S_{\bar{q}}$ are the total spins of the quarks and antiquarks, respectively.[9] The pseudoscalar- and vector-meson states with $S_x=0$ are

$$|P\rangle=\tfrac{1}{2}\sqrt{2}(|q_+\bar{q}_-\rangle+|q_-\bar{q}_+\rangle), \quad (13a)$$

$$|V\rangle=\tfrac{1}{2}\sqrt{2}(|q_+\bar{q}_-\rangle-|q_-\bar{q}_+\rangle), \quad (13b)$$

where the subscript $\pm$ denotes the eigenvalue $S_x=\pm\frac{1}{2}$. The first term on the right-hand side of (13a) has $W_x=+1$; the second term has $W_x=-1$, and similarly for (13b). Thus, neither state is an eigenfunction of W_x. The eigenstates of W_x are

$$|q_+\bar{q}_-\rangle=\tfrac{1}{2}\sqrt{2}(|P\rangle+|V\rangle), \quad W_x=+1 \quad (14a)$$

$$|q_-\bar{q}_+\rangle=\tfrac{1}{2}\sqrt{2}(|P\rangle-|V\rangle), \quad W_x=-1. \quad (14b)$$

Substituting these states for α in Eq. (11), we obtain

$$\langle XY|T|\tfrac{1}{2}\sqrt{2}(|P\rangle+|V\rangle),\beta\rangle =\langle XY|T_{NR}'|\tfrac{1}{2}\sqrt{2}(|P\rangle+|V\rangle),\beta\rangle \times g^M(+1)g^\beta(W_x{}^\beta), \quad (15a)$$

$$\langle XY|T|\tfrac{1}{2}\sqrt{2}(|P\rangle-|V\rangle),\beta\rangle =\langle XY|T_{NR}'|\tfrac{1}{2}\sqrt{2}(|P\rangle-|V\rangle),\beta\rangle \times g^M(-1)g^\beta(W_x{}^\beta), \quad (15b)$$

where the subscript M denotes meson. Combining Eqs. (15a) and (15b), we obtain

$$\langle XY|T|P\beta\rangle = \langle XY|T_{NR}'|P\beta\rangle[g^M(+1)+g^M(-1)]\tfrac{1}{2}g^\beta(W_x{}^\beta) + \langle XY|T_{NR}'|V\beta\rangle[g^M(+1)-g^M(-1)] \times\tfrac{1}{2}g^\beta(W_x{}^\beta). \quad (16)$$

Thus, the scattering amplitude for a process with a pseudoscalar meson in the initial state is given by a linear combination of the modified nonrelativistic amplitudes T_{NR}' for the corresponding scattering processes with *both pseudoscalar- and vector-meson* initial states. This effect arises because the spin form factors change the relative phase of the two components of the quark-antiquark wave function and thus mix the singlet and triplet spin states.

VI. CONCLUSIONS

We thus arrive at the following conclusions regarding the validity of predictions obtained from the nonrelativistic model.

(A) The following types of predictions obtained from the nonrelativistic model are unaffected by the spin form factors and are valid in a relativistic treatment: (i) relations which follow from the selection rule (2); (ii) relations between baryon-baryon scattering processes which all have the same initial polarization state with respect to an axis normal to the scattering plane; and (iii) relations between meson-baryon processes which all have the same initial polarization state with respect to an axis normal to the scattering plane, and which also hold when the pseudoscalar meson in the initial state is replaced by a vector-meson state with $S_x=0$.

(B) The following relations are affected by the spin form factors and *should not be expected to hold for relativistic momentum transfers*: (i) relations between meson-baryon and baryon-baryon scattering; (ii) relations which use the relations (6) and (8) obtained from time-reversal invariance between the two-body scattering amplitudes; and (iii) relations between cross sections for unpolarized beams or unpolarized targets which are obtained by averaging and which do not hold for each individual polarization state.

For forward scattering processes, both elastic and inelastic, the nonrelativistic results can be cast into a relativistic form by using the full W-spin group to specify the states. The invariance of W spin under collinear Lorentz transformations ensures the validity of the prescription that the W spin of the spectator quarks is unchanged. Thus, there are no inconsistencies in nonrelativistic calculations of total cross sections from the forward amplitude, or in calculations of three-point functions which are always collinear in a suitable frame. At small momentum transfers the nonrelativistic approximation may still be valid. It would be of interest to check the predictions of type B above which are expected to fail in the relativistic region to see if they agree with experiment at low momentum transfers where they can be expected to be valid and fail at higher momentum transfers where they can be expected to break down. They can also be compared with the type-A predictions which still are expected to hold in the relativistic range and should be good at higher momentum transfers.

It is also possible to use models or to assume an ansatz to obtain explicit expressions for the spin form factors. One possible ansatz is that the spin form factor for a given quark varies only by a phase factor with a change in the eigenvalue of W_x. This is consistent with Lorentz invariance since Lorentz transformations in the scattering plane can only produce Wigner rotations about the x axis. Such rotations change the spin state of a spectator quark only by a phase factor if it is initially in an eigenstate of W_x. For this case all those predictions from the nonrelativistic case are valid which (a) do not use the relations (6) and (8) which follow from time-reversal invariance, and (b) do not relate the relative phases of amplitudes having different eigenvalues of W_x. All relations between baryon-baryon scattering processes with unpolarized initial states are therefore valid. The meson-baryon relations still have the complications of the mixing of pseudoscalar- and vector-meson states. With this ansatz it is also possible to make use the relations (6) and (8) which follow from time-reversal invariance in a weaker form. An equality between two of the amplitudes f_i implies that the corresponding modified amplitudes f_i' differ only by a phase factor and have the same absolute magnitude. This condition may lead to useful relations.

It is unfortunate that the relativistic corrections appear to invalidate the relations between meson-baryon and baryon-baryon reactions. These are just the predictions which might otherwise provide crucial tests of the quark description as they are not easily obtainable from other models. Significant tests may still be possible at small momentum transfers, where the nonrelativistic approximation is valid.

The predictions of simple branching ratios[13,14] and relations for neutral meson production[15] are all of type A(iii) and are still valid. These include the ρ^0/ω and Λ/Σ^0 branching-ratio predictions in strangeness-exchange reactions. However, these are also obtained from other models.[13]

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