Production of Λ^0 and Σ^0 Hyperons by Pions of Beam Momenta 1.12–1.32 BeV/c*†

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(Received 6 November 1968)

Total cross sections, angular distributions, and polarization distributions have been measured for the reactions $\pi^- p \to \Lambda^0 K^0$ and $\pi^- p \to \Sigma^0 K^0$ at beam momenta of 1.12, 1.23, 1.27, and 1.32 BeV/c. Legendre-polynomical expansion coefficients for angular distributions and polarization distributions are presented and compared with other data. Two approximate cross-section equalities are demonstrated among $I=\frac{1}{2}$ total cross sections over a wide range of energies: $\sigma_{I=1/2}(\Sigma K) \cong \sigma(\Lambda K)$, and $\sigma(\eta N) \cong \sigma(\Lambda K) + \sigma_{I=1/2}(\Sigma K)$ where $\sigma(\eta N)$ is the total cross section for $\pi^- p \to \eta n$. Hyperon decay asymmetries observed in the reaction $\pi^- p \to \Lambda K^0$ are difficult to reconcile with a value of $\alpha_\Lambda = 0.66$.

RESULTS of a study of the reactions $\pi^- + p \to \Lambda^0 + K^0$ and $\pi^- + p \to \Sigma^0 + K^0$ in the LRL Alvarez 72-in. hydrogen bubble chamber for beam momenta of 1128, 1235, 1277, and 1326 MeV/c are reported here. The reaction $\pi^- p \to \Sigma^- K^+$ in the same film has been measured by Kofler.[1] The beam was designed by Crawford.[2] Beam momentum width was $\pm 1\%$ and each momentum interval represents an average over approximately ± 20-MeV/c incident momentum because of beam energy loss in the chamber.

Film was scanned twice for the neutral V decays $\Lambda^0 \to p + \pi^-$ and $K^0 \to \pi^+ + \pi^-$, with an average scanning efficiency of 97% for each scan (minimum neutral track length 0.2 cm projected, approximately 0.35 cm in space). Track reconstruction, geometrical cutoffs, and kinematic fitting were done within the PANG-KICK system.[3] Strange-particle decays were rejected under the following conditions:

(1) The decay vertex lay outside a fiducial volume selected for decays.

(2) The interaction occurred outside an interaction fiducial volume (slightly smaller than the decay volume).

(3) The neutral track length was less than 0.7 cm in space.

(4) The opening angle of the V was less than 3° or greater than 165°, or the lab momentum of the π^- from decay was less than 34 MeV/c (0.8-cm range).

(5) For the decay $K^0 \to \pi^+ + \pi^-$, the c.m. cosine of the production angle of the associated hyperon was greater than 0.6.

Criterion 4 eliminated configurations which might be difficult to detect in scanning. Criteria 2 and 5 eliminated events for which detection efficiency was small and which were, therefore, not corrected for adequately in the event-by-event weighting for escape corrections. Detection-efficiency corrections included effects of all these cutoffs.

Events were rejected if the frame contained more than 35 tracks, if the incident beam track failed the template test for track count,[1] or if beam azimuth, dip, or momentum varied from mean values by more than three standard deviations. These selection criteria were chosen to conform to procedures for evaluating total track length.

Total pion track length L was estimated from

$$L = \bar{n}\bar{l}(N_f - N_{TMT})(1-f_c-f_r)(1-f_s), \qquad (1)$$

where $\bar{n}$ is the average number of tracks per frame, estimated from a track count with a template[1]; $\bar{l}$ is the average pion track length in the fiducial volume, corrected for attenuation[1]; N_f is the number of scannable frames; N_{TMT} is the number of frames with more than 35 tracks, estimated from the track count; f_c [approximately $(7\pm2)\%$] is the fraction of beam contamination by leptons, obtained from a scan for knock-on electrons too energetic to have come from pions incident[1]; f_r [approximately $(5\pm1)\%$] is the fraction of pions included in the track count which failed tighter selection criteria for beam tracks of events (taken from $\pi^- + p \to \Sigma^- + K^+$ events)[1]; f_s is the fraction of satellite beam (which entered when the chamber was insensitive), measured by gap-counting in the film section most affected, and scaled for other film sections by estimates from the film edit. Table I contains values of quantities for calculation of total track length at each beam momentum.

Limits on χ^2 for acceptance of decay and production hypotheses were taken from events in which both K^0

* Much of this work was done in partial fulfillment of the requirements for the Ph.D. at University of Wisconsin by Thomas O. Binford.

† Work supported in part by the U. S. Atomic Energy Commission.

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[1] M. L. Good and R. R. Kofler, following paper, Phys. Rev. 183, 1142 (1969). Details of beam normalization carried out in common to Kofler's and this experiment are given by him.

[2] S. E. Wolf, N. Schmitz, L. J. Lloyd, W. Laskar, F. S. Crawford, Jr., J. Button, J. A. Anderson, and G. Alexander, Rev. Mod. Phys. 33, 439 (1961).

[3] A description of PANG-KICK in the form PACKAGE is contained in A. H. Rosenfeld and W. E. Humphrey, Ann. Rev. Nucl. Sci. 13, 103 (1963).

TABLE I. Data relevant to total-cross-section determination.

Beam momentum (MeV/c)	1128.6	1235	1277	1326
c.m. energy (MeV)	1742	1797	1823	1842
Frames accepted, (N_f-N_{TMT})	48 200	14 490	41 720	23 770
Average tracks/frame, $\bar{n}$	16.56±0.13	18.46±0.25	8.81±0.13	8.66±0.17
Fraction of tracks accepted, $(1-f_c-f_r)(1-f_s)$	0.889	0.850	0.862	0.844
Events per mb denominator for total cross section	2885	968.9	1370	796.6
Corrected total events				
$\pi^-+p\to\Lambda^0+K^0$	1707	470	612	293
$\pi^-+p\to\Sigma^0+K^0$	756	256	314	168
Total cross section (μb)				
$\pi^-+p\to\Lambda^0+K^0$	592±22	485±35	447±28	367±33
$\pi^-+p\to\Sigma^0+K^0$	262±15	264±25	229±20	209±25

and Λ^0 were observed (double V events). Approximately 3% of strange-particle decays failed these criteria. Since readily identifiable background was only 1%, barely failing events were included after examination; 1% of decays failed repeated remeasurement. A few of each of the following classes of events were accepted after inspection and diagonostic tests:

(a) identified[4] three-body decays of Λ^0,

(b) events consistent with strange-particle two-body decays followed by a small-angle scatter in one track,

(c) events which could not be measured.

These categories included all the remaining events.

Just before final selection, the entire selection process was checked. All rejected events were examined. Less than 1% of the events had been lost. These events were restored and a correction of ½% was made for system efficiency.

Separation of double-V and single-K^0 events into the production hypotheses $\pi^-+p\to\Lambda^0+K^0$ and $\pi^-+p\to\Sigma^0+K^0$ or into three-body production modes was unambiguous on the basis of χ^2. Separation of single Λ events was made on the basis of missing-mass squared

TABLE II. Three-body production.

Momentum (MeV/c)	Single Λ^0	Single K^0 Double V	$\Lambda^0K^+\pi^-$
	Number of events		
1128.6	none	none	none
1235	2	1	none
1277	2	2	none
1326	6	2	2

Total cross-section estimates[a] (μb)

Momentum (MeV/c)	$\pi^-+p\to\Lambda^0+K^0+\pi^0$ $\sigma_{\min}$	σ	$\sigma_{\max}$	$\pi^-+p\to\Lambda^0+K^++\pi^-$ $\sigma_{\min}$	σ	$\sigma_{\max}$
1235	1	4	21	0	0	6
1277	2	4	23	0	0	5
1326	3.3	15	38	1.2	4	15

[a] Minimum and maximum cross sections are based on Poisson statistics on the single-K^0 and double-V events, with 2.5% probability of exceeding either extreme.

MM² of the neutral system recoiling from the Λ. Figure 1 shows the spectrum of MM². A handful of three-body production events were identified beyond the kinematic limit of MM² for $\pi^-+p\to\Sigma^0+K^0$; corrections were made for the fraction of the three-body missing-mass spectrum buried within these kinematics limits. Events selected for $\pi^-+p\to\Sigma^0+K^0$ were required to have two consistent measurements. The selection procedure contained an automatic measurement of the mistaken identification of the two hypotheses, since Λ's from double-V events were analyzed with single-V events. Relevant quantities for total cross sections for $\pi^-+p\to\Lambda^0+K^0$ and $\pi^-+p\to\Sigma^0+K^0$ are displayed in Table I. In Table II, data on three-body

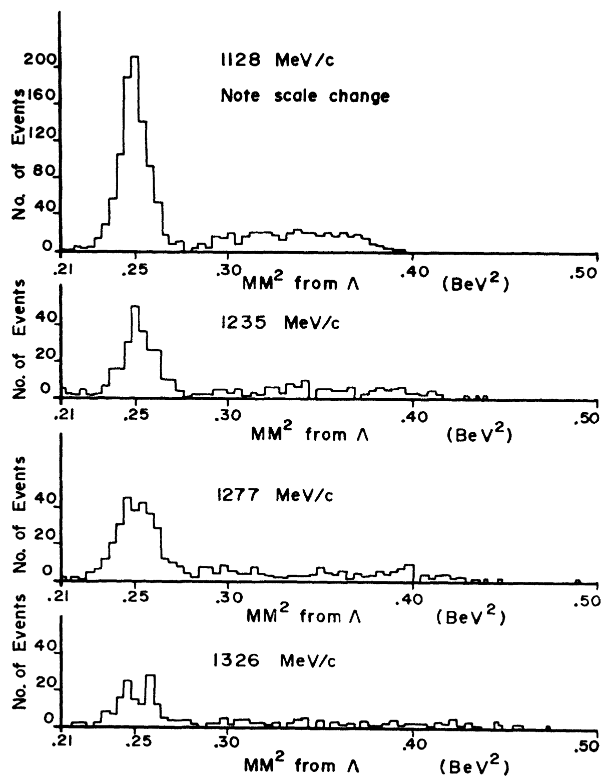

FIG. 1. Histograms of MM² recoiling from Λ.

[4] V. G. Lind, T. O. Binford, M. L. Good, and D. Stern, Phys. Rev. 136, B1483 (1964).

production modes

$$\pi^- + p \rightarrow \Lambda^0 + K^+ + \pi^-,$$
$$\pi^- + p \rightarrow \Lambda^0 + K^0 + \pi^0$$

are given. Because the highest beam momenta are just above threshold for $\pi^- + p \rightarrow \Sigma^0 + K^0 + \pi^0$, events containing a $\Lambda^0 K^0 \pi^0$ final state are assigned to $\pi^- + p \rightarrow \Lambda^0 + K^0 + \pi^0$. Upper and lower limits on cross sections are based on Poisson statistics for single-K^0 and double-V events, assuming 2.5% probability for exceeding each limit.

ANGULAR DISTRIBUTIONS AND POLARIZATIONS

Histograms of the angular distribution in the c.m. system, corrected for detection efficiency, are shown in Fig. 2 for the reaction $\pi^- + p \rightarrow \Lambda^0 + K^0$. A projection method due to Crawford[5] has been used to obtain coefficients for a Legendre expansion of the angular distribution,

$$\frac{d\sigma}{d\Omega}(\theta) = \sum_n A_n P_n(\cos\theta),$$

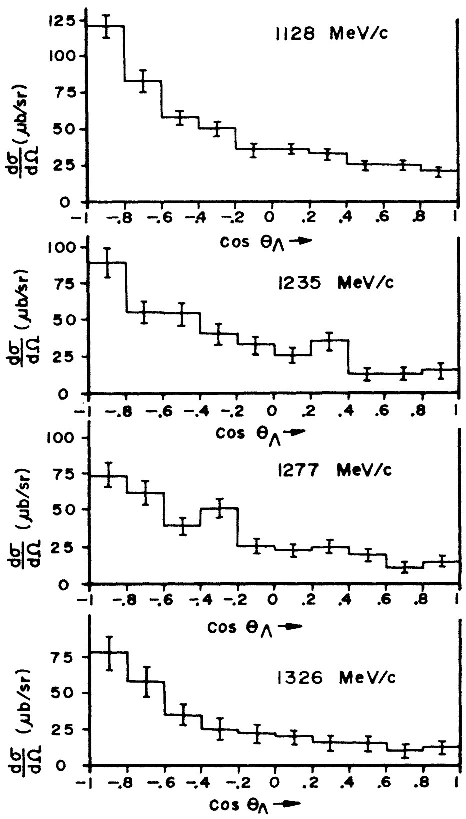

FIG. 2. c.m. angular distribution of the Λ in the reaction $\pi^- + p \rightarrow \Lambda^0 + K^0$.

[5] These results are contained in a series of unpublished reports by F. S. Crawford, Jr.

and the polarization angular distribution,

$$\frac{d\sigma}{d\Omega}(\theta) \alpha P(\theta) = \sum_n B_n P_n^{(1)}(\cos\theta),$$

where $P(\theta)$ is the hyperon polarization and $P_n(\cos\theta)$ and $P_n^{(1)}(\cos\theta)$ are Legendre polynomials of order n. We define $\mu = \cos\theta$. We adopt the convention that the asymmetry parameter for hyperon decay, α, is defined in terms of the pion asymmetry. The coefficients and error matrices were determined from sums over observed events:

$$A_n = C(2n+1) \sum_i P_n(\mu_i)/E_i, \tag{2a}$$

$$\delta A_n \delta A_m = C^2(2n+1)(2m+1) \sum_i P_n(\mu_i) P_m(\mu_i)/E_i^2, \tag{2b}$$

$$B_n = \frac{C3(2n+1)}{n(n+1)} \sum_i \cos\phi_i\, P_n^{(1)}(\mu_i)/\epsilon_i, \tag{2c}$$

$$\delta B_n \delta B_m = \frac{C^2 9(2n+1)(2m+1)}{n(n+1)m(m+1)} \sum_i \cos^2\phi_i \times P_n^{(1)}(\mu_i) P_m^{(1)}(\mu_i)/\epsilon_i^2. \tag{2d}$$

E_i and ϵ_i are over-all weight factors for the event for angular distribution and polarization, respectively; C^{-1} is the number of events per microbarn, as determined by the track count (see Table I); and $\cos\phi_i$ is the decay cosine of the pion in the hyperon rest frame, $\cos\phi_i = \hat{n}_\Lambda \cdot \hat{\pi}_{\text{decay}}$, with $\hat{n}_\Lambda = (\hat{\pi}_{\text{beam}} \times \hat{\Lambda}) / |\hat{\pi}_{\text{beam}} \times \hat{\Lambda}|$. Contributions to the error matrix from c.m. angle errors were ignored. These contributions are quite small through order 3.

Detection efficiency D_i for individual Λ and K^0 decays has the form

$$D_i = B_i (e^{-L_m/c\tau_i\beta_i\gamma_i} - e^{-L_f/c\tau_i\beta_i\gamma_i}) d_i, \tag{3}$$

where d_i is the fraction of decays accepted by criterion 4 for opening angle and momentum (d_i is approximately 0.99), D_i is the branching ratio for charged decay, $L_m = 0.7$ cm is the minimum acceptable decay length, and L_f is the potential path length in the fiducial volume. τ_i is the proper lifetime of the particle, and $\gamma_i\beta_i = P_i/m_i$ are expressed in terms of the particle momentum P_i and mass m_i.

We have used, for angular distribution, $E_\Lambda = D_\Lambda$ for single Λ's (which corrects for events with unobserved Λ and unobserved K^0), $E_K = 1$ for single-K^0 events, and $E_{2V} = 1$ for double-V events. For polarization, we have used $\epsilon_\Lambda = D_\Lambda = \epsilon_{2V}$ and $1/\epsilon_K = 0$. These are slight modifications of the usual relationships. This and related results can be readily obtained by picturing the classes of events as areas on the square of unit probability. One axis represents Λ's, a fraction D_Λ of which are observed; the other axis represents K^0's, a fraction D_K of which are observed. The lines D_Λ and D_K define four areas on

TABLE III. Legendre coefficients of angular distribution for $\pi^-+p\rightarrow\Lambda^0+K^0$. $d\sigma/d\Omega=\Sigma_n A_n P_n(\cos\theta)$.

Incident momentum (MeV/c)	A_0 (μb/sr)	A_1	A_2	A_3	A_4	A_5	A_6
1128	47.1±1.45	−44.4±2.82	27.3 ±3.65	−16.1 ±4.34	11.7 ±4.92	−5.20±5.45	7.55±5.84
1235	38.6±2.32	−39.0±4.38	14.0 ±5.63	−5.32±6.35	3.76±6.97	2.86±7.83	−5.45±8.84
1277	35.6±1.88	−33.6±3.51	14.69±4.50	−7.90±5.38	2.48±6.08	−9.19±6.77	6.35±7.15
1326	29.2±2.25	−31.3±4.48	22.6 ±5.67	−10.3 ±6.38	2.60±7.05	7.31±7.28	−8.78±7.89

TABLE IV. Legendre coefficients of $\sigma\alpha P$ for $\pi^-+p\rightarrow\Lambda^0+K^0$. $\alpha P d\sigma/d\Omega=\Sigma_n B_n P_n^{(1)}(\cos\theta)$.

Incident momentum (MeV/c)	B_1 (μb/sr)	B_2	B_3	B_4	B_5
1128	41.8±3.16	−15.2 ±2.57	7.65±2.14	−3.22±1.94	0.81 ±1.79
1235	29.4±5.11	−15.0 ±4.00	6.49±3.38	3.64±3.29	−0.259±2.84
1277	24.0±3.99	−13.0 ±3.06	4.33±2.65	1.61±2.36	0.585±2.09
1326	25.4±5.16	−8.89±4.22	5.59±3.60	−1.22±3.20	−3.22 ±2.82

the unit square which are equal to the detection probability of single-Λ^0, single-K^0, double V events, and undetected events. Each observed event corresponds to a known area on the unit square, and the problem is to correct for the unobserved events by weighting of observed events.

The polarization $\alpha_\Lambda P_\Lambda$ as a function of c.m. cosine is shown in Fig. 3, where $\alpha_\Lambda P_\Lambda\equiv 3[\sum_i(\cos\phi_i)/\epsilon_i]/[\sum_i 1/\epsilon_i]$. Coefficients obtained from the data for the Legendre polynomial expansion are given in Tables III and IV, along with diagonal errors. Expansions in cosine powers are sensitive to the order of fit which means that much information lies in the correlation coefficients. In the projection method, Legendre coefficients are independent of the order of the expansion, and even with least-squares determination, coefficients of the Legendre expansion and their errors change little with the order of fit for any reasonable fit. Interpretation is simpler with Legendre coefficients; to a good approximation, correlations between coefficients can be neglected.

ANALYSIS OF $\pi^-+p\rightarrow\Sigma^0+K^0$

The angular distribution for Σ^0K^0 is shown in Fig. 4.

Extraction of Legendre expansion coefficients for the reaction $\pi^-+p\rightarrow\Sigma^0+K^0$ is similar to the projection method expressed in Eqs. (2a)–(2d), for events for which the Σ-direction is determined (single-K^0 and double-V events). The Λ decay distribution is asymmetric along $\mathbf{P}_\Lambda$, the Λ polarization vector:

$$f(\theta_\Sigma,\phi)=\sigma(\theta_\Sigma)(1+\alpha_\Lambda|\mathbf{P}_\Lambda|\cos\phi),$$

where θ_Σ is the c.m. angle of the Σ^0, $\cos\phi=\hat{\pi}_{\rm decay}\cdot\mathbf{P}_\Lambda/|\mathbf{P}_\Lambda|$, and α_Λ is the Λ-decay asymmetry parameter. For the decay at rest, the decay orbital angular momentum has no component along the Λ momentum vector $\hat{\Lambda}^\Sigma$. The component of Σ^0 polarization $\mathbf{P}_\Sigma$ along

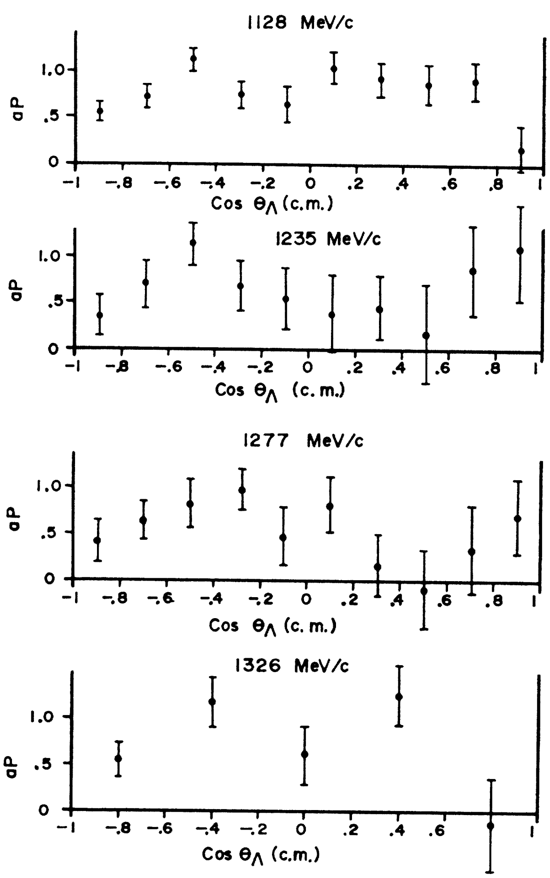

FIG. 3. Angular distribution of Λ polarization in the reaction $\pi^-+p\rightarrow\Lambda^0+K^0$.

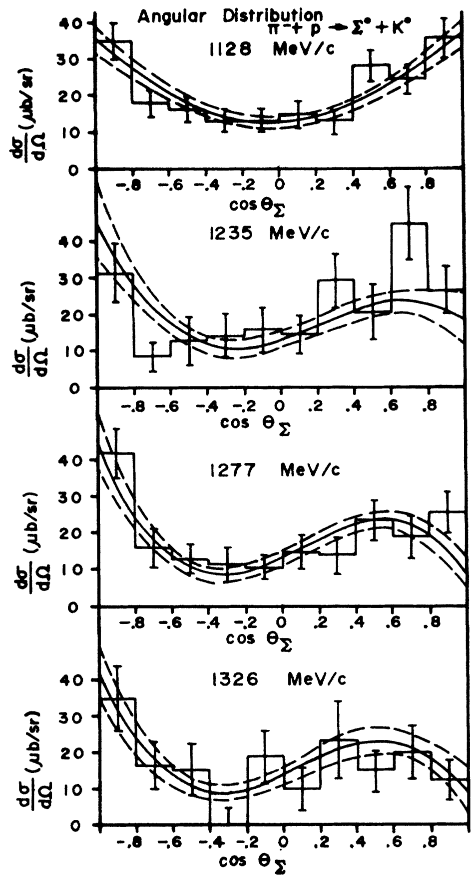

FIG. 4. c.m. angular distribution of the Σ^0 in the reaction $\pi^-+p\rightarrow\Sigma^0+K^0$.

the Λ momentum[6] is

$$\mathbf{P}_\Sigma\cdot\hat{\Lambda}^\Sigma=|\mathbf{P}_\Sigma|(\hat{n}_\Sigma\cdot\hat{\Lambda}^\Sigma)\equiv|\mathbf{P}_\Sigma|\cos\beta,$$

where $\hat{n}$ is the normal to the plane of production. Conservation of angular momentum implies that the Λ spin must be opposite that of the Σ^0, and the photon spin along the spin of the Σ^0. Thus, the distribution function becomes

$$f(\theta_\Sigma,\phi)=\sigma(\theta_\Sigma)(1-\alpha_\Lambda|\mathbf{P}_\Sigma(\theta_\Sigma)|\cos\beta\cos\phi). \quad (4)$$

Accordingly, the projections for B_n and the error matrix are

$$B_n=\frac{C3(2n+1)}{n(n+1)}\sum_i \cos\beta_i\cos\phi_i\,P_n^{(1)}(\mu_{\Sigma i})/\epsilon_i, \quad (5a)$$

$$\delta B_n\delta B_m=\frac{C^29(2n+1)(2m+1)}{n(n+1)m(m+1)}\sum_i \cos^2\beta_i\cos^2\phi_i\,P_n^{(1)}(\mu_{\Sigma i})P_m^{(1)}(\mu_{\Sigma i})/\epsilon_i^2. \quad (5b)$$

For single-Λ events, the Σ direction is not known; a different treatment was used for this sample as described below. Weight factors were chosen so that the two samples provided separate estimates of the expansion coefficients. The double-V class was expanded slightly; if one V was accepted, the other was rejected only if it lay outside the decay fiducial volume. From (3), the detection probability becomes

$$D_i'=e^{-L_f/c\tau_i\gamma_i\beta_i}B_i$$

for the second V if one V was accepted. We have used $E_K=D_K$, $1/\epsilon_K=0$ for single-K^0 events and

$$E_{2V}=[D_\Lambda D_K+D_\Lambda(D_K'-D_K)+D_K(D_\Lambda'-D_\Lambda)]/D_\Lambda'$$

for double-V events. For the polarization, we have

$$\epsilon_{2V}=[D_\Lambda D_K+D_\Lambda(D_K'-D_K)+D_K(D_\Lambda'-D_\Lambda)].$$

For single-Λ events, we can extract only statistical information about the Λ. To obtain expansion coefficients for the Σ, we use a method from Crawford.[5] The observed Λ angular distribution can be written

$$n(\mu_\Lambda)\equiv\sum_n A_n'P_n(\mu_\Lambda)=\int\frac{\partial^2N}{\partial\mu_\Lambda\partial\mu_\Sigma}d\mu_\Sigma$$
$$\equiv\int\tfrac{1}{2}d\mu_\Sigma w(\mu_\Lambda,\mu_\Sigma)n(\mu_\Sigma),$$

where $w(\mu_\Lambda,\mu_\Sigma)$ represents the decay transformation $\Sigma^0\rightarrow\Lambda^0+\gamma$. We have

$$A_n'=(2n+1)\int\tfrac{1}{2}d\mu_\Lambda n(\mu_\Lambda)P_n(\mu_\Lambda)$$
$$=\sum_m A_m\int\tfrac{1}{2}d\mu_\Lambda\int\tfrac{1}{2}d\mu_\Sigma P_m(\mu_\Sigma)P_n(\mu_\Lambda)w(\mu_\Lambda,\mu_\Sigma), \quad (6)$$

which is just a matrix transformation between the coefficients $\{A_m\}$ and $\{A_n'\}$. We obtain A_m from the matrix inversion:

$$A_n'=\sum_m M_{nm}A_m \quad \text{implies} \quad A_m=\sum_n (M^{-1})_{mn}A_n'. \quad (7)$$

The matrix of products of Legendre polynomials can be simply evaluated numerically from the kinematics of Σ^0 decay.[7] Determination of A_n' was identical to (2a),

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[6] The following notation has been used. Superscripts refer to the Lorentz frame in which the vector is defined, using the usual proper sequence of Lorentz transformations. Subscripts refer to the particle momentum defining the vector; some particle labels are self-evident. Thus, $\hat{\Lambda}^\Sigma$ is the unit vector along the $\hat{\Lambda}$ momentum in the Σ rest frame.

[7] Legendre coefficients for the observed Λ distribution are independent; the Σ distribution coefficients determined from them are not independent. However, the matrix of transformation is nearly diagonal and diagonal terms are near unity. Consequently, Legendre coefficients for the Σ distribution are nearly independent. We have determined coefficients through order 3. From the $I=\frac{3}{2}$ channel, we would expect terms up to order 6, but those above order 3 are not above the level of measurement errors: In $\pi^-+p\rightarrow\Sigma^0+K^0$ the contribution would be $\frac{2}{9}$ that of the $I=\frac{3}{2}$ channel, hence negligible. Only very large higher-order terms from $I=\frac{1}{2}$ would change these coefficients.

$A_n'=C(2n+1)\sum_i P_n(\mu_{\Lambda i})/E_i$, where $\mu_{\Lambda i}$ is the c.m. cosine of the ith Λ from Σ^0 decay. The detection efficiency $E_i=D_{\Lambda i}(1-D_K')$ was calculated with an average D_K' for the unobserved K^0. Since the average probability for K^0 decay outside the fiducial volume was less than 1%, D_K' is nearly equal to B_K, the K^0 branching ratio, and the procedure gave a good approximation for D_K'.

Legendre coefficients of $\sigma\alpha P$ were calculated by another projection method, our extension of Crawford's results.[5] There remains a nonvanishing average polarization of the Λ along the normal defined by the incident pion and the Λ, $\hat{n}_\Lambda=(\hat{\pi}_{\text{beam}}\times\hat{\Lambda})/|\hat{\pi}_{\text{beam}}\times\hat{\Lambda}|$. A much larger average polarization remains in $\pi^-+p\to\Lambda^0+K^0$ along this direction; to eliminate the effect of contamination, events which might conceivably have been $\pi^-+p\to\Lambda^0+K^0$ events were removed from the following calculations. The distribution function is given by

$$f(\mu_\Sigma,\xi)=n(\mu_\Sigma)(1+\alpha_\Lambda P_\Lambda\xi)=n(\mu_\Sigma)\times[1-\alpha_\Lambda P_\Sigma(\hat{n}_\Sigma\cdot\hat{\Lambda}^\Sigma)(\hat{\Lambda}^\Sigma\cdot\hat{n}_\Lambda)\xi], \quad (8)$$

where P_Λ and P_Σ are Λ and Σ polarizations, $\xi=\hat{\pi}_{\text{decay}}\cdot\hat{n}_\Lambda$ is the decay cosine of the pion with respect to $\hat{n}_\Lambda$, and $\hat{n}_\Sigma$ is the unit vector normal to the production plane. As above, $\hat{\Lambda}^\Sigma$ is the unit vector along the Λ velocity in the Σ^0 rest frame. We write $\hat{\Lambda}^\Sigma=\cos\gamma\,\hat{\Sigma}+\sin\gamma(\cos\phi\,\hat{x}+\sin\phi\,\hat{n}_\Sigma)$ in terms of the polar angle γ, the azimuthal angle ϕ, and the triplet $(\hat{x},\hat{\Sigma},\hat{n}_\Sigma)$, where $\hat{x}=\hat{\Sigma}\times\hat{n}_\Sigma$. Then $\hat{n}_\Sigma\cdot\hat{\Lambda}^\Sigma=\sin\gamma\sin\phi$ defines one of the dot products in Eq. (8), and[8]

$$(\hat{\Lambda}^\Sigma\cdot\hat{n}_\Lambda)=\hat{\Lambda}^\Sigma\cdot(\hat{\pi}_{\text{beam}}\times\hat{\Lambda}^c)/\sin\theta_\Lambda=\frac{1}{P_\Lambda{}^c\sin\theta_\Lambda}\times\left\{\hat{\Lambda}^\Sigma\cdot\hat{\pi}_{\text{beam}}\times\left[\hat{\Lambda}^\Sigma+\boldsymbol{\eta}_\Sigma\left(\frac{\boldsymbol{\eta}_\Sigma\cdot\hat{\Lambda}^\Sigma}{\gamma_\Sigma+1}+E_\Lambda{}^\Sigma\right)\right]\right\}, \quad (9)$$

$$(\hat{\Lambda}^\Sigma\cdot\hat{n}_\Lambda)\simeq\sin\gamma\sin\phi\sin\theta_\Sigma E_\Lambda{}^\Sigma\eta_\Sigma/(P_\Lambda{}^c\sin\theta_\Lambda), \quad (10)$$

defines the other in terms of the energy $E_\Lambda{}^\Sigma$ of the Λ in the Σ^0 rest frame $P_\Lambda{}^c$, the Λ momentum in the c.m. (an observable), with $\cos\theta_\Lambda\equiv\hat{\pi}_{\text{beam}}\cdot\hat{\Lambda}^c$ in the c.m. system in terms of observables, and η_Σ, known from the Σ c.m. momentum. The very small second term from (9) has been dropped. It could be included but is quite negligible. We have, finally,

$$(\hat{\Lambda}_\Sigma\cdot\hat{n}_\Lambda)(\hat{\Lambda}_\Sigma\cdot\hat{n}_\Sigma)=\sin^2\gamma\sin^2\phi\sin\theta_\Sigma\,\eta_\Sigma E_\Lambda{}^\Sigma/P_\Lambda{}^c\sin\theta_\Lambda.$$

We then define a projection operator

$$g=\frac{3\xi\sin\theta_\Lambda P_\Lambda{}^c}{\eta_\Sigma E_\Lambda{}^\Sigma}. \quad (11)$$

We evaluate the projection in terms of elementary integrals:

$$C\int dN\,g=-\alpha_\Lambda 3\int_{-1}^{1}\tfrac{1}{2}d\mu_\Sigma\int_{-1}^{1}\tfrac{1}{2}d\cos\gamma\int_0^{2\pi}\frac{d\phi}{2\pi}\int_{-1}^{1}\tfrac{1}{2}d\xi\times\sin^2\phi\sin^2\gamma\sin\theta_\Sigma\,n(\mu_\Sigma)\xi^2P_\Sigma(\mu_\Sigma),$$

to find

$$C\int dN\,g=-\tfrac{1}{3}\alpha_\Lambda\int_{-1}^{1}\tfrac{1}{2}d\mu_\Sigma[\Sigma_nB_nP_n{}^{(1)}(\mu_\Sigma)]\times P^{(1)}(\mu_\Sigma)=-\tfrac{1}{3}\alpha_\Lambda\tfrac{2}{3}B_1, \quad (12)$$

$$C\int dN\,g=-\tfrac{1}{9}\alpha_\Lambda 2B_1. \quad (13)$$

In the same way, we define $h\equiv 3g\mu_\Lambda P_\Lambda{}^c/\eta_\Sigma E_\Lambda{}^\Sigma$, and find

$$\alpha_\Lambda B_2=-\frac{15}{6}C\int dNh. \quad (14)$$

As above, the projection consists of evaluating the integrals as sums over the data.

Results from the single-Λ and the double-V-plus-single-K^0 events were combined according to the least-squares result:

$$A=(H_1{}^{-1}+H_2{}^{-1})^{-1}(H_1{}^{-1}A_1+H_2{}^{-1}A_2),$$

where A_1 and A_2 are the two independent sets of coefficients, and H_1 and H_2 are the corresponding error matrices.

The resulting Legendre coefficients are displayed in Tables V and VI.

We have compared the data in this study to the results of other experiments in the energy region. Data which are in good agreement are (1) $\pi^-+p\to\Lambda^0+K^0$

TABLE V. Legendre coefficients of angular distribution for $\pi^-+p\to\Sigma^0+K^0$. $d\sigma/d\Omega=\Sigma_nA_nP_n(\cos\theta)$.

Incident momentum (MeV/c)	A_0 (μb/sr)	A_1	A_2	A_3
1128	20.8±1.2	0.74±2.0	15.2±2.7	−6.3±3.5
1235	21.0±2.0	−1.7 ±3.6	13.5±4.4	−12.9±5.1
1277	18.2±1.6	−1.2 ±2.5	9.9±3.2	−16.9±3.8
1326	16.6±2.0	−1.8 ±3.1	8.1±3.9	−13.9±4.7

TABLE VI. Legendre coefficients of $\sigma\alpha P$ for $\pi^-+p\to\Sigma^0+K^0$. $\alpha Pd\sigma=\Sigma_nB_nP_n{}^{(1)}(\cos\theta)$.

Incident momentum (MeV/c)	B_1 (μb/sr)	B_2	$\bar{P}$
1128	−0.4±4.2	2.2±5.4	0.32±0.32
1235	4.7±6.1	11.5±6.7	1.1 ±0.7
1277	6.9±4.8	−7.52±6.0	0.7 ±0.5
1326	10.2±7.0	6.7 ±7.6	1.7 ±0.75

[8] R. Hagedorn, *Relativistic Kinematics* (W. A. Benjamin, Inc., New York, 1963), p. 9.

angular distribution at 1123 MeV/c,[9,10] polarization at 1170 and 1320 MeV/c [11] and the general trend of the polarization,[12,13] total cross section at 1123 and 1220 MeV/c,[9] and (2) $\pi^- + p \to \Sigma^0 + K^0$ angular distribution at 1123 MeV/c [10] and polarization at 1320 MeV/c.[11] For all data excepting those mentioned below, we have $\chi^2 = 30$ for 26 constraints, testing equality of the sets of Legendre coefficients. We find mild disagreement with the total cross-section values for $\pi^- + p \to \Sigma^0 + K^0$ from the results of the Berkeley group,[9] and with total cross sections of the Pisa data at 1330 MeV/c.[10] Our data are in striking disagreement with the Michigan data[10] for 1220 MeV/c, with respect to both angular distributions and total cross sections. The disagreement with some previous results is not surprising, since they disagree among themselves.

DISCUSSION

First, we note that the values of $\alpha_\Lambda P_\Lambda$ in the reaction $\pi^- + p \to \Lambda + K^0$ are uncomfortably large to be accommodated by the accepted value $\alpha_\Lambda = 0.663 \pm 0.022$.[14] This value of α_Λ would imply $\alpha_\Lambda P_\Lambda \leq 0.663$ everywhere, a result that does not seem compatible with Fig. 3. We have not been able to convert this feeling into a quantitative statement about α_Λ, because of lack of knowledge of polarization and its angular dependence.

To make comparison of phases of the associated production partial waves with phases in pion-nucleon scattering, we must change the signs of the c.m. cosine and the normal to the plane of production. The tables have been prepared with the c.m. cosine of the hyperon, and the normal defined by $\hat{n}_Y = (\hat{\pi}_{\text{beam}} \times \hat{Y}) / |\hat{\pi}_{\text{beam}} \times \hat{Y}|$, where $\hat{Y}$ is a unit vector along the hyperon momentum. Change of sign of the c.m. cosine changes the sign of the odd A_n and even B_n, while changing the sign of the normal to the plane of production changes the sign of all B_n. As a final result, odd A_n and odd B_n change sign.

The reactions $\pi^- + p \to \Sigma^- + K^+$ and $\pi^- + p \to \Sigma^0 + K^0$ are isospin mixtures of $I = \frac{1}{2}$ and $I = \frac{3}{2}$. The $I = \frac{1}{2}$ part can be extracted for the total cross section, and is given by

$$\sigma(\Sigma K, I = \tfrac{1}{2}) = \tfrac{3}{2}(\sigma_{\Sigma^- K^+} + \sigma_{\Sigma^0 K^0} - \tfrac{1}{3}\sigma_{\Sigma^+ K^+}), \quad (15)$$

where $\sigma_{\Sigma^- K^+}$, $\sigma_{\Sigma^0 K^0}$, and $\sigma_{\Sigma^+ K^+}$ are the total cross sections for $\pi^- + p \to \Sigma^- + K^+$, $\pi^- + p \to \Sigma^0 + K^0$, and $\pi^+ + p \to \Sigma^+ + K^+$. World supply data for these quantities are plotted in Fig. 5.[2,10–30] (Sources are identified in Table VII.) The $I = \frac{1}{2}\, \Sigma K$ cross section climbs very rapidly at threshold, and is approximately equal to the ΛK cross section (pure $I = \frac{1}{2}$) at all energies above threshold, as shown in Fig. 6. The data indicate that the equality which we suggest is only approximate; it appears not to hold within the errors, but the near equality over a wide range of energies is striking.

In Fig. 7, we also show the comparison between the two $I = \frac{1}{2}$ associated production cross sections and the $I = \frac{1}{2}$ reaction $\pi^- + p \to \eta + n$.[31,32] The sum of the $I = \frac{1}{2}\, \Lambda K$ and ΣK cross sections is equal, within errors, to the cross section for $\pi^- + p \to \eta + n$. We have plotted one entirely unjustified point below the ΣK threshold, a point with value twice the ΛK cross section. This point

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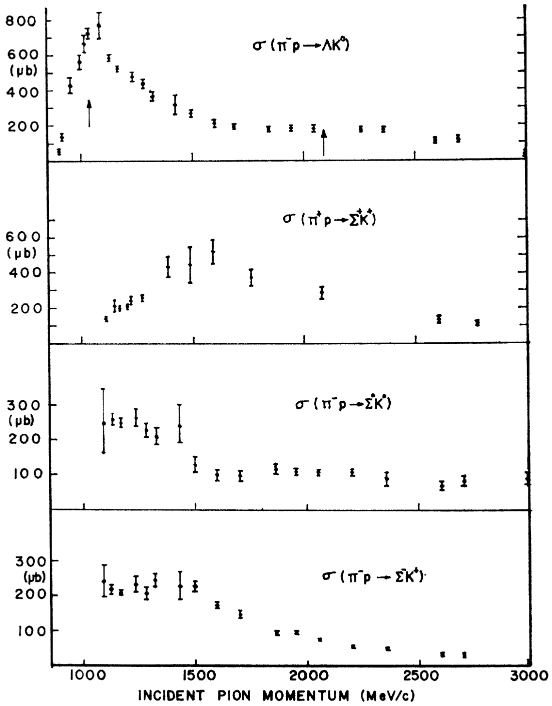

FIG. 5. Total cross sections. The arrows on the K^0 plot denote $N^*(1688)$ and $N^*(2190)$, respectively.

also is equal to the ηn cross section at that energy. We have, therefore, two empirical cross section relations:

$$\sigma(\pi^-+p\to\eta^0+n)\cong\sigma(\pi^-+p\to\Lambda^0+K^0)+\sigma(\pi^-+p\to\Sigma K, I=\tfrac{1}{2}) \quad (16)$$

and

$$\sigma(\pi^-+p\to\Lambda^0+K^0)\cong\sigma(\pi+p\to\Sigma K, I=\tfrac{1}{2}). \quad (17)$$

The reactions $\pi^-+p\to\Lambda^0+K^0$ and $\pi^++p\to\Sigma^++K^+$ are states of pure isospin $\frac{1}{2}$ and $\frac{3}{2}$, respectively. A prominent feature of the total cross sections for these reactions are the peaks at the known $I=\frac{1}{2}$ and $I=\frac{3}{2}$ pion-nucleon resonances. In the $I=\frac{1}{2}$ associated pro-

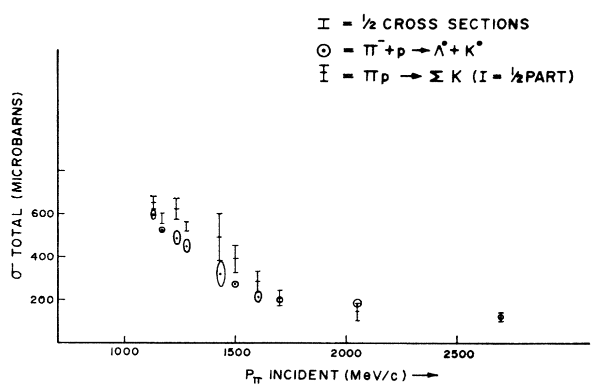

FIG. 6. Momentum dependence of $I=\frac{1}{2}$ cross sections: $\pi^-+p\to\Lambda^0+K^0$, $\pi p\to\Sigma K(I=\frac{1}{2}$ part).

TABLE VII. Sources for total-cross-section data.

Beam momentum (MeV/c)	Reference Σ^+K^+	ΛK^0	Σ^0K^0	Σ^-K^+
905	...	23	...	...
922	...	23	...	...
959	...	23	...	...
1000	...	23	...	...
1020	...	24	...	...
1035	...	13	...	...
1090	...	10	10	10
1111	30	...	...	...
1128	...	This expt	This expt	1
1155	22	...	...	...
1170	15	12	12	12
1206	30	...	...	...
1220	16	...	...	...
1235	...	This expt	This expt	1
1268	30	...	...	...
1277	...	This expt	This expt	1
1326	...	This expt	This expt	1
1390	16, 19	...	...	...
1430	...	10	10	10
1490	17	...	...	...
1500	...	25, 29	25, 29	25
1590	18	26	...	...
1600	...	25	25	25
1700	...	25	25	25
1760	19	...	...	...
1860	...	25	25	25
1950	...	25	25	25
2050	...	25	25	25
2080	19	...	...	...
2200	...	25	25	25
2350	...	25	25	25
2600	(av of 2.35, 2.62, 2.90) 20	25	25	25
2700	...	27	27	27
2770	21	...	...	...
3000	...	28	28	28

duction, channel, there is a peak at 1690-MeV c.m. energy with a width of about 100-MeV c.m. and possibly a bump at about 2200-MeV c.m., with a width of the order of 200-MeV c.m. These correspond to $N^*_{1688}(\frac{5}{2}^+)$ and $N^*_{2190}(\frac{7}{2}^-)$, respectively. In the $I=\frac{3}{2}$ channel, there

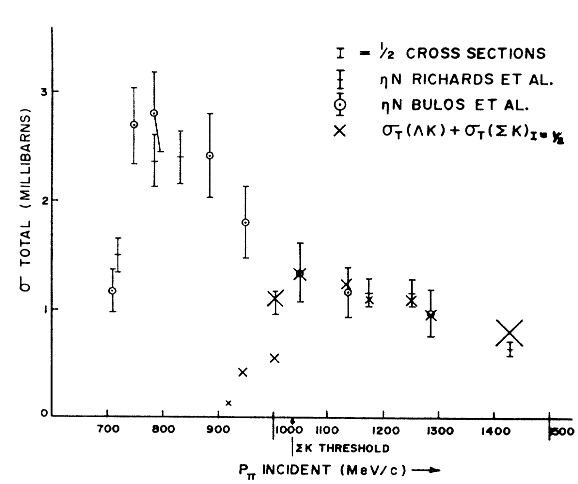

FIG. 7. Momentum dependence of $I=\frac{1}{2}$ cross sections for the sum $\sigma_T(\pi^-+p\to\Lambda^0+K^0)+\sigma_T(\pi^-+p\to\Sigma+K)_{I=1/2}$ and $\pi^-+p\to\eta^0+n$.

is a peak at 1950-MeV c.m. with a width of about 200-MeV c.m., corresponding to $\Delta_{1920}(\frac{7}{2}^+)$. Any influence from the $I=\frac{3}{2}$ resonance $\Delta_{2420}(\frac{11}{2}^+)$ is much smaller than the peak at 1950 MeV. The Σ^+K^+ data at high energies are too few to give more than a hint of the energy dependence. The near equality of $I=\frac{1}{2}$ ΛK and ΣK cross sections might suggest that the energy behavior of the $I=\frac{1}{2}$ ΣK channel is also dominated by resonances, but the strong backward peaking of Λ and, at higher energies of Σ^+, certainly suggests a strong K^* exchange contribution.

It is probable that no one mechanism is dominant.

A comparison of the data presented in this paper with that of Kofler[1] on $\pi^-p\rightarrow\Sigma^-K^+$ and Carayannopoulos *et al.*[30] on $\pi^+p\rightarrow\Sigma^+K^+$, to test the triangle inequalities of charge independence, has been made.[33]

ACKNOWLEDGMENTS

The authors want to thank the Alvarez group, in particular, Dr. F. S. Crawford, Jr., for making the exposure possible, and our scanners and measurers for their tireless efforts. Dr. W. F. Fry and Dr. U. Camerini participated in early states of the experiment. Finally, great assistance with computing problems came from Dr. R. W. Hartung.

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PHYSICAL REVIEW VOLUME 183, NUMBER 5 25 JULY 1969

Study of $\pi^-p\rightarrow\Sigma^-K^+$ from 1.12 to 1.32 BeV/c*

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(Received 6 November 1968)

Total cross sections, angular distributions, and decay asymmetries have been measured for the reaction $\pi^-p\rightarrow\Sigma^-K^+$ at pion beam momenta of 1.12, 1.23, 1.27, and 1.32 BeV/c. Legendre-polynomial expansion coefficients for the angular distributions are presented and compared with other data. Preliminary results of a partial-wave analysis are discussed.

I. INTRODUCTION

THE purpose of this paper is to present data obtained in a study of the reaction $\pi^-+p\rightarrow\Sigma^-+K^+$ at incident pion momenta of 1128, 1235, 1277, and 1325 MeV/c. At each of these momenta, total cross sections, angular distributions, and Σ^--decay asymmetries have been measured. The data are compared with other data[1–5] at different beam momenta. The data have already been combined with Σ^0 and Σ^+ data to test the charge independence hypothesis in strong interactions.[6] It is further hoped that these data, when combined with data from similar experiments, will aid in understanding those processes responsible for hyperon production.

II. EXPERIMENTAL DETAILS

A. General Procedure

The Σ^-K^+ events were produced in the LRL 72-in. liquid-hydrogen bubble chamber. The pion-beam transport system has been described in the literature.[2] A total of 140 000 triad photographs were used for this experiment. Scanners were instructed to search the photographs for any event which had the typical appearance of a Σ^-K^+ production: a two-prong topology with a kink in the negative track corresponding to the decay of the Σ^- hyperon,

$$\Sigma^-\rightarrow n+\pi^-. \tag{1}$$

The events found as a result of two complete scans of the film were measured on digitized microscopes and the measurements were analyzed using the PANG and KICK programs.[7] The scanning, measuring, and computing process produced 2325 Σ^-K^+ events. In order to maintain consistency between the sample of events and the

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