is a peak at 1950-MeV c.m. with a width of about 200-MeV c.m., corresponding to $\Delta_{1920}(\frac{7}{2}^+)$. Any influence from the $I=\frac{3}{2}$ resonance $\Delta_{2420}(\frac{11}{2}^+)$ is much smaller than the peak at 1950 MeV. The Σ^+K^+ data at high energies are too few to give more than a hint of the energy dependence. The near equality of $I=\frac{1}{2}$ ΛK and ΣK cross sections might suggest that the energy behavior of the $I=\frac{1}{2}$ ΣK channel is also dominated by resonances, but the strong backward peaking of Λ and, at higher energies of Σ^+, certainly suggests a strong K^* exchange contribution.

It is probable that no one mechanism is dominant.

A comparison of the data presented in this paper with that of Kofler[1] on $\pi^-p \rightarrow \Sigma^-K^+$ and Carayannopoulos *et al.*[30] on $\pi^+p \rightarrow \Sigma^+K^+$, to test the triangle inequalities of charge independence, has been made.[33]

ACKNOWLEDGMENTS

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[33] T. O. Binford, R. R. Kofler, and M. L. Good, second following paper, Phys. Rev. 183, 1148 (1969).

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Study of $\pi^-p \rightarrow \Sigma^-K^+$ from 1.12 to 1.32 BeV/c*

Myron L. Good† and Richard R. Kofler‡
University of Wisconsin, Madison, Wisconsin 53706
(Received 6 November 1968)

Total cross sections, angular distributions, and decay asymmetries have been measured for the reaction $\pi^-p \rightarrow \Sigma^-K^+$ at pion beam momenta of 1.12, 1.23, 1.27, and 1.32 BeV/c. Legendre-polynomial expansion coefficients for the angular distributions are presented and compared with other data. Preliminary results of a partial-wave analysis are discussed.

I. INTRODUCTION

THE purpose of this paper is to present data obtained in a study of the reaction $\pi^-+p \rightarrow \Sigma^-+K^+$ at incident pion momenta of 1128, 1235, 1277, and 1325 MeV/c. At each of these momenta, total cross sections, angular distributions, and Σ^--decay asymmetries have been measured. The data are compared with other data[1–5] at different beam momenta. The data have already been combined with Σ^0 and Σ^+ data to test the charge independence hypothesis in strong interactions.[6] It is further hoped that these data, when combined with data from similar experiments, will aid in understanding those processes responsible for hyperon production.

II. EXPERIMENTAL DETAILS

A. General Procedure

The Σ^-K^+ events were produced in the LRL 72-in. liquid-hydrogen bubble chamber. The pion-beam transport system has been described in the literature.[2] A total of 140 000 triad photographs were used for this experiment. Scanners were instructed to search the photographs for any event which had the typical appearance of a Σ^-K^+ production: a two-prong topology with a kink in the negative track corresponding to the decay of the Σ^- hyperon,

$$\Sigma^- \rightarrow n+\pi^-. \quad (1)$$

The events found as a result of two complete scans of the film were measured on digitized microscopes and the measurements were analyzed using the PANG and KICK programs.[7] The scanning, measuring, and computing process produced 2325 Σ^-K^+ events. In order to maintain consistency between the sample of events and the

* Work supported in part by the U. S. Atomic Energy Commission.

† Present address: State University of New York, Stony Brook, N. Y.
‡ Present address: University of Massachusetts, Amherst, Mass.

[1] J. Steinberger, in *Proceedings of the 1958 International Conference on High Energy Physics* (CERN Scientific Information Service, Geneva, 1958), p. 148.

[2] S. E. Wolf, N. Schmitz, L. J. Lloyd, W. Laskar, F. S. Crawford, Jr., J. Button, J. A. Anderson, and G. Alexander, Rev. Mod. Phys. 33, 439 (1961).

[3] F. S. Crawford, Jr., in *Proceedings of 1962 International Conference on High Energy Physics*, edited by J. Prentki (CERN, Geneva, 1962), p. 270.

[4] J. A. Schwartz, University of California Radiation Laboratory Report No. UCRL 11360, 1964 (unpublished).

[5] J. A. Anderson, F. S. Crawford, Jr., and J. C. Doyle, University of California Radiation Laboratory Report No. UCRL 16861, 1965 (unpublished); Bull. Am. Phys. Soc. 10, 467 (1965).

[6] T. O. Binford, M. L. Good, and R. R. Kofler, following paper, Phys. Rev. 183, 1148 (1969).

[7] A general discussion of bubble-chamber film analysis, with specific reference to the PANG–KICK system called PACKAGE, can be found in A. H. Rosenfeld and W. E. Humphrey, Ann. Rev. Nucl. Sci. 13, 103 (1963).

total-track-length determination, and also in order to reduce biases, it was found necessary to impose the following selection criteria:

(1) Film edit: All film considered by a physicist to be unusable was discarded from the experiment.

(2) Angle tests: The beam track for each event was required to agree with nominal values within 3 standard deviations.

(3) Momentum test: The beam track momentum for each event was required to agree with the nominal values.

(4) Thin-window test: The beam track for each event was required to have entered the chamber through the thin entrance window.

(5) Fiducial volume tests: Both the production and the decay vertices were required to be within a fiducial volume of the chamber. The fiducial volume for the decay vertex was chosen larger in order to keep escape corrections finite.

(6) Σ-length test: To alleviate the obvious bias against short Σ tracks, the Σ^- track for each event was required to be longer than $L_{\min}=6$ mm.

(7) Σ-decay angle: The space angle between the Σ^- hyperon and its daughter π^- was required to be greater than 12.5°. The above tests 2–7 were made by computer, and an event was rejected if it failed any one of them. There were no detectable biases for either production-plane or decay-plane orientations in the chamber.

B. Estimate of Number of Events

Corrections for selection criteria 1–4 did not have to be made to the event sample because the determination of total pion track length incorporated these criteria also. Corrections for selection criteria 5 and 6 were made by calculating for each event the probability that the Σ^- track would have a length between $L_{\min}$ and $L_{\max}$, where $L_{\min}=6$ mm and $L_{\max}$ was the projected length along the Σ trajectory to the limits of the decay fiducial volume. This probability is given by

$$P_L=\frac{1}{\tau}\int_{t_{min}}^{t_{\max}} e^{-t/\tau}dt\,, \tag{2}$$

where τ is the Σ^- mean lifetime and $t_{\min}$ and $t_{\max}$ are the proper times in the hyperon rest frame corresponding to the $L_{\min}$ and $L_{\max}$. In general, this time is given by

$$t=mL/pc. \tag{3}$$

Here m is the hyperon rest mass, L is the hyperon length, p is the hyperon momentum in the laboratory rest frame, and c is the velocity of light. The probability for each event P_{Li} then becomes

$$P_{Li}=\exp\left(-\frac{mL_{\min}}{p_ic}\right)-\exp\left(-\frac{mL_{\max\,i}}{p_ic}\right). \tag{4}$$

Next a correction was made for the decay-angle criterion (No. 7 above) by calculating for each event the probability that the hyperon would eject its daughter pion at an angle greater than 12.5° relative to the Σ direction in the laboratory rest frame. This was done by assuming that the decay distribution was isotropic in the hyperon rest frame. The decay-angle correction P_{Ai} was also dependent on the hyperon laboratory momentum. The total "detection probability" was taken as $P_i=P_{Ai}P_{Li}$, and hence the worth or weight factor for each event which passed all the above tests was

$$w_i=1/P_{Ai}P_{Li}. \tag{5}$$

C. Total-Track-Length Determination

Determination of the total track length is, in principle, easy to perform. If one knows the number of frames, N_F, the average number of tracks per frame, N_T, and the average length per track, $\bar{l}$, then the total track length is given simply by

$$L=N_FN_T\bar{l}. \tag{6}$$

1. Number of Frames, N_F

The number of nonblank frames surviving the film-edit criteria is listed in Table I for each beam momentum.

2. Average Track Count N_T

In order to determine the average number of tracks per picture, beam tracks were counted in a random subsample of the film. To facilitate the counting, templates were made for use at the scanning table (Fig. 1). In order to be counted, a beam track had to pass the bottom scale within left and right limits. The

TABLE I. Summary of total-track-length determination.

Nominal momentum (MeV/c)	Actual momentum (MeV/c)	Usable non-blank frames N_F	Av number tracks per frame N_T	Av track length $\bar{l}$ (cm)	Total pion track length L_π (cm)
1125	1128±10	48 820	16.0 ±0.14	109.00±0.37	$(7.62\pm0.15)\times10^7$
1225	1235±9	17 699	17.45±0.23	118.33±0.30	$(3.25\pm0.08)\times10^7$
1275	1284±15	26 193	6.71±0.16	120.25±0.37	$(3.96\pm0.09)\times10^7$
	1284±14	5 296	8.61±0.39	120.41±0.17	
	1266±15	11 419	13.38±0.25	118.88±0.58	
1325	1326±0	24 470	8.62±0.17	118.84±0.54	$(2.23\pm0.07)\times10^7$

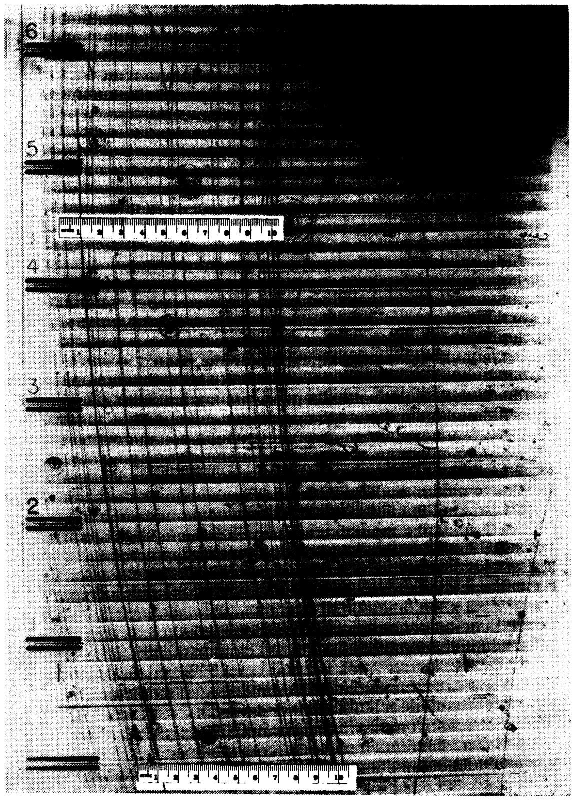


FIG. 1. An example of the beam templates used for counting beam tracks on the scanning table. Beam tracks were counted at the bottom scale. Tracks intersecting the two scales at different readings were not counted if the discrepancy was greater than a preset tolerance. The upper scale was expanded slightly to properly account for the angular dispersion of the beam across the chamber.

left limit was imposed in order t o insure that the beam track remains in the visible region of the chamber, and the right limit to insure that the beam track passed within the rightmost limits of the chamber entrance window. Beam tracks were furthermore required to intersect the top scale at a reading within a preset tolerance of the reading on the lower scale. The latter requirement assured that counted tracks were close to the correct angles for beam tracks. The upper scale was expanded by an appropriate amount to take into account the angular dispersion of the beam. All Σ^-K^+ events were required to have beam tracks which passed the same template test. In addition to counting tracks in a random subsample of the film, track counts were made in all frames which contained an event. This is a biased sample; however, the unbiased average number of beam tracks per frame can be calculated from the biased sample.[8] The final average track count was obtained by taking the weighted average of these two independent determinations. The results are found in Table I.

[8] F. S. Crawford, Jr., Rev. Sci. Instr. 30, 1096 (1959).

3. *Average Track Length* $\bar{l}$

The average track length, corrected for attenuation due to all beam interactions, was found by using

$$\bar{l}=(1/\sigma\eta)(e^{-\sigma\eta s_1}-e^{-\sigma\eta s_2}), \qquad (7)$$

where σ is the total π^- interaction cross section obtained from the literature,[9,10] η is the number of nucleons per unit volume, s_1 and s_2 are the distances from the position at which tracks were counted to the positions at which tracks entered and left the fiducial volume, respectively.

4. *Contamination Estimates*

A correction to the track count had to be made for muon and electron contamination in the beam. The method of Δ-ray analysis[11] was used, and an extensive study was made in the 1125-MeV/c film. The combined muon and electron contamination was found to be $f_c=(7.6\pm1.5)\%$. Estimates of the contamination at the higher momenta were made by scaling the value at 1125 MeV/c inversely with the beam momentum. Errors introduced by this scaling were expected to be small compared with the statistical limitations of the data.

5. *Nonbeam Correction*

A final correction had to be made to the track count to correct for the fact that a small fraction f_R of tracks which were counted on the template would have been rejected by the more stringent computer tests for beam momentum, beam angles, and chamber window entrance. This fraction was determined by measuring a sample of beam tracks and applying the computer-made criteria to them. The value obtained was $f_R\approx0.05$ in most of the film.

The total corrected pion track length was then found by

$$L_\pi=N_FN_T\bar{l}(1-f_c)(1-f_R). \qquad (8)$$

The results are tabulated in Table I.

The scanning biases and analysis problems are essentially different in the topologies of this reaction and in the Σ^0K^0 reaction studied in the same film[12]; hence

TABLE II. Results for the $\pi^-p\to\Sigma^-K^+$ cross section.

Nominal momentum MeV/c	Number of events	Sum of weight factors	Total cross section (μb)
1125	465	597.3±27.7	218±11
1225	216	275.1±18.7	235±17
1275	237	297.5±19.3	209±14
1325	156	197.2±15.8	245±21

[9] J. Brisson, J. Detoeuf, P. Falk Vairant, L. Van Rossum, and G. Valladas, Nuovo Cimento 19, 210 (1961).

[10] P. Falk Vairant and G. Valladas, Rev. Mod. Phys. 33, 362 (1961).

[11] The method was developed and described by F. S. Crawford, Jr., in Lawrence Radiation Laboratory Memo No. 165, 1960 (unpublished).

[12] T. O. Binford, M. L. Good, V. G. Lind, D. Stern, R. Krauss, and E. Dettman, preceding paper, Phys. Rev. 183, 1134 (1969).

the treatment of track and frame criteria was not the same for both. For example, the exclusion of satellite beam tracks and frames with too many tracks was not felt necessary for studying the $\Sigma^- K^+$ reaction, for which the tracks are all connected and thus the scanning is easier. Both procedures were, of course, an attempt to yield the correct final cross sections, and we believe the final results are independent of these procedural choices.

III. RESULTS

A. Total Cross Sections

The total number of events used at each momentum and the sum of the weight factors are listed in Table II together with the resulting cross-section values at each momentum. The sums of weight factors were corrected for an over-all scanning efficiency of 97%. Cross sections were calculated according to

$$\sigma = WA/\rho N_0 L_\pi , \tag{9}$$

where W is the sum of the weight factors (listed in Table II), A is the atomic weight of hydrogen (1.008), ρ is the density of liquid hydrogen (0.0598 g/cm^3),[13] N_0 is Avogadro's number, and L_π is the total pion track length (listed in Table I).

B. Angular Distributions

Differential cross sections were also calculated and results are displayed in the graphs of Fig. 2. The angular

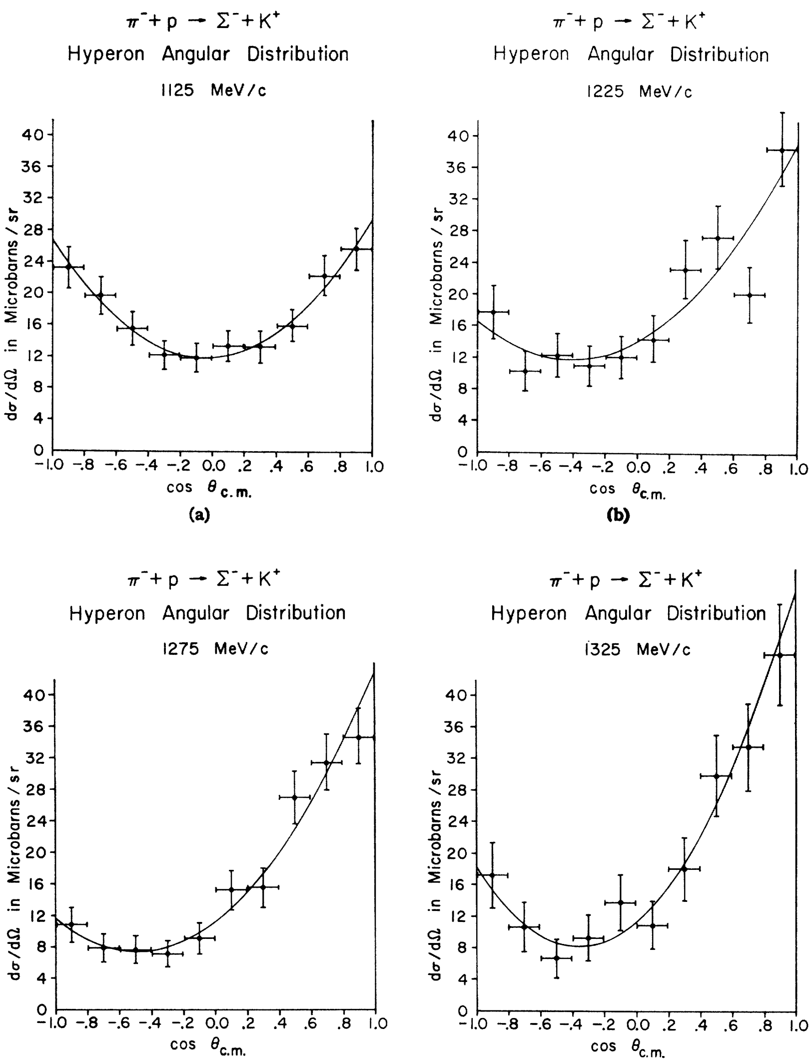


FIG. 2. Angular distributions in the $\pi^- p$ c.m. system. Cos$\theta_{c.m.}$ is the cosine of the angle between the outgoing hyperon and the incoming pion. The solid lines are the results of least-squares fit of the data to a power series of the form $d\sigma/d\Omega = \sum_{n=0}^{4} A_n P_n(\cos\theta)$, where the P_n are Legendre polynomials.

[13] J. A. Anderson (private communication).

TABLE III. Legendre coefficients for $d\sigma/d\Omega = \sum_{n=0}^{n_{max}} A_n P_n(\cos\theta)$. (Units are μb.)

Nominal momentum (MeV/c)	A_0	A_1	A_2	A_3	A_4	χ^2
1125	17.3±0.9	1.5±1.3	10.7±1.7	0.0±2.0	−0.6±2.5	1.26
1225	18.7±1.4	10.8±2.1	9.7±2.8	−1.8±3.4	4.3±3.8	9.05
1275	16.6±1.1	15.3±1.6	9.6±2.1	−2.8±2.4	−2.0±2.8	2.79
1325	19.5±1.7	17.0±2.7	16.3±3.5	−1.2±4.0	2.2±4.7	3.08

distributions were fitted by the method of least squares to a power series in Legendre polynomials

$$\frac{d\sigma}{d\Omega} = \sum_{n=0}^{n_{max}} A_n P_n(\cos\theta). \qquad (10)$$

At all beam energies used in this experiment, it was found that a good fit to the data could be obtained by keeping terms only as high as $n_{max} = 2$. The fits do not improve appreciably by including higher-order terms. However, there is evidence from the reaction $\pi^+ p \rightarrow$

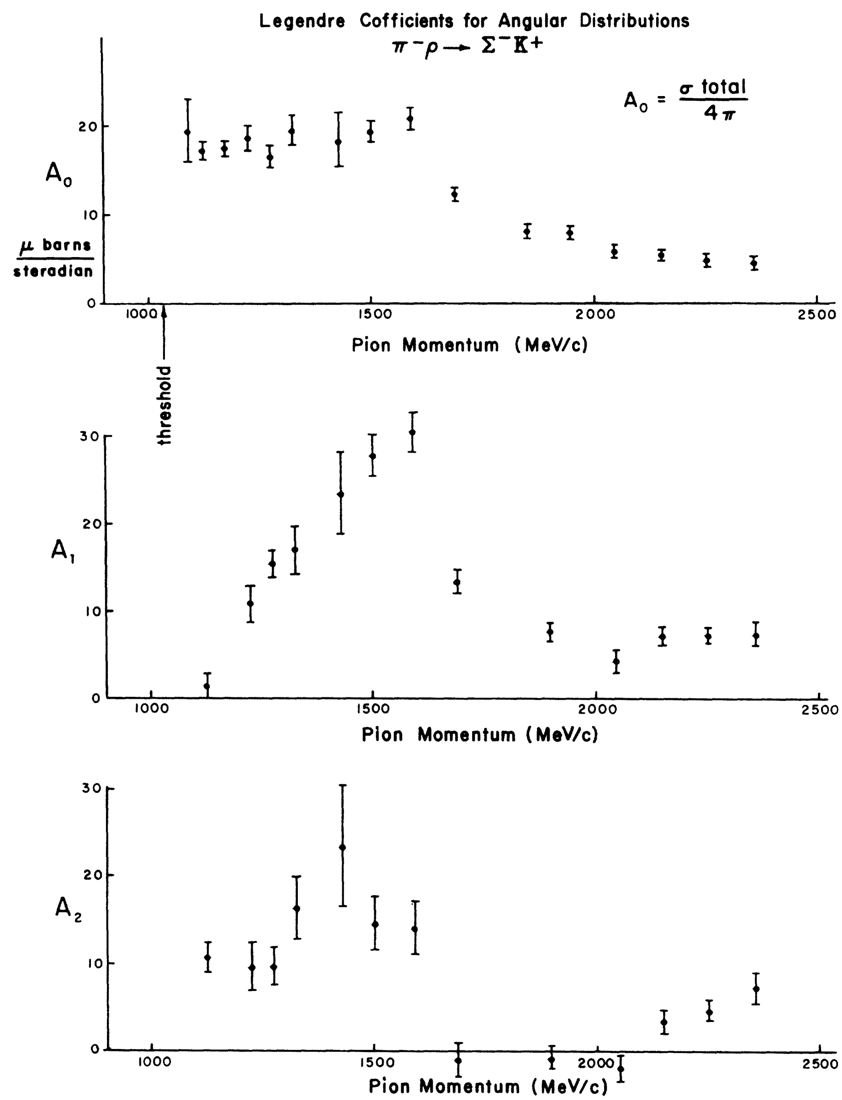


FIG. 3. Legendre polynomial coefficients versus beam momentum. The results of this, as well as other experiments (Refs. 1–5), are shown.

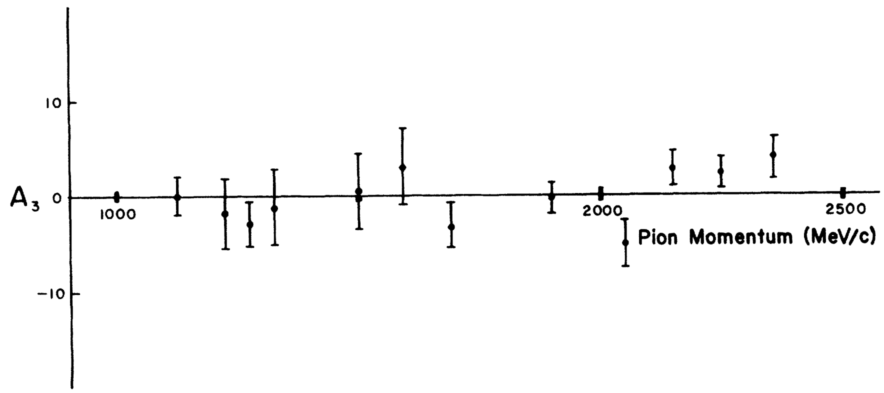
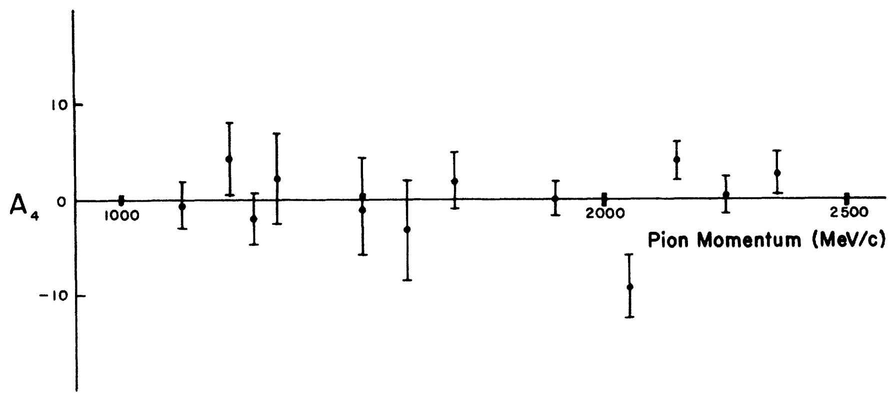


FIG. 3 (*continued*).



Σ^+K^+ at these same energies[14,15] that at least D waves are required. The Σ^+K^+ channel has a reaction amplitude which is pure isospin $\frac{3}{2}$. Since Σ^-K^+ production has a reaction amplitude which is part isospin $\frac{1}{2}$ and part isospin $\frac{3}{2}$, D waves are also expected here. Accordingly, we have kept terms in the power series up to $n_{max}=4$. The results are listed in Table III.

C. Hyperon-Decay Asymmetry

The hyperon-decay distribution has the form

$$I(\phi)d\phi=\tfrac{1}{2}(1+\alpha\langle P\rangle\cos\phi)d\phi\,. \qquad (11)$$

Here, ϕ is the angle between the decay pion and the normal to the hyperon-production plane in the hyperon rest frame, α is the usual hyperon-decay asymmetry parameter, and $\langle P\rangle$ is the average polarization of the parent hyperon. The product $\alpha\langle P\rangle$ was measured according to

$$\alpha\langle P\rangle\pm\delta(\alpha\langle P\rangle)=3\langle\cos\phi\rangle\pm\left(\frac{3-\langle\cos\phi\rangle}{N}\right)^{1/2}, \qquad (12)$$

where the angular brackets indicate an average, and N is the total number of events in the measurement sample.

The results obtained were

Beam momentum (MeV/c)	$\alpha\langle P\rangle\pm\delta(\alpha\langle P\rangle)$
1125	-0.051 ± 0.058
1225	0.021 ± 0.086
1275	-0.114 ± 0.067
1325	-0.096 ± 0.099

IV. DISCUSSION

Figure 3 displays the coefficients A_n in the Legendre polynomial fits to the angular distributions. The results are plotted for this experiment along with data from other experiments (Refs. 1–5) in the region from threshold to 2.5 BeV/c. The coefficient A_0 is related to the total cross section as shown, hence, the behavior of A_0 gives the momentum dependence of the total cross section. The cross section rises rapidly[3] from threshold to a plateau value $\sigma\approx220\,\mu$b and gradually diminishes above 1.5 BeV/c. Within the rather limited data, the total cross-section behavior is almost featureless. The term A_1 shows a rise from threshold to a maximum value at 1.59 BeV/c followed by a decrease. That A_1 is always positive is a reflection of the fact that the hyperon travels preferentially forward in the production c.m. system. Within errors, the terms A_3 and A_4 are zero throughout this momentum region. It would be tempting to conclude that D waves and higher do not contribute to the reaction amplitude; however, the

[14] F. S. Crawford, Jr., F. Grard, and G. A. Smith, Phys. Rev. **128**, 368 (1962).

[15] N. L. Carayannopoulos, G. W. Tautfest, and R. B. Willmann, Phys. Rev. **138**, B433 (1965).

presence[14] of D waves in the reaction $\pi^+p \rightarrow \Sigma^+K^+$ (pure isospin-$\frac{3}{2}$) as low as 1170 MeV/c indicates that the simplicity of the angular distributions here is accidental.

The fact that the decay asymmetry parameter for Σ^- is small makes it difficult to measure the hyperon polarization in the production process. Lack of polarization makes a phase-shift analysis difficult. However, it has been shown[16] that by combining the Σ^-K^+ production data with Σ^0K^0 and Σ^+K^+ data, a phase-shift analysis is possible, in principle. A partial-wave analysis was attempted at 1125 MeV/c using Σ^-K^+ data from this experiment, Σ^0K^0 data from the experiment of Binford *et al.*,[12] and Σ^+K^+ data from the experiment of Carayannopoulos *et al.*[15] The analysis was complicated by large errors due to the limited statistics and also the fact that the uncorrected data violate charge independence slightly for backward hyperons in the production c.m. system.[6] Although the analysis is not complete, the results show that the isospin-$\frac{1}{2}$ S-wave ($S_{1/2}$) amplitude is very small. The threshold experiment,[3] on the other hand, shows the S-wave amplitude rising rapidly from threshold (at 1030 MeV/c). Because the $S_{3/2}$ amplitude, as determined by the Σ^+K^+ reaction, is small near threshold,[3] one concludes that the $S_{1/2}$ amplitude rises rapidly from threshold at 1030 MeV/c but has fallen off sharply before 1125 MeV/c. This observation supports the idea of an S_{11} resonance at $\sim$1715 MeV as reported at Vienna.[17] At higher momenta, phase-shift analysis is made difficult further by the presence of D waves; results are not available.

The predominance of forward hyperons throughout this momentum region suggests that the reaction might be dominated by baryon exchange in the u channel. Such a model would require the exchange of neutral baryons with strangeness$=-1$. Barger[18] has shown that the quantitative features of the reaction at 3 BeV/c can be understood using baryon exchange alone. At lower momenta, however, resonances in the s channel are also expected to play a role. The absence of backward hyperons in the c.m. system suggests that meson exchange in the t channel is relatively unimportant. In fact, meson exchange in the Σ^-K^+ reaction would require the exchange of a doubly charged K^*; such a particle has never been detected.

[16] M. L. Good, University of Wisconsin High Energy Physics Notes, No. 38 (unpublished).

[17] Reported by A. Donnachie, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968* (CERN Information Service, Geneva 23, Switzerland, 1968), p. 142.

[18] V. Barger, Rev. Mod. Phys. **40**, 129 (1968).

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Charge Independence in Σ Hyperon Production*

Thomas O. Binford,† Myron L. Good,‡ and Richard R. Kofler§
Physics Department, University of Wisconsin, Madison, Wisconsin 53706
(Received 6 November 1968)

The triangle inequality of charge independence is evaluated with the data of the preceding papers on the reactions $\pi N \rightarrow \Sigma K$. In the small-momentum-transfer region, the inequality is satisfied by the data only after allowing for experimental errors. Thus, the fit lies at the edge of the region allowed by charge independence.

It has been shown[1] that the three reactions

$$\pi^+ + p \rightarrow \Sigma^+ + K^+, \qquad (1)$$

$$\pi^- + p \rightarrow \Sigma^0 + K^0, \qquad (2)$$

$$\pi^- + p \rightarrow \Sigma^- + K^+ \qquad (3)$$

have amplitudes $[f^+(\theta), f^0(\theta), f^-(\theta)]$ which are not independent. The assumption of charge independence in strong interactions forces the reaction amplitudes to obey the relationship

$$\sqrt{2}f^0(\theta) + f^-(\theta) = f^+(\theta), \qquad (4)$$

which corresponds to a triangle in the complex plane

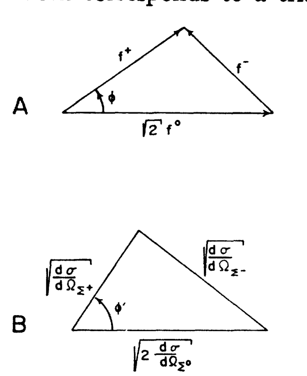

FIG. 1. Charge independence in strong interactions forces the reaction amplitudes for ΣK production to form the triangle (A) in the complex plane. Because the differential cross sections are the squares of amplitudes summed over spin states, the appropriate square roots of the cross sections must also form a closed triangle (B). This leads to three "triangle inequalities" among the cross sections.

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† Present address: Massachusetts Institute of Technology, Cambridge, Mass.

‡ Present address: State University of New York, Stony Brook, N. Y.

§ Present address: University of Massachusetts, Amherst, Mass.

[1] J. J. Sakurai, Phys. Rev. **107**, 908 (1957).

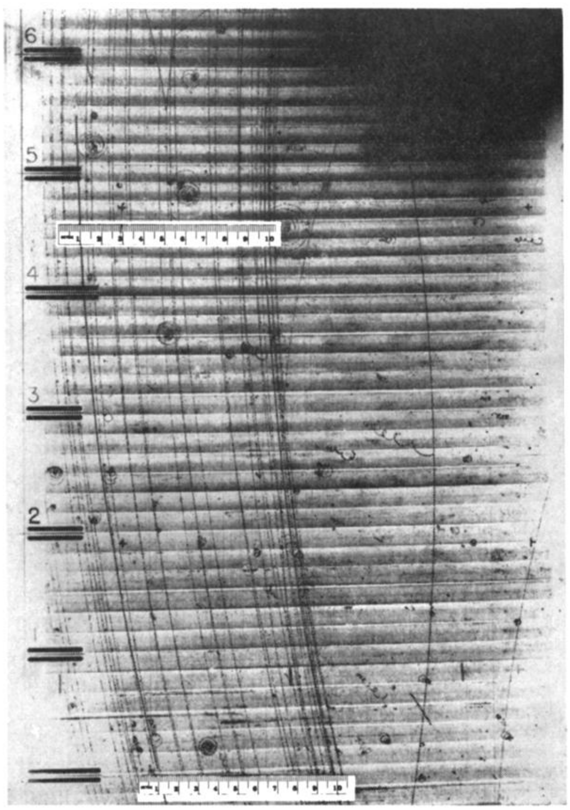

FIG. 1. An example of the beam templates used for counting beam tracks on the scanning table. Beam tracks were counted at the bottom scale. Tracks intersecting the two scales at different readings were not counted if the discrepancy was greater than a preset tolerance. The upper scale was expanded slightly to properly account for the angular dispersion of the beam across the chamber.