Resolution of the $T=0$ Fermi-Yang Ambiguity in K-Nucleon Scattering*

A. K. RAY,† R. W. BURRIS, H. E. FISK,‡ AND R. W. KRAEMER
Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213

AND

D. G. HILL AND M. SAKITT
Brookhaven National Laboratory, Upton, New York 11973

(Received 27 February 1969)

From an exposure of the BNL 30-in. deuterium-filled bubble chamber to a 600-MeV/c K^+ beam, the angular distribution (14 912 events) and polarization (853 double-scattered events) for the charge-exchange reaction have been obtained. The results of partial-wave analyses of the data indicate the dominance of a positive $P_{\frac{1}{2}}$ phase shift, and consequently establish a $T=0$ Yang-type solution.

PREVIOUS research on K^++p and K^++d interactions, at momenta between 230 and 812 MeV/c, has led to the following results[1–3]: K^++p elastic scattering can be described by a Columb amplitude and a repulsive S-wave phase shift whose magnitude increases with increasing beam momentum; charge-exchange and non-charge-exchange K^++d scattering require S, P, and possibly D waves at these momenta; and there exists, for the $T=0$ phase shifts, a Fermi-Yang ambiguity. The predicted polarizations for the Fermi-Yang ambiguous solutions are large and of opposite sign. Hence, a polarization measurement will decide between the two.

The angular distribution data come from 14 912 examples of the charge-exchange reaction

$$K^++d\rightarrow K^0+p+p_{\mathrm{sp}}, \qquad (1)$$

which occurred, with subsequent decay of the K^0 into π^+ and π^-, in the BNL 30-in. deuterium-filled bubble chamber, exposed to a 600-MeV/c separated K^+ beam at the Brookhaven AGS.

Our analysis assumes the simple impulse approximation. In reaction (1), the impulse spectator p_{sp} is defined as the proton in the final state with the smaller momentum. Events are accepted in which the spectator momentum is less than 300 MeV/c. This cut results in a 5% loss of events when a loss of $\leq 2\%$ is expected. A comparison of the momentum distribution for events below 300 MeV/c and the prediction from the Hulthén wave function gives a χ^2 of 32.5 for 19 degrees of freedom.

The center-of-mass scattering angle θ_1^* has been calculated assuming (a) an initially stationary neutron and (b) a neutron of initial momentum equal and opposite to the spectator proton. The angular distribution, normalized to a charge-exchange cross section of 6.6 ± 0.6 mb, is shown in Fig. 1(a).[4] The data obtained according to (a) and (b) are shown as dashed and solid lines, respectively. Phase shifts obtained in separately fitting either of these data sets are consistent within errors.

The polarization data are obtained from 853 selected events in which the nonspectator proton from (1) scatters via one of the following reactions:

$$p+d\rightarrow p+d, \quad 377 \text{ events}, \qquad (2)$$

$$p+d\rightarrow p+n+p_{\mathrm{sp}}, \quad 241 \text{ events}, \qquad (3)$$

$$p+d\rightarrow p+p+n_{\mathrm{sp}}, \quad 144 \text{ events}, \qquad (4)$$

$$p+d\rightarrow p+n+p. \quad 91 \text{ events}. \qquad (5)$$

In reactions (3) and (4) the spectator nucleon must have a momentum which is less than 250 MeV/c and also less than 2 standard deviations smaller than either of the momenta of the other final-state nucleons. If these two conditions are not satisfied, the event is classified as an example of reaction (5). To ensure reasonable analyzing power, to eliminate possible bias, and to reduce kinematical overlap in the above reactions, the following criteria have been imposed in obtaining the 853 events:

(i) The laboratory kinetic energy E_2 of the proton before scattering must be ≥ 30 MeV and the nucleon-nucleon center-of-mass scattering angle θ_2^* was required to be $\leq 90°$.

(ii) The proton must scatter at least 10° in the laboratory system.

(iii) The distance from the charge-exchange vertex to the second vertex must be at least 1 cm.

(iv) The length of the scattered proton after the second vertex must be greater than 5 cm if it does not stop in the chamber.

* Work supported in part by the U. S. Atomic Energy Commission.

† Present address: Department of Physics, Visva-Bharati, Santiniketan, West Bengal, India.

‡ On leave; present address: Rutherford High Energy Laboratory, Chilton, Didcot, Berkshire, England.

[1] S. Goldhaber, W. Chinowsky, G. Goldhaber, W. Lee, T. O'Halloran, T. F. Stubbs, G. M. Pjerrou, D. H. Stork and H. K. Ticho, Phys. Rev. Letters **9**, 135 (1962).

[2] W. Slater, D. H. Stork, H. K. Ticho, W. Lee, W. Chinowsky, G. Goldhaber, S. Goldhaber, and T. O'Halloran, Phys. Rev. Letters **7**, 378 (1961).

[3] V. J. Stenger, W. E. Slater, D. H. Stork, H. K. Ticho, G. Goldhaber, and S. Goldhaber, Phys. Rev. **134**, B1111 (1964).

[4] A charge-exchange cross section of 6.6 ± 0.6 mb has been inferred from the data in Ref. 2.

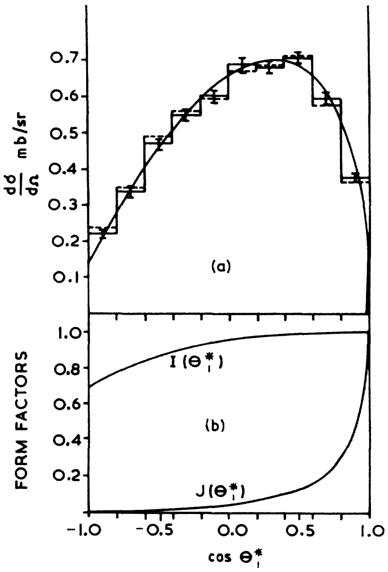

FIG. 1. (a) Angular distribution for reaction (1). The solid curve is solution 12 in Table I. (b) Impulse approximation form factors $I(\theta_1^*)$ and $J(\theta_1^*)$. See text and Ref. 8.

If $\hat{n}_1 = \mathbf{K}^+ \times \mathbf{p}_1 / |\mathbf{K}^+ \times \mathbf{p}_1|$ and $\hat{n}_2 = \mathbf{p}_1 \times \mathbf{p}_2 / |\mathbf{p}_1 \times \mathbf{p}_2|$, where $\mathbf{K}^+$, $\mathbf{p}_1$, and $\mathbf{p}_2$ are momentum vectors for the K^+, the nonspectator proton, and the proton after scattering, the intensity of scattered protons is

$$I_2(\theta_2^*) = I_0(\theta_2^*)[1 + P_1(\theta_1^*) A(\theta_2^*, E_2) \cos\chi]. \quad (6)$$

$I_0(\theta_2^*)$ is the differential cross section for an unpolarized beam of protons. $P_1(\theta_1^*)$ is the proton polarization acquired in the charge-exchange scattering and $\cos\chi = \hat{n}_1 \cdot \hat{n}_2$. $A(\theta_2^*, E_2)$ is the analyzing power for reactions (2)–(5). In reactions (3) and (4), where an event is classified as a pn or pp scattering, the analyzing power appropriate to that classification is assigned to the event. If the event is not classified uniquely as a quasifree pn or pp scattering [reaction (5)] or if it is an elastic pd scattering [reaction (2)], the analyzing power is taken to be an appropriately weighted average of the pn and pp analyzing power.[5] The pn and pp analyzing power used comes from a parametrization of the tables compiled by Wilson.[6] The average product of the analyzing power and the polarization for our sample is -0.20 ± 0.05.[7]

The polarization and its error shown in Fig. 2 are determined by the maximum-likelihood method, where Eq. (6) is used in constructing the likelihood function. The polarization for $\cos\theta_1^*$ greater than 0.7 is not measured, since this angular region contains most of the events rejected by criterion (i).

The angular distribution and polarization were fitted with

$$\frac{d\sigma}{d\Omega_1^*} = \frac{1}{4K^2}\{[|f_1 - f_0|^2 + \tfrac{2}{3}|g_1 - g_0|^2][I(\theta_1^*) - J(\theta_1^*)] + \tfrac{1}{3}|g_1 - g_0|^2[I(\theta_1^*) + J(\theta_1^*)]\},$$

and

$$\mathbf{P}(\theta_1^*) = -\hat{n}_1 \frac{2\,\mathrm{Im}(f_1 - f_0)(g_1^* - g_0^*)}{|f_1 - f_0|^2 + |g_1 - g_0|^2},$$

where f_T and g_T are the spin-nonflip and spin-flip amplitudes for isotopic spin T. The form factors $I(\theta_1^*)$ and $J(\theta_1^*)$, shown in Fig. 1(b) result from the impulse approximation and the Pauli principle for the two final-state protons.[8]

The partial-wave analysis has been done in two independent ways. The first method, S, is the standard one in which the f_T and g_T are expressed in terms of a fixed number of phase shifts, while the second method, which has been developed by Cutkosky and Deo (CD), takes into account long-range forces and certain analytic properties of the scattering amplitude and consequently generates from a small number of low-angular-momentum amplitudes higher-angular-momentum phase shifts.[9] The utility of the CD method has been demonstrated in a recent analysis of K^+p scattering from 345 to 1450 MeV/c.[10]

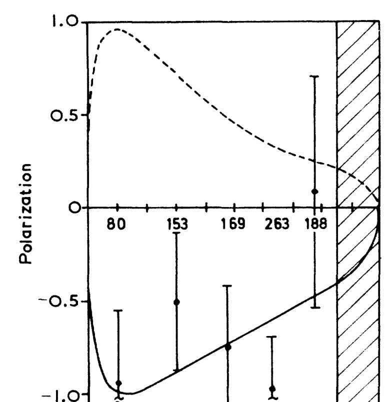

FIG. 2. Polarization for reaction (1). The number of events used in determining the polarization is indicated above each error bar. The dashed curve is the predicted polarization for the $T=0$ Fermi solution as determined from the angular distribution. The solid curve is the polarization given by solution 12 in Table I.

[5] The averaging process is consistent in our angular region with the polarization measurements in pd elastic scattering at 146 MeV by H. Postma and R. Wilson, Phys. Rev. **121**, 1229 (1961).

[6] R. Wilson, *The Nucleon-Nucleon Interaction* (Wiley-Interscience, New York, 1963).

[7] Positive proton polarization is taken to be in the direction of $\hat{n}_1$ and positive analyzing power implies an excess of protons with positive polarization scattered to the left of $\mathbf{p}_1$ in the plane whose normal is $\hat{n}_2$.

[8] The definition and a discussion of $I(\theta_1^*)$ and $J(\theta_1^*)$ are given in Ref. 3 and in J. M. Lévy-Leblond and M. Gourdin, Nuovo Cimento **23**, 1163 (1962); M. Gourdin and A. Martin, *ibid.* **11**, 670 (1959).

[9] R. E. Cutkosky and B. B. Deo, Phys. Rev. Letters **20**, 1272 (1968).

[10] R. E. Cutkosky and B. B. Deo (to be published).

TABLE I. Phase-shift solutions at 600 MeV/c.

Solution	Method	$T=1$ input	χ^2	χ^2 (pol)[a]	No. of parameters	No. of degrees of freedom	σ_{CE}[b] (mb)	σ_{NCE} (mb)	Phase shifts $\delta_{LJ}{}^I$ (deg) $\delta_{S_{1/2}}{}^0$	$\delta_{P_{1/2}}{}^0$	$\delta_{P_{3/2}}{}^0$	$\delta_{D_{3/2}}{}^0$	$\delta_{D_{5/2}}{}^0$
1	S	I	12.7	3.6	5	10	6.6	21.4	10.7 ± 3.4	31.0 ± 6.8	3.8 ± 2.2	-11.1 ± 4.6	1.0 ± 1.2
2	S	I	15.0	3.5	5	10	6.6	21.1	13.2 ± 6.6	31.8 ± 6.4	-2.6 ± 2.4	-2.3 ± 2.4	-0.9 ± 1.0
3	S	I	14.2	3.4	4	11	6.6	21.4	10.4 ± 2.0	33.4 ± 2.7	1.9 ± 1.0	-7.4 ± 1.3	
4	CD	I	16.2	3.6	5	10	6.7	18.4	14.4 ± 7.2	29.2 ± 6.3	-1.2 ± 2.4	-5.1 ± 2.2	1.0 ± 0.7
5	CD	I	26.3	3.4	4	11	7.1	19.3	10.2 ± 4.0	31.0 ± 4.8	3.1 ± 1.8	-12.2 ± 3.3	(1.2)[c]
6	CD	I	29.5	8.4	4	11	6.8	17.4	21.0 ± 6.9	20.1 ± 7.1	-0.8 ± 1.4	-7.0 ± 1.1	(0.7)
7	S	IIF	20.5	7.3	3	12	6.5	17.8	14.7 ± 5.6	21.0 ± 4.6	-2.3 ± 1.0		
8	S	IIF	16.2	3.4	3	12	6.6	21.0	1.2 ± 5.5	36.6 ± 3.6	0.3 ± 1.2		
9	CD	IIF	26.3	4.7	3	12	7.0	15.8	10.9 ± 5.5	26.3 ± 3.7	-2.5 ± 0.7	(0.2)	(0.5)
10	S	IIY	14.2	3.4	4	11	6.6	18.2	12.6 ± 4.0	39.1 ± 5.0	-0.6 ± 2.0	-7.2 ± 2.4	
11	CD	IIY	25.0	3.2	4	11	7.1	18.1	12.3 ± 4.6	37.2 ± 4.9	0.1 ± 1.8	-11.7 ± 3.5	(0.8)
12	S	Variable	16.3	3.5	6	11	6.5	21.5	6.0 ± 7.6	30.3 ± 6.4	-2.7 ± 3.2		
									$\delta_{S_{1/2}}{}^1$	$\delta_{P_{1/2}}{}^1$	$\delta_{P_{3/2}}{}^1$		
									-32.7 ± 3.0	-8.6 ± 3.0	4.2 ± 3.0		

[a] χ^2(pol) is the contribution to χ^2 from fitting the polarization data given in Fig. 2.
[b] σ_{CE} and σ_{NCE} refer to the K^+d charge-exchange and non-charge-exchange cross sections as calculated.
[c] Numbers in parentheses are the higher-angular-momentum phase shifts generated in the CD analysis.

We use as input to our analysis of charge-exchange scattering one of two sets of $T=1$ phase shifts. The first, set I, is $\delta_{S_{1/2}}=-34^0\pm2^0$, $\delta_{P_{1/2}}=0$, and $\delta_{P_{3/2}}=0$, as extrapolated from Goldhaber *et al.*[1] The second, set II, is $\delta_{S_{1/2}}=-30^0\pm2^0$, $\delta_{P_{1/2}}=-11^0\pm2^0$ or $4^0\pm2^0$, and $\delta_{P_{3/2}}=6^0\pm2^0$ or $-2.5^0\pm2^0$, as found by CD.[10] Set II constitutes an approximate Fermi-Yang ambiguity and the solution with $\delta_{P_{1/2}}=-11^0$ and $\delta_{P_{3/2}}=6^0$ will be referred to as the $T=1$ Fermi set (IIF).

We accept only those solutions which have $P(\chi^2)>0.001$ and which give a K^+d non-charge-exchange cross section (σNCE) within 2 standard deviations of the 19 ± 3 mb expected.[11] In fitting the angular distribution and polarization data by the S method, we find, for set I, only two solutions which give a satisfactory S-, P-, and D-wave fit. These two solutions are labeled 1 and 2 in Table I and they are P-wave-dominant Yang-type (large-$P_{1/2}$) phase shifts. A similar solution (3) is also obtained with four parameters, but no acceptable solution is found with S and P waves only.

In the CD analysis of the angular distribution and polarization data, with set I as input, one five-parameter and a pair of four-parameter solutions were found.[12] Two of these, labeled 4 and 5, are similar to 1–3, while the third (6), which gives a poorer fit to the polarization, exhibits a larger $S_{1/2}$ phase shift and smaller P and D waves.

If the $T=1$ Fermi ambiguous solution of CD (set IIF) is used as input for fitting the differential cross section and polarization data, two solutions are obtained with the S method, 7 and 8, and one, 9, with the CD method. These S- and P-wave solutions are qualitatively similar to the S and P wave found in solutions 1–6. The inclusion of a $T=0$ $D_{3/2}$ phase shift gives improved fits with set IIF as input, but results in solutions whose S and P waves are indistinguishable from those of solutions 7–9. No three-parameter fits were obtained for the $T=1$ Yang set (IIY); however, one four-parameter fit was found with either method, and these are labeled 10 and 11 in Table I.

Thus, the inclusion of $T=0$ D waves is optional with the $T=1$ Fermi set as input but is necessary if the $T=1$ Yang set is taken to be the correct solution. At 780 and 910 MeV/c there are K^++p polarization measurements which each favor the $T=1$ Fermi solution of CD by about 2 standard deviations.[13] With the supporting evidence of being able to fit the charge-exchange data with only S and P waves if the $T=1$ Fermi set is assumed to be correct, we conclude that the simplest consistent description of K-nucleon scattering at 600 MeV/c implies S and P waves in both the $T=0$ and $T=1$ channels.

As a further check on this conclusion, we have made a six-parameter fit (method S) using as input our charge-exchange differential cross section and polarization data, the K^+p elastic cross section (12.3 ± 1.1 mb), and the real part of the forward K^+p elastic scattering amplitude obtained from dispersion relations.[14] This fit is labeled 12; the $T=1$ part is identified with set IIF and the $T=0$ phase shifts are consistent with the three-parameter $T=0$ solutions 8 and 9. The angular distribution and polarization for this solution are shown in Figs. 1(a) and 2. Also shown as the dashed curve in Fig. 2 is the polarization predicted from the best $T=0$ Fermi solu-

[11] The non-charge-exchange cross section at 530 MeV/c is 18.9 ± 1.8 mb (Ref. 3). The value reported at 770 MeV/c is 20 ± 2 mb as determined by V. Cook, D. Keefe, L. T. Kerth, P. G. Murphy, W. A. Wenzel, and T. F. Zipf, Phys. Rev. Letters **7**, 182 (1961). From these two numbers we estimate 19 ± 3 mb at 600 MeV/c.

[12] In fitting the charge-exchange data with the CD method, the following coupling constants have been used: $g_{\Lambda Kp}{}^2=14$ and $g_{\Sigma Kp}{}^2=\frac{1}{10}g_{\Lambda Kp}{}^2$ (see Ref. 9).

[13] W. Hirsch and G. Gidal, Phys. Rev. **135**, B191 (1964); S. Feminò, S. Jannelli, F. Mezzanares, L. Monari, and P. Serra, Nuovo Cimento **50A**, 371 (1967).

[14] M. Lusignoli, M. Restignoli, G. Violini, and G. A. Snow, Nuovo Cimento **45A**, 792 (1966). $D_{lab}(0)$ was taken to be -2.2 ± 0.4 (BeV/c)$^{-1}$ at 600 MeV/c.

tion as determined from our angular distribution assuming set I for the $T=1$ phase shifts.

All solutions listed in Table I have been obtained for the moving-neutron angular distribution in the form-factor impulse approximation. We have verified the existence of similar solutions for the moving- and stationary-neutron angular distributions with the closure and form-factor impulse approximations. In all solutions for the moving-neutron angular distributions, a better χ^2 is obtained with the form-factor method. To allow for all the uncertainties caused by the deuteron, the errors estimated from χ^2 minimization have been doubled.

We conclude: Both the standard and CD methods of analysis give equivalent results and establish a Yang-type phase-shift solution for the $T=0$ channel. The data are consistent with S- and P-wave K-nucleon scattering in both the $T=0$ and 1 channels. When fitting the angular distribution alone, the CD method usually gives fewer solutions for a given number of parameters; in particular, the CD method gives no acceptable $T=0$ Fermi solution. The $T=0$ Yang solution is also consistent with the ratio $K_S{}^0p/(\Lambda\pi^+ + 2\Sigma^0\pi^+)$ observed in $K_L{}^0p$ scattering at comparable energies.[15]

It is a pleasure to thank A. Dralle, Dr. J. Canter, Dr. Y. Cho, Dr. O. Skjeggestad, Professor A. Engler, Professor C. Meltzer, and Professor D. K. Robinson for their participation in various stages of this research. We thank Dr. B. Deo and Professor R. Cutkosky for numerous discussions regarding their method of analysis and for the use of many of their computer programs. We also wish to acknowledge the skillful work of our scanning and measuring staffs.

[15] J. A. Kadyk, Y. Oren, G. Goldhaber, S. Goldhaber, and G. H. Trilling, Phys. Rev. Letters **17**, 599 (1966); J. K. Kim, *ibid.* **19**, 1074 (1967).

PHYSICAL REVIEW VOLUME 183, NUMBER 5 25 JULY 1969

Dilatation Covariance and Exact Solutions in Local Relativistic Field Theories*

GERALD ROSEN
Drexel Institute of Technology, Philadelphia, Pennsylvania 19104
(Received 27 February 1969)

Consideration is given to the invariance of field equations under space-time dilatations with (induced) transformation of constant parameters therein. Analysis of such "dilatation covariance" facilitates the determination of rigorous closed-form singularity-free solutions to essentially nonlinear classical field equations. More over, for essentially nonlinear quantized (boson) field theories, dilatation-covariance considerations facilitate the determination of rigorous closed-form solutions to the Schrödinger stationary-state equation.

BECAUSE it has been customary to regard mass constants and coupling constants that appear in Lagrangians as being absolutely fixed, space-time dilatation (scale) transformations have played a minor role in the analysis of local relativistic field theories. Only special classical field theories feature dilatation-transformation invariance with absolutely constant physical parameters in their Lagrangians, and such theories do not ordinarily retain dilatation invariance when subject to quantization.[1] On the other hand, dilatation *covariance* on both the classical and the quantum level is a property of all local relativistic field theories with the appropriate (induced) dilatation transformation of parameters in the field equations. Our purpose in this article is to show that dilatation-covariance considerations not only facilitate the determination of rigorous closed-form solutions to classical field equations, but also facilitate the determination of rigorous closed-form solutions to the Schrödinger stationary-state equation in local relativistic quantum field theories.

We illustrate the general notion and utility of dilatation covariance by discussing a specific model theory, the self-interacting complex scalar field theory based on the Lagrangian density

$$\mathcal{L} = -(\partial\psi^*/\partial x^\mu)(\partial\psi/\partial x_\mu) - m^2\psi^*\psi + g\psi^*\psi \ln\psi^*\psi, \quad (1)$$

where m^2 and g are real physical constants and a system of convenient physical units is employed. The derived classical field equation

$$\partial^2\psi/\partial x^\mu\partial x_\mu - m^2\psi + g(1+\ln\psi^*\psi)\psi = 0 \quad (2)$$

admits solutions $\psi=\psi(x;m^2,g)$ that depend parametrically on m^2 and g with $x=(x^0,x^1,x^2,x^3)$ a point in space-time. For distinct pairs of values of m^2 and g, the manifolds of solutions[2] to Eq. (2) are homologous, the

* Work supported by a National Science Foundation grant.

[1] For the implications of dilatation invariance in a special nonlinear field model, see H. Mitter, Nuovo Cimento **32**, 1789 (1964).

[2] Suitable regularity conditions associated with the existence of global solutions must supplement Eq. (1) for definition of the manifold of classical (c-number) solutions; the existence of global solutions to the generic equation $\partial^2\psi/\partial x^\mu\partial x_\mu - U'(\psi^*\psi)\psi = 0$ has been the subject of numerous recent investigations: K. Jörgens, Math. Z. **77**, 295 (1961); I. Segal, Proc. Symp. Appl. Math. **17**, 210 (1965); C. S. Morawetz, Proc. Roy. Soc. (London) **306A**, 291 (1968).