presence[14] of D waves in the reaction $\pi^+p \to \Sigma^+K^+$ (pure isospin-$\frac{3}{2}$) as low as 1170 MeV/c indicates that the simplicity of the angular distributions here is accidental.

The fact that the decay asymmetry parameter for Σ^- is small makes it difficult to measure the hyperon polarization in the production process. Lack of polarization makes a phase-shift analysis difficult. However, it has been shown[16] that by combining the Σ^-K^+ production data with Σ^0K^0 and Σ^+K^+ data, a phase-shift analysis is possible, in principle. A partial-wave analysis was attempted at 1125 MeV/c using Σ^-K^+ data from this experiment, Σ^0K^0 data from the experiment of Binford *et al.*,[12] and Σ^+K^+ data from the experiment of Carayannopoulos *et al.*[15] The analysis was complicated by large errors due to the limited statistics and also the fact that the uncorrected data violate charge independence slightly for backward hyperons in the production c.m. system.[6] Although the analysis is not complete, the results show that the isospin-$\frac{1}{2}$ S-wave ($S_{1/2}$) amplitude is very small. The threshold experiment,[3] on the other hand, shows the S-wave amplitude rising rapidly from threshold (at 1030 MeV/c). Because the $S_{3/2}$ amplitude, as determined by the Σ^+K^+ reaction, is small near threshold,[3] one concludes that the $S_{1/2}$ amplitude rises rapidly from threshold at 1030 MeV/c but has fallen off sharply before 1125 MeV/c. This observation supports the idea of an S_{11} resonance at $\sim$1715 MeV as reported at Vienna.[17] At higher momenta, phase-shift analysis is made difficult further by the presence of D waves; results are not available.

The predominance of forward hyperons throughout this momentum region suggests that the reaction might be dominated by baryon exchange in the u channel. Such a model would require the exchange of neutral baryons with strangeness $= -1$. Barger[18] has shown that the quantitative features of the reaction at 3 BeV/c can be understood using baryon exchange alone. At lower momenta, however, resonances in the s channel are also expected to play a role. The absence of backward hyperons in the c.m. system suggests that meson exchange in the t channel is relatively unimportant. In fact, meson exchange in the Σ^-K^+ reaction would require the exchange of a doubly charged K^*; such a particle has never been detected.

[16] M. L. Good, University of Wisconsin High Energy Physics Notes, No. 38 (unpublished).

[17] Reported by A. Donnachie, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN Information Service, Geneva 23, Switzerland, 1968), p. 142.

[18] V. Barger, Rev. Mod. Phys. **40**, 129 (1968).

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Charge Independence in Σ Hyperon Production*

THOMAS O. BINFORD,† MYRON L. GOOD,‡ AND RICHARD R. KOFLER§
Physics Department, University of Wisconsin, Madison, Wisconsin 53706

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The triangle inequality of charge independence is evaluated with the data of the preceding papers on the reactions $\pi N \to \Sigma K$. In the small-momentum-transfer region, the inequality is satisfied by the data only after allowing for experimental errors. Thus, the fit lies at the edge of the region allowed by charge independence.

IT has been shown[1] that the three reactions

$$\pi^+ + p \to \Sigma^+ + K^+ , \tag{1}$$

$$\pi^- + p \to \Sigma^0 + K^0 , \tag{2}$$

$$\pi^- + p \to \Sigma^- + K^+ \tag{3}$$

have amplitudes $[f^+(\theta), f^0(\theta), f^-(\theta)]$ which are not independent. The assumption of charge independence in strong interactions forces the reaction amplitudes to obey the relationship

$$\sqrt{2} f^0(\theta) + f^-(\theta) = f^+(\theta) , \tag{4}$$

which corresponds to a triangle in the complex plane

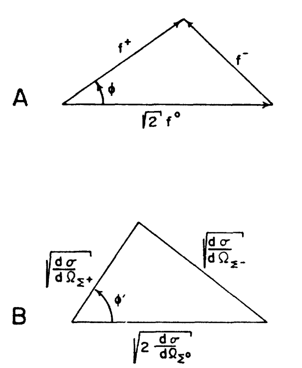

FIG. 1. Charge independence in strong interactions forces the reaction amplitudes for ΣK production to form the triangle (A) in the complex plane. Because the differential cross sections are the squares of amplitudes summed over spin states, the appropriate square roots of the cross sections must also form a closed triangle (B). This leads to three "triangle inequalities" among the cross sections.

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† Present address: Massachusetts Institute of Technology, Cambridge, Mass.

‡ Present address: State University of New York, Stony Brook, N. Y.

§ Present address: University of Massachusetts, Amherst, Mass.

[1] J. J. Sakurai, Phys. Rev. **107**, 908 (1957).

TABLE I. Coefficients in the Legendre polynomial expansion $d\sigma/d\Omega=\sum_n a_nP_n(\cos\theta)$ for the ΣK production differential cross sections.

Pion momentum (BeV/c)	Reaction product	a_0	a_1	a_2	a_3	χ^2	Constraints
1.125	Σ^+K^+	11.5±0.8	11.2±1.4	4.9 ±1.7		2.53	7
	Σ^-K^+	17.4±1.0	1.4±1.4	11.3 ±1.7		1.31	7
	Σ^0K^0	20.8±1.2	0.7±2.7	15.2 ±3.5		6.95	7
1.225	Σ^+K^+	17.1±1.2	11.4±1.7	2.2 ±2.2	−10.8±2.9	6.36	6
	Σ^-K^+	18.7±1.7	11.0±2.0	9.40±2.6	−1.7±3.1	10.34	6
	Σ^0K^0	21.0±2.0	−1.7±3.6	13.5 ±4.4	−12.9±5.1	7.73	6
1.275	Σ^+K^+	21.1±1.6	10.0±2.0	−6.5 ±2.7	−19.4±3.5	7.18	6
	Σ^-K^+	16.5±1.4	15.5±1.6	9.4 ±2.0	−2.3±2.3	3.31	6
	Σ^0K^0	18.2±1.6	−1.2±2.5	9.9 ±3.2	−16.9±3.8	3.50	6

(Fig. 1). Because the differential cross sections are the squares of the amplitudes, this leads to the following "triangle inequalities"[2]:

$$\left(2\frac{d\sigma}{d\Omega}\bigg|_{\Sigma^0}\right)^{1/2}\leq\left(\frac{d\sigma}{d\Omega}\bigg|_{\Sigma^+}\right)^{1/2}+\left(\frac{d\sigma}{d\Omega}\bigg|_{\Sigma^-}\right)^{1/2}, \tag{5}$$

$$\left(\frac{d\sigma}{d\Omega}\bigg|_{\Sigma^+}\right)^{1/2}\leq\left(2\frac{d\sigma}{d\Omega}\bigg|_{\Sigma^0}\right)^{1/2}+\left(\frac{d\sigma}{d\Omega}\bigg|_{\Sigma^-}\right)^{1/2}, \tag{6}$$

$$\left(\frac{d\sigma}{d\Omega}\bigg|_{\Sigma^-}\right)^{1/2}\leq\left(2\frac{d\sigma}{d\Omega}\bigg|_{\Sigma^0}\right)^{1/2}+\left(\frac{d\sigma}{d\Omega}\bigg|_{\Sigma^+}\right)^{1/2}. \tag{7}$$

Previous experimental data for the reactions (1)–(3) have hinted at a possible violation of Eq. (5), especially for hyperons produced backward in the c.m. system.[3–7]

In this paper, we compare recent data[8–10] for the reactions (1)–(3) with the triangle inequality of Eq. (5). The data were obtained from exposures of the LRL 72-in. liquid-hydrogen bubble chamber to incident pion momenta of 1.125, 1.225, and 1.275 BeV/c. In order to smooth the effects of statistical fluctuations in the angular distribution, the data for each reaction at each beam momentum were fitted to a power series of the form

$$\frac{d\sigma}{d\Omega}=\sum_{n=0}^{n_{\max}} a_nP_n(\cos\theta), \tag{8}$$

using the method of least squares. (θ is the hyperon angle relative to the beam direction in the center of mass of the production process.) The resulting Legendre polynomial coefficients and the χ^2 for each fit are presented in Table I. At 1.125 BeV/c, all three charge states require terms only up to $P_2(\cos\theta)$ in the power series, while at the two higher momenta terms up to $P_3(\cos\theta)$ are required. The left and right sides of Eq. (5) were then evaluated at any c.m. production cosine using the power-series representation of the data. Errors were propagated using the full error matrix which resulted from the least-squares fit to the data. The results are shown in the graphs of Fig. 2. It is seen that at all three energies the inequality of Eq. (5) is not violated significantly. The closest the data come to violating charge independence is at 1.125 BeV/c for production angles near 180° (i.e., low four-momentum transfer to the baryon). Here the central values for the cross sections violate the inequality; however, they are within 1.4 standard deviations of satisfying charge independence.[11] We find no violations of the inequalities of Eqs. (6) or (7) using this same approach. We conclude that, within the statistical limitations, the data for ΣK production are consistent with charge independence.

It is interesting to note, however, that for all three incident pion energies presented here, the data come close to violating the inequality of Eq. (5) in the interval $-1.0\leq\cos\theta\leq-0.6$. Presumably, what we have here is a nearly flat triangle (i.e., $\phi^0\approx0°$ in Fig. 1),

[2] Equation (4) holds for each spin state; thus, (5)–(7) hold for each spin state and also, it is easy to show, after summing over spins. Thus, the square roots of the spin-summed differential cross sections must also form a triangle, as in Fig. 1(B).

[3] J. Brown, D. Glaser, D. Meyer, M. Perl, J. Van der Velde, and J. W. Cronin, Phys. Rev. **107**, 906 (1957).

[4] F. S. Crawford, Jr., R. L. Douglass, M. L. Good, G. R. Kalbfleisch, M. L. Stevenson, and H. K. Ticho, Phys. Rev. Letters **3**, 394 (1959).

[5] J. A. Anderson, F. S. Crawford, B. B. Crawford, R. L. Golden, F. Grard, L. J. Lloyd, G. W. Meisner, L. R. Price, and G. A. Smith, in *Proceedings of the International Conference on High Energy Physics, Geneva, 1962*, edited by J. Prentki (CERN, Geneva, 1962), p. 270.

[6] F. S. Crawford, F. Grard, and G. A. Smith, in *Proceedings of the International Conference on High Energy Physics, Geneva, 1962*, edited by J. Prentki (CERN, Geneva, 1962), p. 270.

[7] J. R. Albright, T. O. Binford, U. Camerini, W. F. Fry, M. Foster, M. L. Good, R. Hartung, R. Kofler, V. Lind, R. Matsen, C. Murphy, M. Peters, D. Reeder, G. Tautfest, and R. Willman, in *Proceedings of the International Conference on High Energy Physics, Geneva, 1962*, edited by J. Prentki (CERN, Geneva, 1962), p. 276.

[8] For Σ^+ data: N. L. Carayannopoulos, G. W. Tautfest, and R. B. Willman, Phys. Rev. **138**, B433 (1965).

[9] For Σ^0 data: T. O. Binford *et al.*, second preceding paper, Phys. Rev. **183**, 1134 (1969).

[10] For Σ^- data: M. L. Good and R. R. Kofler, preceding paper, Phys. Rev. **183**, 1142 (1969).

[11] One might worry that the high values for $\alpha_\Lambda P_\Lambda$ observed by Binford *et al.* (Ref. 9) might reflect a detection bias, and hence a loss of Λ's, and by inference a loss of Σ^0's. We would point out, however, that an experimental bias that would prefer left-handed events as opposed to right-handed ones is difficult to invent; and the trouble with the closure of the charge-independence triangle is not a shortage of Σ^0's, but an excess. As for the possibility of losing Λ's into the Σ^0 category, this received a very reassuring check (Ref. 9) by analyzing double V's, in which the K^0 alone identifies the reaction, as single Λ's. This showed the misidentification of the production reaction for single Λ's to be negligible.

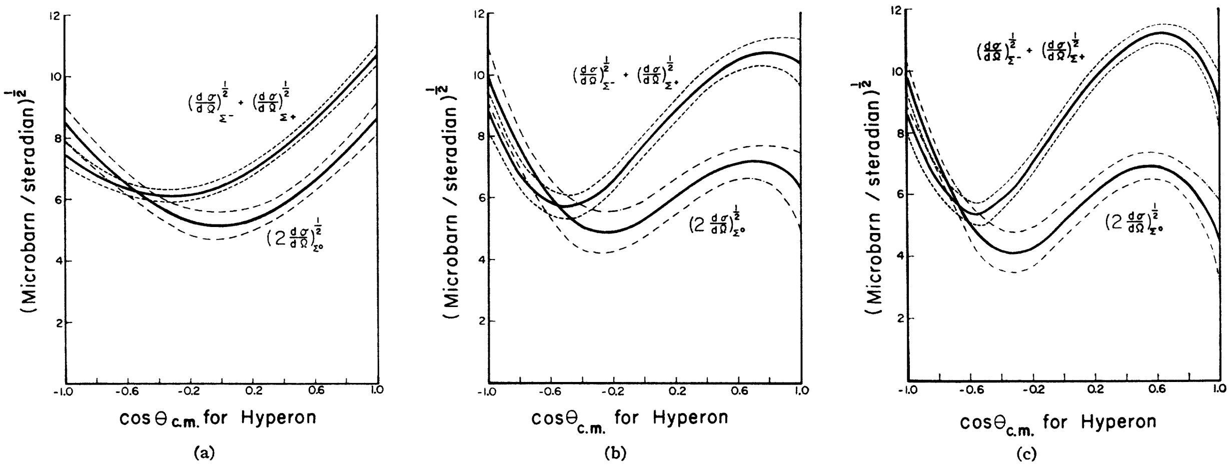

FIG. 2. Graphs (a)–(c) show the test of the "triangle inequality": $(d\sigma/d\Omega)_{\Sigma^+}{}^{1/2}+(d\sigma/d\Omega)_{\Sigma^-}{}^{1/2}\leq(2d\sigma/d\Omega)_{\Sigma^0}{}^{1/2}$ at incident pion momenta of 1.12, 1.23, and 1.27 BeV/c, respectively. The solid lines represent least-squares fits of the data to power series in Legendre polynominals. The dashed lines are the 1-standard-deviation limits on the fits, and were obtained using the full error matrices resulting from the fits.

which in the limiting case leads to equality for Eq. (5) instead of an inequality. Michel has shown[12] that in the limiting case of a "flat triangle," the hyperon polarization must be equal in all three charge states. If the triangle is not quite flat, but the opening angle of the triangle is small, the Michel analysis places limits on the degree of inequality of the hyperon polarizations.

The Σ^+K^+ reaction is the only reaction for which significant polarization data is currently available. Using it alone, the Michel analysis can be applied to set limits on the permissible polarizations in the Σ^0 and Σ^- channels. Because the polarization predictions degenerate rapidly as the triangle opens up, the most interesting place to study the data is the interval $(-1.0\leq\cos\theta\leq-0.6)$ in the 1.125-BeV/c results. The average polarization of the Σ^+ for this pion momentum

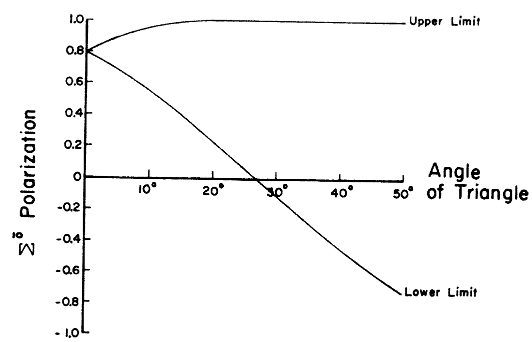

FIG. 3. Limits on the polarization of Σ^- and Σ^0 hyperons are shown as a function of the angle ϕ', the opening angle of the charge-independence triangle. The limits shown are those for a Σ^+ polarization of 0.80.

[12] L. Michel, Nuovo Cimento **22**, 203 (1961).

and production angles is[7]

$$P_{\Sigma^+}\approx0.80.$$

The graph of Fig. 3 shows the limits placed on the Σ^0 and Σ^- polarizations as a function the opening angle of the triangle [ϕ' in Fig. 1(B)] by the Michel analysis. Integrating the differential cross sections over that same region yields the values

$$\sigma_{\Sigma^+}(-1.0\leq\cos\theta\leq-0.6)=12.3\pm2.5\ \mu\text{b}, \tag{9a}$$

$$\sigma_{\Sigma^0}(-1.0\leq\cos\theta\leq-0.6)=69.1+7.5\ \mu\text{b}, \tag{9b}$$

$$\sigma_{\Sigma^-}(-1.0\leq\cos\theta\leq-0.6)=53.6\pm4.0\ \mu\text{b}. \tag{9c}$$

Denoting these values by $\langle\sigma\rangle$, we thus have

$$\langle\sigma_{\Sigma^+}\rangle^{1/2}=\ 3.51\pm0.36\ (\mu\text{b})^{1/2}, \tag{10a}$$

$$\langle2\sigma_{\Sigma^0}\rangle^{1/2}=11.76+0.64\ (\mu\text{b})^{1/2}, \tag{10b}$$

$$\langle\sigma_{\Sigma^-}\rangle^{1/2}=\ 7.32\pm0.27\ (\mu\text{b})^{1/2}. \tag{10c}$$

These values violate the "triangle inequality" of Eq. (5) somewhat, as we expected. If we "stretch" the experimentally obtained amplitudes within errors so as to minimize χ^2 while satisfying Eq. (5), then charge independence is barely satisfied and the triangle is flat. The best values for a flat triangle are then

$$\langle\sigma_+\rangle_{\text{flat}}{}^{1/2}=\ 3.69, \tag{11a}$$

$$\langle2\sigma_0\rangle_{\text{flat}}{}^{1/2}=11.12, \tag{11b}$$

$$\langle\sigma_-\rangle_{\text{flat}}{}^{1/2}=\ 7.43. \tag{11c}$$

The probability that the experimental values [Eqs. (10)] violate Eq. (5) by as much as they do if the triangle is actually flat is

$$P_{\text{flat}}=0.13,$$

corresponding to a lack of closure by 1.11 standard deviations. If we now stretch the data further, say twice as far as above on each of the amplitudes, then the triangle opens up and we have the values

$$\langle\sigma_+\rangle_{\text{open}}^{1/2} = 3.87\,, \tag{12a}$$

$$\langle 2\sigma_0\rangle_{\text{open}}^{1/2} = 10.48\,, \tag{12b}$$

$$\langle\sigma_-\rangle_{\text{open}}^{1/2} = 7.59\,. \tag{12c}$$

Similarly, the probability that the experimental values [Eqs. (10)] are consistent with these "open" values [Eqs. (12)] is $P_{\text{open}}=0.013$ (2.22 std. dev.). The values of Eqs. (12) yield an opening angle of $\phi'=34°$ for the triangle of Fig. 1(B). The probability that the experimental values from Eqs. (10) are consistent with the "open" values of Eqs. (12), subject to the condition that charge independence is not violated, is given by

$$P_{\text{open, CI good}} = P_{\text{open}}/P_{\text{flat}} = 0.10\,.$$

It is accordingly 90% probable that the opening angle of the triangle is less than 34°. The graph of Fig. 3 shows that one can say very little about the Σ^- and Σ^0 polarizations from the Michel analysis except that the polarizations are "probably" greater than -0.2. If it could be shown conclusively that the opening angle of the triangle were less than, say, 15°, then more restrictive limits could be placed on the Σ^- and Σ^0 polarizations. Experimentally the Σ^- polarization cannot be measured via the usual decay asymmetry method, since the Σ^- does not appreciably mix parity states in its decay. Measurement of the Σ^0 polarization is complicated by its electromagnetic decay $\Sigma^0\rightarrow\Lambda^0+\gamma$. At 1.17 BeV/$c$ (slightly different from the 1.125 BeV/c we have analyzed here), the Σ^0 polarization measured by Crawford *et al.*[13] is $\bar{P}_\Sigma\approx+0.6\pm0.5$ for $-1.0\leq\cos\theta\leq-0.6$. This value, although not statistically conclusive, is consistent with the charge-independence limits shown in Fig. 3.

Finally, we want to draw attention to the fact that the flatness of the triangle extends over a considerable range of energy.

The reactions (1)–(3) have been studied near threshold.[14] The angular distributions were found to be consistent with pure S wave. The charge-independence triangle for the total cross sections was found to be "flat," and in the same sense as here, i.e., Eq. (5) (with σ_t instead of $d\sigma/d\Omega$) is barely satisfied.

Doyle, Crawford, and Anderson[15] have published a test of the triangle at 1170 MeV/c, and also find it flat in the backward direction.

Thus, one can say that from threshold up to 1275 MeV/c, where at least D waves are present, the triangle is flat in the small-momentum-transfer direction. At higher energies, 1.59 GeV/c and above, it is known that the backward Σ^- production is nearly zero ($d\sigma/d\Omega\leq2$ μb/sr, $-1<\cos\theta<-0.8$),[16] while the Σ^+ and Σ^0 production show a large backward peak (typically 60 μb/sr for Σ^+),[17] due presumably to exchange of strange mesons such as $K^*(890)$ and $K^*(1400)$.

Inspection of Eqs. (4) and (5) shows that in these circumstances ($|f^-|\approx0$) one has $\sqrt{2}f_0\approx f_+$ and so Eq. (5) will again be barely satisfied, i.e., the triangle will again be flat, with $\phi'\leq\tan^{-1}[(2\,\mu\text{b})^{1/2}/(60\,\mu\text{b})^{1/2}]\approx10°$.

Dominance of K^* exchange cannot be the reason for flatness of the triangle at low momenta, since the backward Σ^- hyperon production, which cannot occur by K^* exchange, is quite large. Also 1.6 GeV/c, where the ward Σ^- cross section is already small ($\sim2\,\mu$b/sr, Goussu *et al.*[16]), is less than one half-width above the center of the $\Delta(1920)$ resonance, so that one would hardly expect complete K^* exchange dominance.

The physical significance of the flatness of the triangle is that the two isospin amplitudes are relatively real; and, for Eq. (5), in particular, to be barely satisfied, they must differ in phase by $\sim180°$. The data show that in the small-momentum-transfer direction, this condition is approximately fulfilled wherever measurements have been made. This includes threshold (1030 MeV/c), closely spaced energies (1125, 1170, 1225, 1275 MeV/c) up to where at least D waves contribute, a point (1590 MeV/c) in the vicinity of the $\Delta(1920)$ resonance, and the entire region above 1590 MeV/c. Only in the last-mentioned region is there an obvious explanation, in terms of K^* exchange.

Whether this curious behavior is an accident, or a sign of some underlying simplicity, we do not know; however, it is extremely interesting to note that the charge-independence triangle for π-nucleon elastic and charge-exchange scattering is also flat over the same energy region and in the same angular region, i.e., for forward mesons. This has been pointed out by Tornqvist.[18]

[13] J. A. Anderson, F. S. Crawford, Jr., and J. C. Doyle, University of California Radiation Laboratory Report No. UCRL 16861, 1965 (unpublished).

[14] F. Crawford, F. Grard, and G. Smith, in *Proceedings of the International Conference on High Energy Physics, Geneva, 1962*, edited by J. Prentki (CERN, Geneva, 1962), p. 270.

[15] J. C. Doyle, F. S. Crawford, Jr., and J. A. Anderson, Phys. Rev. **165**, 1483 (1968).

[16] J. Schwartz, Ph.D. thesis, Berkeley (unpublished); T. Wangler, A. Erwin, and W. Walker, Phys. Rev. **137**, B414 (1965); O. Goussu *et al.*, Nuovo Cimento **42A**, 606 (1966).

[17] For example, R. Kofler, R. Hartung, and D. Reeder, in *Proceedings of the Thirteenth International Conference on High Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967); T. Wangler *et al.*, Ref. 14; P. Daronian *et al.*, Nuovo Cimento **41A**, 503 (1966).

[18] N. Tornqvist, Phys. Rev. **161**, 1591 (1967), see especially **Fig.** 15.