Muon-Nucleon Inelastic Scattering at High Momentum Transfers

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The inelastic interactions of high-energy muons in nuclear emulsions have been investigated using a monoenergetic beam of 5-GeV/c muons produced at the Brookhaven A. G. S. The projected angular divergence of the incident beam was found to be less than 2°, and the contamination by pion-induced events is estimated at 0.2%. A total of 134 events were located by area scanning of the emulsion pellicles. The energies of the scattered muon and, whenever possible, the shower particles were measured by the method of multiple scattering. These tracks were then identified by the well-known g^*-$|\mathbf{p}|\beta c$ method. The spectrum of single mesons produced agrees with predictions of the Williams-Weizsäker method and previous electron scattering data. The integral distributions of transferred energy and four-momentum transferred are shown to be in agreement with quantum electrodynamics. No evidence was found for an anomalous scattering in pion production previously reported by cosmic-ray workers.

I. INTRODUCTION

During the past decade, electron-nucleon scattering experiments have provided convincing evidence on nucleon structure. Early measurements of the elastic electron-proton cross section have been carried out at Stanford[1,2] and recently at other laboratories.[3-10] The precise measurements of the elastic electron-proton cross section have been of interest both because of their fundamental significance and because they provide standard experimental quantities against which the results of other electron scattering measurements are frequently normalized.

More recently,[11-17] muon-nucleon electromagnetic interactions have received considerable interest. The study of the elastic scattering of (structureless) muons by protons is described in quantum electrodynamics in the same way as the elastic scattering of (structureless) electrons by protons, that is, by means of the Rosenbluth formula. In this expression, a single virtual photon is exchanged between the electron and proton or between the muon and proton; and the structure of the proton is represented by two electromagnetic form factors which are functions of q^2, the square of the four-momenta carried by the virtual photon. The form factors are important phenomenological properties of the nucleons and are used to interpret the results of many other experiments involving the electromagnetic interactions of the nucleon. Although the μ-n and e-n scattering are alike for the same four-momentum transfer, muon scattering requires a much smaller radiative correction than electron scattering because of the larger muon mass. This extra advantage of the muon over the electron has a significant effect in experiments at very high momentum transfers. Consequently, the studies of elastic muon scatterings by Cool *et al.*[15] implies that muons would make better probes than electrons in the search for a breakdown in quantum electrodynamics. Similarly, the muon should be better suited for studying the inelastic electromagnetic interactions, which are currently being studied by electron scattering. These studies will help us to understand the differences between the muon and the electron beyond their known mass difference. As a matter of fact, the search for electron-muon differences might be better studied with muon inelastic rather than elastic scattering measurements.

The original experiments on particle production by muons were with cosmic rays as the source of muons.

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[15] R. Cool, A. Maschke, L. Lederman, M. Tannenbaum, R. Ellsworth, A. Melissinos, J. Tinlot, and T. Yamanouchi, in *Proceedings of the International Conference on High-Energy Physics, Dubna, 1964* (Atomizdat, Moscow, 1965); Phys. Rev. Letters **14**, 724 (1965).

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[17] J. A. Kirk, D. M. Cottrell, J. J. Lord, and R. J. Piserchio, Nuovo Cimento **40A**, 523 (1965).

Many of these experiments implied the existence of an anomalous interaction in addition to the expected electromagnetic scattering. Their data fitted the curves for scattering by a point nucleus without the finite-charge-extension effects which were found in electron scattering.[11,12] The cosmic-ray work in general, however, is subject to several difficulties:

(a) It is difficult to discriminate against the contamination due to incoherent elastic scattering.

(b) The pion contamination is uncertain and hence difficult to correct for.

(c) The incoming energy of individual muons is not known.

Moreover, beam experiments[13–19] have reported no anomalous behavior in muon scattering. The existence at the Brookhaven A. G. S. of a pure (pion contamination $\sim 10^{-7}$) muon beam[15] from which a monoenergetic beam (~ 5 GeV/c) of muons could be separated led us to investigate the inelastic muon interactions in nuclear emulsion. Certain preliminary results of this experiment were reported previously.[19] We are reporting here the full details of the experiment along with its complete results and discussion. The form factors used were $[1/(1+q^2/\Lambda^2)]^2$, where Λ is related to the radius of the proton charge distribution, and G_{MV}^2, where G_{MV} (the magnetic isotopic vector form factor) is given by[20]

$$G_{MV}=2.35\left(\frac{-1.0}{1+q^2/(1.16\ \mathrm{GeV}^2/c^2)}+\frac{2.0}{1+q^2/(0.563\ \mathrm{GeV}^2/c^2)}\right). \quad (1)$$

They were specifically chosen to allow direct comparison with the cosmic-ray data of Higashi *et al.*[12] and the calculation of inelastic electron scattering by Hand *et al.*[20] We have found no indication in our analysis of any anomalous behavior. In fact, the data show agreement with the Williams-Weiszäcker theory,[22] and the photoproduction experiments similar to those found for pion production by electrons.[21]

II. THEORY

We are interested in inelastic collisions of negatively charged muons with complex emulsion nuclei of the type

$$\mu^-+A\rightarrow\mu^-+B+C\cdots, \quad (2)$$

where A is the target nucleus and B, C, etc., are particles produced in the interaction or in the subsequent breakup of the nucleus. Such interactions may be explained by several possible mechanisms:

(a) Elastic scattering from free protons in the emulsion.

(b) Incoherent elastic scattering The incident muon scatters elastically off an individual nucleon where the recoil of the target causes the subsequent breakup of the nucleus.

(c) The muon produces a new particle in the field of an individual nucleon. Usually the particle produced is a pion, but any of the strange particles can also be produced. The produced particle may emerge from the nucleus and appear as a shower particle (real production), or the particle may be absorbed before it leaves the nucleus (virtual production).

Until recently the inelastic electromagnetic interaction was discussed in terms of the Williams-Weiszäcker approach.[22] This semiclassical theory, sometimes known as the method of virtual quanta, exploits the similarity between the fields of a rapidly moving charged particle and fields of a pulse of radiation. The main idea of this approach is to decompose the electromagnetic field produced by the incoming muon at the target into a spectrum of equivalent photons, so that the cross section of the muon-nucleon interaction can be obtained by integration of the known photoproduction cross section over this spectrum.

$$\sigma_\mu(E_1)=\int N(E_1,\epsilon)\sigma_{\mathrm{photo}}(\epsilon)d\epsilon, \quad (3)$$

where $N(E_1,\epsilon)$ is the number of virtual photons of energy ϵ in the equivalent photon spectrum of a muon of incident energy E_1; $\sigma_{\mathrm{photo}}(\epsilon)$ is the total photoproduction cross section; and $\sigma_\mu(\epsilon)$ is the corresponding total muon-production cross section. Kessler and Kessler[23] have derived the following expression for $N(E_1,\epsilon)$:

$$N(E_1,\epsilon)=\frac{2\alpha}{\epsilon\pi}[\ln(E_1/m)-\tfrac{1}{2}]\left(1-\frac{\epsilon}{E_1}+\frac{\epsilon^2}{2E_1^2}\right), \quad (4)$$

where α is the fine-structure constant. Their derivation is based on the assumption that the square of the four-momentum transfer, q^2, is very small. This is equivalent to an approximation for small scattering angles θ, since

$$q^2\simeq 4E_1(E_1-\epsilon)\sin^2(\tfrac{1}{2}\theta). \quad (5)$$

[18] T. Konishi, O. Kusumoto, S. Ozaki, M. Teranaka, T. Wada, Y. Watase, and M. Ohta, Nuovo Cimento 54, 781 (1968).

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[22] W. F. Weiszäcker, Z. Physik 88, 612 (1934); H. Heitler, *Quantum Theory of Radiation* (Oxford University Press, New York, 1954), p. 147.

[23] D. Kessler and P. Kessler, Nuovo Cimento 4, 601 (1956); P. Kessler, *ibid.* 17, 809 (1960).

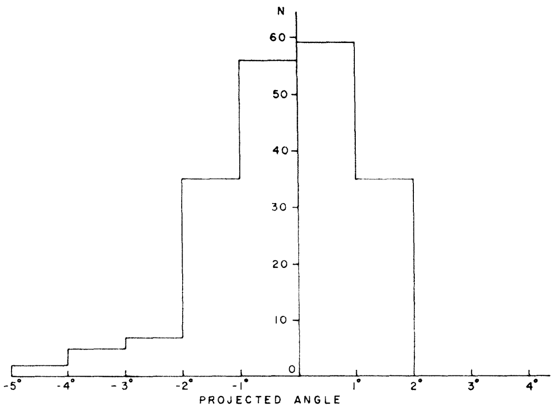

FIG. 1. Angular spread of muons in the beam.

In order to study effects of nucleon structure, however, it is necessary to have a theory[24–26] valid for large values of q^2. It has been shown recently[25,26] that the cross section for the general electromagnetic interaction can be expressed in terms of the two unknown Lorentz- and gauge-invariant functions L and L' as follows:

$$\partial^2\sigma_\mu/\partial q^2\partial\epsilon=(\alpha/8\pi^2|\mathbf{p}_1|^2q^4)\times\{L[(E_1^2+E_2^2)q^2-2m^2\epsilon^2-\tfrac{1}{2}q^4]+L'(2m^2-q^2)q^2\}. \quad (6)$$

The expressions for L and L' have been obtained for the case of point nucleons by evaluating the real photoproduction cross section in terms of the same invariant functions, resulting in the relations

$$\left.\begin{array}{l} L=(4\pi/\epsilon)\sigma_{\mathrm{photo}}(\epsilon) \\ L'=0 \end{array}\right\}\text{(point nucleon)}. \quad (7)$$

As an approximation to the expression for the finite charge distribution (and also to allow us to compare our data with the most recent cosmic-ray[12] data), we can multiply the expressions for point charge by the form factor which best agrees with the electron elastic scattering data.

$$L(q^2,\epsilon)=(4\pi/\epsilon)\sigma_{\mathrm{photo}}(\epsilon)[\Lambda^2/(q^2+\Lambda^2)]^2 \quad \text{(finite size)}. \quad (8)$$

The correct choice of a form factor for a specific process making up the electromagnetic interaction is difficult, since the theory has not been sufficiently developed to assign form factors to each process. Since this experiment deals with a combination of distinct processes, the choice of a suitable form factor is still more difficult. However, since a large number of the events are single-pion production through the $(\frac{3}{2},\frac{3}{2})$ resonance, perhaps a better choice would be the form factor G_{MV}^2, which was calculated by Hand *et al.*[20] from pion electroproduction data, and is given by Eq. (1).

III. EXPERIMENTAL PROCEDURE

A stack of ten Kodak NTB-4 nuclear emulsion pellicles was exposed to a 5-GeV/c beam of negative muons of flux density 2×10^5/cm^2 at the Brookhaven A. G. S. The beam prepared by Cool *et al.*[15] was produced by allowing about 10% of the pions to decay into muons and then filtering out the remaining pions through 32 ft of light concrete absorber. Since the pion-proton scattering cross section is known to be about 10^4 times larger than the muon-proton cross section, it necessitates a very high effective-beam purity. The pion contamination in this beam was found[15] to be less than 1 part out of 10^7 and was considered negligible in this experiment. The particles were then subjected to momentum analysis in a deflecting magnet to produce a monoenergetic beam. Our stack was exposed to 5-GeV/c muons with the plane of the emulsion parallel to the beam. The momentum spread of the beam was calculated to be $\sim$10% and the projected angular divergence of the incident beam in the emulsion is shown in Fig. 1. The spread in the dip angle was about the same.

The emulsions were processed at the Brookhaven National Laboratory using standard techniques. A 1-mm grid was printed on the bottom of each pellicle to facilitate the recording and the relocation of events.

The emulsions were area-scanned with Leitz and with Bausch and Lomb binocular microscopes under 100× or 150× magnifications for muon stars. All events with a beam track passing through the center of the star were listed by the scanners as a possible muon interaction. By area scanning, only events with at least one dark or grey track were found in a pellicle. All these scanned events were reexamined by a physicist for possible background stars produced by cosmic rays where one of the light tracks belonging to the event happened to be parallel to the primary beam. For that reason we checked the dip angle of primary tracks in each event before we accepted any muon-produced event in the stack.

The selected events were classified according to the formalism N_h+N_s, where N_h is the number of black and grey tracks in the interaction and N_s is the number of light tracks. Thus N_h+N_s gives the total number of charged particles after the interaction. The nomenclature black, grey, or light was established as follows:

(i) light tracks: those in which $g_s\leq1.5g_0$, where g_s is the number of grains per 100 μ of the secondary track and g_0 is the number of grains per 100 μ of the primary track;

(ii) grey tracks: those in which $1.5g_0<g_s\leq2.5g_0$; and

(iii) black tracks: those in which $g_s>2.5g_0$.

Table I shows the prong distribution for 134 events.

[24] M. Gourdin, Nuovo Cimento 21, 1094 (1961).

[25] K. Daiyasu, K. Kobayakawa, T. Murota, and T. Nakano, J. Phys. Soc. Japan 17, Suppl. A-III, 344 (1962).

[26] S. D. Drell and J. D. Walecka, Ann. Phys. (N. Y.) 28, 18 (1964).

Selection criteria. The following selection criteria were used to distinguish between valid muon interactions and background events:

(a) The incident light track should not make an angle greater than 3° with the direction of the primary beam.

(b) The light track was traced back 1000 μ to make sure that it was flat and straight and had the same dip angle as the primary beam.

(c) The grain count g_0 of the primary track was measured to make sure that it corresponded to other primary tracks measured at that depth in the emulsion.

(d) At least one of the secondary particles should be identified as possibly being the emerging muon.

Using these very stringent criteria, only 134 possible inelastic interactions out of about 1000 stars were found to be produced by the primary muon beam in our stack. The rest of the stars were produced either by cosmic rays or by the secondary tracks produced in other interactions.

The grain density g_s and the product of three-momentum and velocity $|\mathbf{p}|\beta c$ were measured for all grey and light secondary tracks in each of the events as long as there was a sufficient length of track in the emulsion ($\sim$5000 μ). The grain density g_0 of the primary track was measured separately for each event in order to eliminate any error due to nonuniformity of the blob density from place to place in the emulsion stack. The blob count was performed over a length of 1000 μ for the primary as well as (when possible) for the secondary track. The measurement of the blob count on the primary track also helped us to check if that track was parallel to the other primary tracks in the same field of view. In this way we eliminated any coincident-background star. A Koristka scattering microscope, to which was attached a filar micrometer that could be read to an accuracy of 0.02 μ, was used for the measurements of $|\mathbf{p}|\beta c$. The shower particles were identified by a well-known g^*-$|\mathbf{p}|\beta c$ technique[27] by using the Sternheimer energy-loss relation and the measurements of relative grain density and momentum, or by the range-energy relation wherever it was possible. Theoretical curves for g^* versus $|\mathbf{p}|\beta c$ were drawn for each particle (μ, π, k, etc.) produced in muon interactions at 5-GeV/c incoming momentum. In the use of these theoretical curves there are certain ranges of g^* and $|\mathbf{p}|\beta c$ in which identification is ambiguous. When the values of g^* and $|\mathbf{p}|\beta c$ for a given shower particle occurred in one of these "regions of confusion," i.e., where different curves for g^* versus $|\mathbf{p}|\beta c$ for different particles overlap, certain preferences were followed. If the light track of interest was that which made the smallest angle with respect to the primary direction, preference in identification was given to muon over pion. For other shower particles, preference was given to pion over muon and kaon, and proton over Σ. There were 19 events for which the lightly ionizing track making the smallest angle with the continuation of the primary direction had insufficient track length for accurate energy determination—leading in many cases to misidentification of these particles as other than muons. The angular distribution of these 19 events was compared to the angular distribution of the remaining 115 events and no difference was found. The distributions of four-momentum transferred and energy transferred in the interactions were also examined with and without the 19 doubtful events, assuming the particle to be a muon of the measured energy. As expected, only the energy spectrum was changed significantly by their removal and hence they were left out in the energy spectrum. The projected angle and dip angle that the shower particles made with the direction of the incident muon were measured using a Leitz microscope with 53× oil-immersion objective. The angle in space was then calculated on the computer. Because of the uncertainty, primarily in the dip angle measurement, the error in the space angle is $\sim\frac{1}{2}°$. The observed angular distribution of the outgoing muons in the laboratory system was given before[19] and was discussed in terms of the theory of Kessler and Kessler[23] in which use was made of the virtual-photon spectrum introduced by Williams and Weizsäcker.[22] The angular distribution of all the grey and black prongs is isotropic in the laboratory system.

TABLE I. Nomenclature of events.

Type of event (N_h+N_s)	Total No.
1+1	40
2+1	26
3+1	15
4+1	13
5+1	11
6+1	8
7+1	2
8+1	4
9+1	0
10+1	2
11+1	1
5+2	3
6+2	1
8+2	1
12+2	1
2+3	1
3+3	2
5+3	1
6+3	1
13+4	1
Total No. of events	134

IV. RESULTS AND DISCUSSION

In Table I is shown the prong distribution for 134 possibly inelastic events with at least one grey or black outgoing track. One can see that about 30% of these events are of the 1+1 type and events with large multiplicity are due to evaporation tracks from heavy

[27] W. H. Barkas, *Nuclear Research Emulsions* (Academic Press Inc., New York, 1963), p. 264.

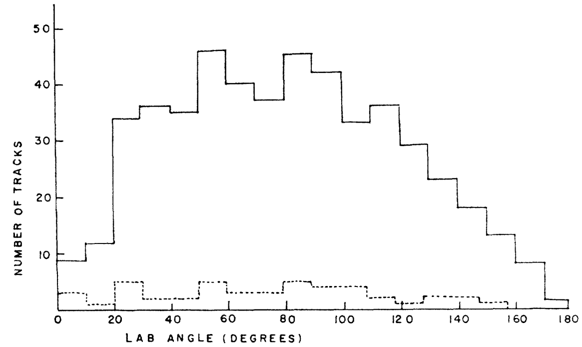

FIG. 2. Angular distribution of black and grey tracks for all events. Dashed curve is only (1+1)-type events.

emulsion nuclei after their interaction with the incoming muons. Figure 2 shows the angular distribution of the grey and the black prongs from all the events in the laboratory system. The angular distribution of the single secondary proton from events of type 1+1 is also shown by the dashed line in the same figure. The distribution for all events shows a Gaussian shape with a maximum around 90°.

So far, we have considered all 134 muon-produced events as inelastic events produced under process (c) of Sec. II. There may be a few events which might have been produced under the processes (a) and (b) of Sec. II. This is discussed in the following section.

Contamination due to Quasi-Elastic Scattering

In this experiment, we were interested in events corresponding to process (c) of Sec. II. Events corresponding to (a) and (b), therefore, are a contamination which must be accounded for or removed. We have the following relation from kinematics of elastic and inelastic events:

$$2\epsilon M_T - q^2 = M_F^2 - M_T^2, \quad \text{(inelastic)} \qquad (9a)$$

$$2\epsilon M_T - q^2 = 0, \quad \text{(elastic)} \qquad (9b)$$

where M_F is the invariant mass of final system not including the muon and M_T is the invariant mass of target nucleon. Since the energy transferred ϵ and the four-momentum transferred q^2 can be calculated for each event from measurements of $|\mathbf{p}|\beta c$ and the scattering angle of the emerging muon, the value of $M_F^2 - M_T^2$ can be calculated for each event. Figure 3 shows the number of events plotted against $M_F^2 - M_T^2$. The threshold for pion production is ~ 0.3 BeV^2. A peak appears at the lower end of the histogram and is partially due to a contamination of events of types (a) and (b). All 1+1 events were checked for coplanarity. There were three events which satisfied the coplanarity test within the experimental limits and were in the peak. They were considered definitely elastic or quasi-elastic scattering events and were subsequently removed from all calculations. Because of the error inherent in scattering measurements ($\sim$15–20%), difficulty arose in distinguishing quasi-elastic scattering, where the recoil proton scattered before leaving the nucleus, from the events which were caused by virtual-pion production. However, range estimates made on black prongs set lower limits on the energy transferred in a given elastic interaction. Then, using Eqs. (5) and (9), a lower limit on the elastic scattering angle (which can be more accurately measured) is established. For example, assuming an elastic interaction and taking $\epsilon_{\min} = 15$ MeV, we get $q_{\min}^2 = 0.03$ (GeV/c)^2 and $\theta_{\min} = 2°$. Therefore, we can expect the contamination due to process (b) to occur at low values of ϵ and high values of q^2. By normalizing our energy-transfer data at $\epsilon = 1$ GeV/c (see Fig. 4), we effectively eliminate any serious contamination due to process (b). In our previous paper,[19] we gave the integral q^2 distribution with

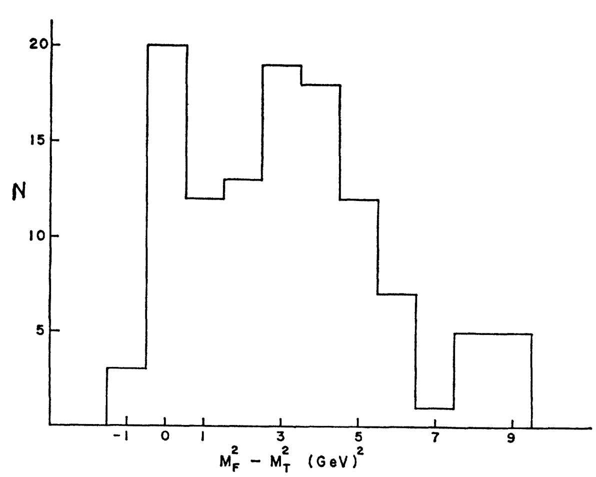

FIG. 3. Distribution of $M_F^2 - M_T^2$.

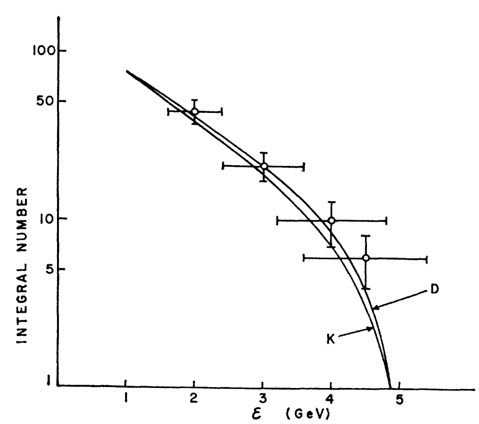

FIG. 4. Integral spectrum of the transferred energy distribution (ϵ). Curve K is the semiclassical theory of Kessler and Kessler, while curve D is based on the more rigorous treatment of Daiyasu *et al.*, with $\Lambda^2 = 0.365$ (GeV/c)^2.

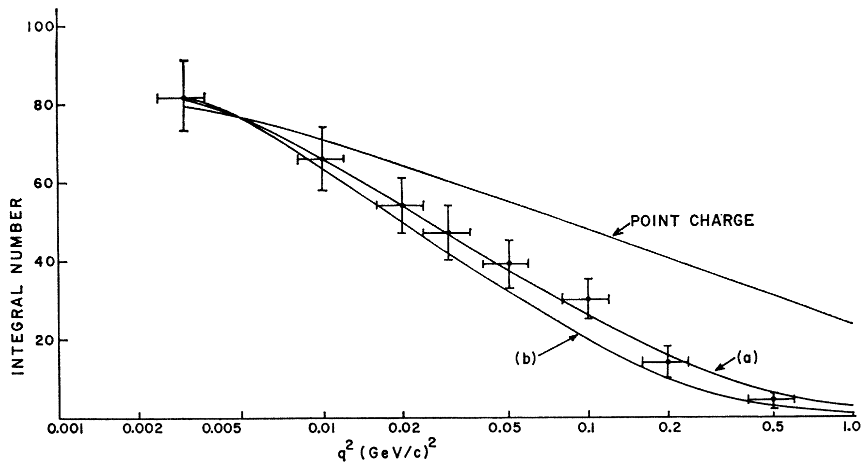

FIG. 5. Integral spectrum of the square of the transferred four-momentum after removal of only the elastic events. The curves were calculated for different values of Λ^2: 0.365 $(\text{GeV}/c)^2$ [curve (b)] and 0.71 $(\text{GeV}/c)^2$ [curve (a)]. The curves were normalized to the data at $q^2=0.005$ $(\text{GeV}/c)^2$ corresponding approximately to the average nuclear radius in the emulsion. The datum point at $q^2=0.003$ $(\text{GeV}/c)^2$ demonstrates that the fit is not sensitive to the normalization point.

no attempt to separate out events of type (b). In a subsequent examination of the data, we found 12 events (ten 1+1 and two 2+1) whose minimum energy transfer (estimated from the ranges of the black prongs) is approximately equal to that allowed by Eq. (9b). But in the q^2 spectra given here (see Fig. 5), the data are limited to events in which the energy transferred is too large to admit the possibility of elastic scattering at that angle. Figure 6 gives the kinematical space of q^2 versus ϵ. The kinematical relationship is $\epsilon=q^2/2M_T$, where M_T is the mass of target particle. The solid curve represents scattering by a free nucleon, while the dashed curve represents the kinematics for a bound particle with an effective mass of $\frac{1}{3}$ the proton rest mass. Events with $q^2<0.001$ $(\text{GeV}/c)^2$ or $\epsilon<0.1$ GeV are not shown; nor are the 19 events for which it was not possible to obtain an accurate scattering measurement.

TABLE II. Different cross-section values for muons in emulsion.

Process	σ (μb/nucleon)
Inelastic muon-nucleon interaction	3.6 $\pm$0.30
Real-pion production	0.70$\pm$0.18
Real-strange-particle production	0.18$\pm$0.15

The total cross sections for the various processes are given in Table II. The cross section labeled "Inelastic interactions" includes all events which satisfied the selection criteria (see Sec. III), including the 19 events mentioned above. The remaining events could be identified as either elastic and quasi-elastic events or inelastic muon-nucleon events. The real-pion and

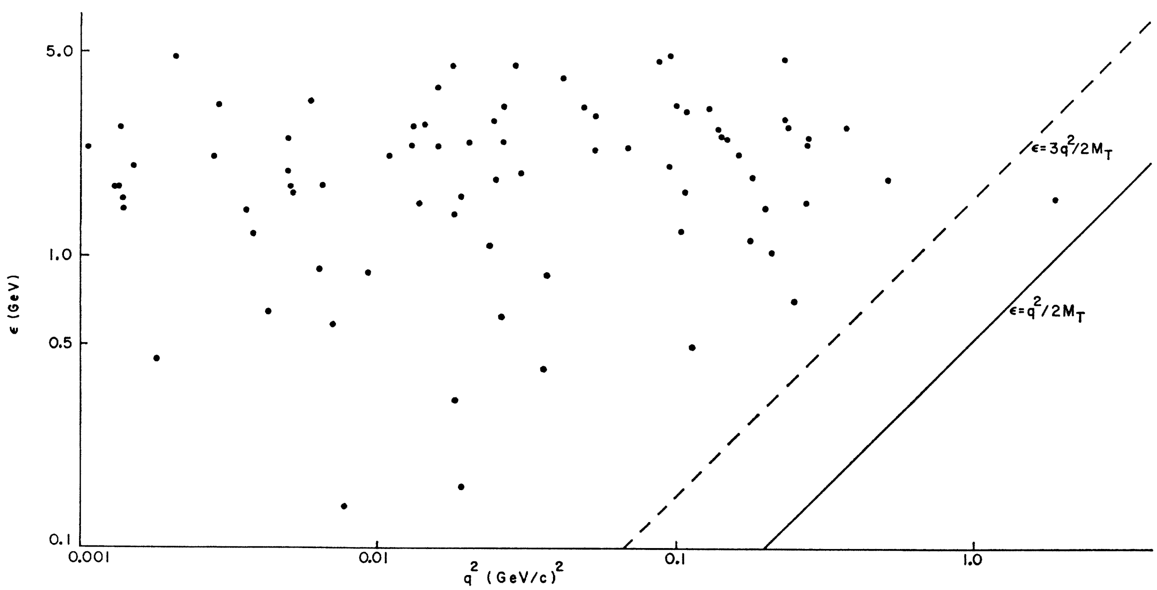

FIG. 6. Plot of the square of the transferred four-momentum q^2 versus transferred energy ϵ. The solid line ($\epsilon=q^2/2M_T$) is for elastic interaction with a free proton. The dashed line ($\epsilon=3q^2/2M_T$) corresponds roughly to scattering by a proton bound in a nucleus.

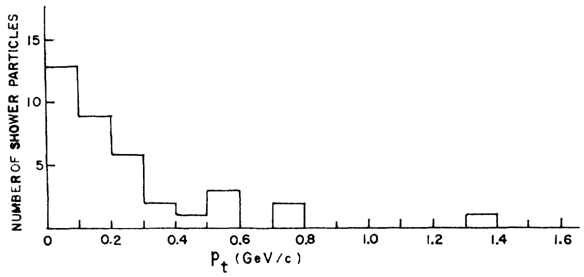

FIG. 7. Transverse momentum distribution of shower particles.

strange-particle production cross sections include only those events in which the identity of the shower particle could be determined.

The transverse momentum p_t, where $p_t = |\mathbf{p}| \sin\theta$, has been calculated for all shower particles wherever a scattering measurement was possible. The value of the momentum of the shower particle was obtained directly from the value of $|\mathbf{p}|\beta c$, which was determined by the relation

$$|\mathbf{p}|\beta c = |\mathbf{p}|^2c^2/(|\mathbf{p}|^2c^2+m_s^2c^2)^{1/2}, \tag{10}$$

where m_s is the mass of the shower particle. The value of m_s used for each of the shower particles depended upon the identification obtained by the g^*-$|\mathbf{p}|\beta c$ method. The most probable value of the p_t of secondary particles produced in strong interactions in previous experiments[28] was found to be independent of the primary energy, the angle of emission, and the longitudinal momentum of the secondary particle. In particular, the average value of the transverse momentum of the pions and of the strange particles was found to be nearly independent of the incident energy, p_t for the heavy particles being only slightly larger than for the lighter ones. The distribution of p_t for all shower particles is given in Fig. 7. The average value of p_t was calculated to be 244 ± 35 MeV/c, which agrees with previous values,[28] for strong interactions. The agreement between the value of $\langle p_t\rangle$ from this experiment on electromagnetic interactions and the values calculated previously for particles produced in strong interactions indicates that the constant value of $\langle p_t\rangle$ is also independent of the type of interaction producing the particle.

The angular distribution of the scattered muons with quasi-elastic events removed is substantially the same as that reported in our preliminary results.[19] The energy spectrum of single pions arising from muon-nuclear interactions is shown in Fig. 8. The theoretical curve[11,19] shown has a general agreement with the experiment except at the higher values of kinetic energies, since it was calculated for higher muon energy (10-GeV muons). A similar agreement between photoproduction experiments and pion production by electrons scattered at small angle has been reported by Panofsky and co-workers.[21] Single-pion photoproduction has been shown to be principally through the $(\frac{3}{2},\frac{3}{2})$ resonance. The angular distribution of the photoproduced pions shows a strong dependence upon the energy of the incident photon. But production that is backward in the c.m. system predominates at all energies corresponding to this resonance.[29] There is a backward enhancement in the angular distribution (in the c.m. system) of single pions produced by muons in inelastic interactions of the target nucleon with the virtual photon that is associated with the incoming muon. This distribution is shown in Fig. 9. The production of pions in the backward direction is in general agreement with the photoproduction data.[29]

Following the Williams-Weizsäcker approach, where we assume that we can express the muon-nucleon cross section entirely in terms of the photon-nucleon cross section, we substitute the expression for W_1 and W_2 from Eq. (8) into Eq. (6), which gives

$$\frac{\partial^2\sigma_u}{\partial(q^2)\partial\epsilon}=\frac{\alpha\sigma_{\text{photo}}(\epsilon)}{2\pi\epsilon|p_1|^2q^4} \times\{[E_1^2+(E_1-\epsilon)^2]q^2-2m^2\epsilon^2-\tfrac{1}{2}q^4\}\left(\frac{\Lambda^2}{\Lambda^2+q^2}\right)^2. \tag{11}$$

From recent experiments on multiple-pion and strange-particle production by photons,[30] we can see that $\sigma_{\text{photo}}(\epsilon)$ can be expected to have a complicated de-

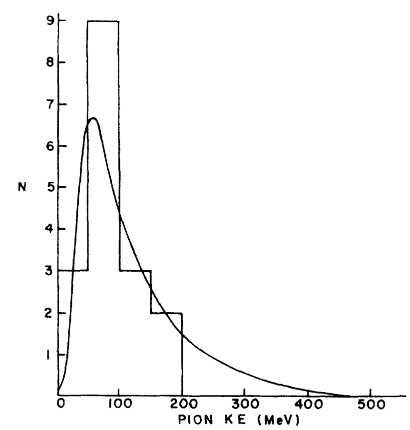

FIG. 8. Energy spectrum of pions produced from the interaction of muons with nucleons. In the calculated curve the effect of pion absorption and nucleon motion was taken into consideration.

[28] O. Minakawa *et al.*, Nuovo Cimento Suppl. **II**, 125 (1959); B. Edwards *et al.*, Phil. Mag. **3**, 237 (1958); L. F. Hansen *et al.*, Phys. Rev. **118**, 812 (1960); M. Schein *et al.*, *ibid.* **116**, 1238 (1954); P. L. Jain, *ibid.* **125**, 679 (1962); Nuovo Cimento **24**, 698 (1962); **32**, 873 (1964).

[29] E. H. Bellamy, Progr. Nucl. Phys. **8**, 239 (1960).

[30] Y. Eisenberg, P. Bastien, B. T. Field, V. K. Fischer, L. A. Pless, A. Rogers, C. Rogers, L. Rosenson, T. L. Watts, R. K. Yamamoto, N. Widgoff, A. M. Shapiro, R. E. Lanou, A. E. Brenner, M. E. Law, E. E. Ronat, K. Strauch, J. C. Street, J. J. Szymanski, J. D. Teal, L. Guerriers, G. A. Salandin, and G. E. Fisher, Phys. Rev. Letters **13**, 636 (1964).

pendence upon the energy transferred by the virtual photon; however, it appears that setting $\sigma_{\text{photo}}(\epsilon)$ equal to a constant is a reasonable approximation.

The integrated spectrum of the transferred energy is shown in Fig. 4. The curve labeled K is the Kessler-Kessler form of the Williams-Weizsäcker theory and is obtained by integrating Eq. (4). The curve labeled D is based on the more exact theory of Daiyasu *et al.*[25] as given in Eq. (11), with $\Lambda^2=0.365$ (GeV/c)^2, and corresponds to integration over the following limits:

$$q_{\min}^2=M^2\epsilon^2/E_1(E_1-\epsilon)\geq 0.005\ (\text{GeV}/c)^2,$$
$$q_{\max}^2=2M_T\epsilon$$

and

$$\epsilon_{\min}=0.15\ \text{GeV},$$
$$\epsilon_{\max}=4.8\ \text{GeV}.$$

The lower limit on the q^2 integration was chosen to correspond roughly to the average nuclear radius in the emulsion, using

$$q^2=\hbar^2c^2/(1.2\langle A^{1/3}\rangle\times 10^{-13}\ \text{cm})^2=0.005\ (\text{GeV}/c)^2.$$

Changing the lower limit to

$$q^2=\hbar^2c^2/(1.2\langle A\rangle^{1/3}\times 10^{-13}\ \text{cm})^2=0.003\ (\text{GeV}/c)^2$$

was tried and it did not produce any significant change. The results of Daiyasu *et al.*[25] and of Kessler and Kessler.[23] are in agreement with experiment and with each other. The transferred-energy spectrum unfortunately does not yield any distinction between different values of the parameter Λ^2 [$\Lambda^2=0.365$ or 0.71 (GeV/c)^2] of the form factor corresponding to the charge density. The point-charge form of the theory of Daiyasu *et al.* is almost exactly the same as the results of the semi-classical theory.

Some discrimination among the various parameters of the charge density can be obtained from the q^2 distribution. The complete integrated q^2 distribution was shown in our preliminary results,[19] where we normalized to a q^2 [$=0.002$ (GeV/c)^2] corresponding roughly to the radius of the heavier emulsion nuclei. In Fig. 5 we show the integrated q^2 distribution with quasi-elastic scatterings removed (14 events). Comparison of the two spectra shows that quasi-elastic scattering contaminates the data in the region $q^2\sim 0.1$ (GeV/c)^2 and was therefore quite serious. The data are shown normalized to the theory of Daiyasu *et al.*[25] at $q^2=0.005$

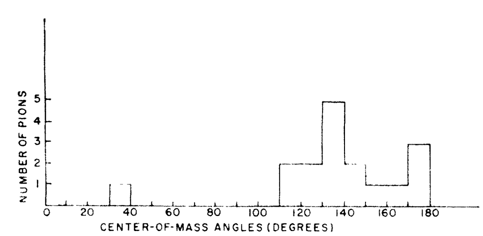

FIG. 9. Angular distribution of pions in the c.m. system.

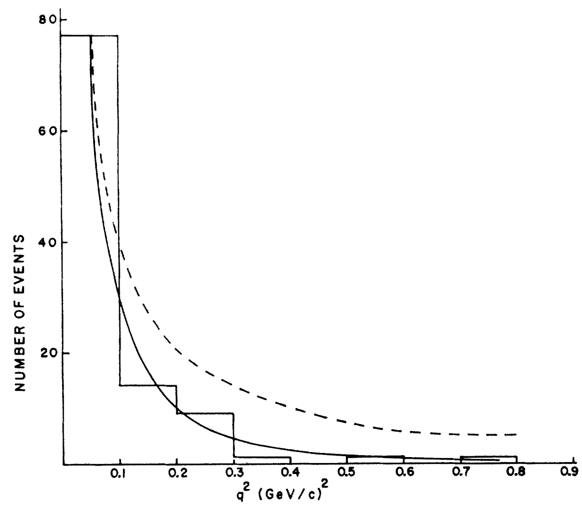

FIG. 10. Histogram of q^2 distribution with point-charge (dashed curve) and an extended-charge (solid curve) distribution. For the extended-charge distribution, the form factor G_{MV}^2 was used from Eq. (1) in the text.

(GeV/c)^2, which roughly corresponds to the average nuclear radius. The data and theoretical representations are also extended to $q^2=0.003$ (GeV/c)^2 (Fig. 5) to show that the choice of a normalization point does not particularly affect the fit of the data to the theory. The data obviously do not fit the point-charge theory. Two curves are given using different values of Λ^2 in the form factors; one corresponds to the radius of a free nucleon [$\Lambda^2=0.365$ (GeV/c)^2], while the other [$\Lambda^2=0.71$ (GeV/c)^2] represents a radius of 0.57×10^{-13} cm. Examination of Fig. 5 indicates a better fit of the data by the curve representing the smaller proton radius ($\Lambda^2=0.71$).

The form factor used in the preceding discussion, $[1/(1+q^2/\Lambda^2)]^2$, was for the charge distribution of the proton and was specifically chosen for comparison with cosmic-ray data. Since this experiment deals with a combination of distinct processes, the choice of a suitable form factor is difficult. Hand *et al.*,[20] on examining the inelastic scattering of electrons through the $(\frac{3}{2},\frac{3}{2})$ resonance, found the factor G_{MV} [Eq. (1)] in good agreement with the data. Moreover, the single-pion spectrum[19] from this experiment showed good agreement with predictions based on the $(\frac{3}{2},\frac{3}{2})$ resonance. Figure 10 gives a histogram of the q^2 distribution of events where G_{MV}^2 is used as the form factor in calculating the theoretical expression (dashed curve). The fit is good but better statistics are necessary for any quantitative determination of the correct form factors to be used.

The primary purposes of the present effort has been to search for an anomalous scattering of muons. The spectrum of pions produced singly in these interactions shows good agreement with predictions based upon photoproduction studies of the $(\frac{3}{2},\frac{3}{2})$ resonance and it also agrees with the electron scattering results. The spectrum of multiply produced pions and strange particles could not be treated because of the limited statistics. The good agreement of the angular and transferred-energy distributions with the semiclassical forms of the Williams-Weiszäcker technique probably results from the fact that in this type of experiment small values of the muon scattering angle predominate. The anomalous scattering definitely does not show up in the q^2 distribution (given here in both differential and integral form). The muon behaves as a "heavy electron" at 5-GeV/c primary momentum. It appears that the muon can safely (and in many respects more easily) be used for the study of electromagnetic interactions at higher energies.

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Resonance Production in Multiparticle Final States in K^+p Interactions at 9 GeV/c*

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We present results on the five- and six-body final states $K^0p\pi^+\pi^+\pi^-$ and $K^0p\pi^+\pi^+\pi^-\pi^0$ in K^+p interactions at 9 GeV/c. We observe copious production of known resonances such as $N^{*++}{}_{1236}$, $K^*{}_{890}$, $K^*{}_{1420}$, ρ, and ω. In addition, there are significant mass enhancements in the A_1 and A_2 mass regions with mass and width values of $M_{A_1}=1060\pm20$ MeV, $\Gamma_{A_1}=160\pm20$ MeV and $M_{A_2}=1290\pm20$ MeV, $\Gamma_{A_2}=50\pm10$ MeV, respectively.

I. INTRODUCTION

It has been pointed out that, because of the angular momentum barrier, high-mass high-spin resonances are expected to decay primarily through several steps involving lower-mass lower-spin resonances rather than to decay directly into two or three nonresonating particles.[1] Specifically, high-mass high-spin K^* mesons are expected to decay through lower-mass and lower-spin resonances, such as $K^*{}_{890}$ or ρ mesons, rather than directly into $K\pi$ or $K\pi\pi$ states. Such K^* resonances, if they exist, could appear in the four- or five-meson mass distributions of the five- and six-body final states in K^+p interactions. At the same time, it is of interest to study the roles of the intermediate mass and spin resonances, such as K^*, ρ, or the "A" mesons, in multiparticle final states. In this paper we present results of a study of the five- and six-body final states involving a visible K^0 meson produced in K^+p reactions at 9 GeV/c.

The experiment has been carried out in about 200 000 exposures of the Brookhaven National Laboratory 80-in. liquid-hydrogen bubble chamber, to a 9-GeV/c rf-separated K^+ beam of the Brookhaven AGS. The events used in this work were four prongs and four prongs associated with a visible K^0 decay. The events were measured on the LRL flying-spot digitizer (FSD), and remeasurements were carried out on conventional digitizing machines. The events were reconstructed in space and kinematically fitted in the program SIOUX. The four-prong plus visible K^0 events were fitted to the following hypotheses:

$$K^0p\pi^+\pi^+\pi^-, \tag{1}$$

$$K^0p\pi^+\pi^+\pi^-\pi^0, \tag{2}$$

$$K^0n\pi^+\pi^+\pi^+\pi^-. \tag{3}$$

Kinematic ambiguities among these hypotheses were resolved on the scan table. In addition, those four-pronged events (without a visible K^0 decay) which did not fit a four-constraint hypothesis ($K^+p\rightarrow K^+p\pi^+\pi^-$ or $K^+pK^+K^-$) were fitted to the following hypotheses:

$$K^+p\pi^+\pi^-\pi^0, \tag{4}$$

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[1] See, for example, R. H. Dalitz, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967), p. 215; G. Goldhaber, CERN Report No. 67-24, 1967, Vol. III (unpublished).