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Simulated Extensive Air Showers with a Medium-Strong Interaction*

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It is proposed that anomalous measurements of cosmic-ray pions and muons above a characteristic energy of 10^{12} eV can be explained, at least in part, by the presence of the medium-strong contact interaction between protons. That has been suggested by Abarbanel, Drell, and Gilman. The results of a programmed simulation of extensive air showers in a simple model have the correct behavior to explain the pion measurements, and they show a slight tendency in the correct direction for an explanation of the measurements of muons. Some remarks are made on the difficulties associated with theories of direct production of muons.

I. INTRODUCTION

RECENT measurements of cosmic-ray events, where the primary particle may have an energy of 10^{14} eV or greater, have revealed two apparently new phenomena. First, neutral pions produced early in extensive air showers have unusually large components of momentum perpendicular to the axis of the shower.[1] Second, there is a widely discussed report[2] of an observation of muons in the energy range 10^{12}–10^{13} eV and with a distribution that is almost independent of zenith angle. It is an attractive proposition to consider the possibility that the two effects may have a single cause. Previous suggestions for the parentage or identity of the particles in the Utah experiment[2] have included heavy muons,[3] integer-charged quarks,[4] intermediate vector bosons,[5] and magnetic monopoles, but with one exception these suggestions have not been related explicitly to the less-well-known pion observations.[1] The exception is a qualitative discussion[6] of the evidence for the existence of an intermediate vector boson. However, there have been no quantitative estimates by means of simulation of cosmic-ray showers.

The hypothesis of heavy muons is contradicted by existing measurements of the cosmic radiation.[7] Theories based on monopoles share with theories which modify the weak interaction at high energies[8] the difficulty that cross sections contain coupling constants which are too small to allow significant effects in simulation of showers. This leaves intact theories of quarks or intermediate vector bosons, provided that some new and strong coupling is assumed—without any other experimental evidence—in order to obtain approximate agreement with measurements.

Is there a chance of finding a theory with a suitably strong coupling whose value has been proposed already on quite different experimental grounds? There is a clear affirmative answer to the question: Abarbanel, Drell, and Gilman[9] have put forward a possible proton-proton contact interaction to explain a simple relation between nucleon form factors and proton-proton elastic scattering data, and have determined a medium-strong coupling constant for the interaction in a later paper.[10] Their theory has the useful feature that shrinkage of the diffraction peak in proton-proton scattering does not occur in the usual way. Therefore, the geometrical picture of the evolution of a cosmic-ray shower with the contact interaction built in differs from one based on a multiperipheral model or diffraction scattering alone. In particular, the mean angle which a secondary particle makes with the axis of a shower must be increased. Since this is precisely the type of behavior that one should expect to explain the observations[1] of high-energy cascades, it is of interest to make numerical estimates with a simple model of extensive air showers. If they are promising, then there is a case for the large amount of computational work required for a more detailed simulation of the showers.

Calculations based on a simple model are presented in this paper. In addition to the question of pion cascades, the consequences of the model for high-energy muons are examined. The theory of the contact interaction is outlined in Sec. II. The design of the program for simulation is explained in Sec. III, and the results are reviewed in Sec. IV. Subsequent sections are devoted to remarks on the model and the results.

II. THEORY OF PROTON-PROTON SCATTERING

The discussion will follow closely that of Abarbanel, Drell, and Gilman.[10] Let the "driving" terms for the proton-proton T matrix be

$$g^2G^2(t)\bar{u}(p_4)\gamma_\mu u(p_2)\bar{u}(p_3)\gamma_\mu u(p_1)+B(s,t)\,, \qquad (1)$$

where g is the coupling constant for the contact inter-

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[10] H. D. I. Abarbanel, S. D. Drell, and F. J. Gilman, Phys. Rev. **177**, 2458 (1969).

action, G is the magnetic form factor of the proton, p_1 and p_2 are initial four-momenta, p_3 and p_4 are final four-momenta, $s=(p_1+p_2)^2$, $t=(p_1-p_3)^2$, and $B(s,t)$ represents all the scattering agencies except the contact interaction. The form (1) is an approximation, valid if $s\gg -t\gg M^2$, where M is the proton mass, but the condition on s and t is well fulfilled in the early stages of extensive air showers.

Construction of a unitary S matrix at cosmic-ray energies is not possible in our present state of knowledge, particularly about the available inelastic channels in proton-proton scattering, but a method of Blankenbecler and Goldberger[11] for satisfying the requirements of unitarity in an approximate manner can be used. Consider the Fourier-Bessel representation

$$T_{mn}(s,t)=\int_0^\infty bdb\, J_0(b\sqrt{-t})H_{mn}(b^2,s)\,, \qquad (2)$$

with an impact parameter b, for the T-matrix element for scattering from an initial state of n particles to a final state of m particles. In the limit where s is much larger than the square of the invariant mass of any intermediate state, which is certainly justifiable for high-energy cosmic-ray showers, the implication of unitarity for (2) is that

$$\mathrm{Im}H_{mn}(b^2,s)=\sum_i H_{mi}(b^2,\, s+i\epsilon)\rho_i(s)H_{in}(b^2,\, s-i\epsilon) +O(s^{-1})\,, \qquad (3)$$

where the sum extends over all i-particle intermediate states whose density with respect to s is $\rho_i(s)$. To make the calculation workable, it is now assumed[10,11] that the sum in (3) is restricted to $i=2$. There may be some extra support here for the assumption, because the construction of fireball-type cosmic-ray models can proceed quite satisfactorily with only two-particle intermediate states. Then, if $q=(-t)^{1/2}$, the inversion of (2) is

$$H_{22}(b^2,s)=\int_0^\infty qdq\, J_0(bq)T_{22}(s,\, -q^2)\,. \qquad (4)$$

It is advantageous to split the Fourier-Bessel transform of (1) into a part H_c from the contact interaction and a part H_d, which refers to diffraction scattering and is generated by $B(s,t)$. The transformed version of (1) is thus the sum of two parts,

$$H_c(b^2,s)=g^2\int_0^\infty qdq\, J_0(bq)G^2(-q^2) \qquad (5)$$

and

$$H_d(b^2,s)=\int_0^\infty qdq\, J_0(bq)B(s,\, -q^2)\,, \qquad (6)$$

[11] R. Blankenbecler and M. L. Goldberger, Phys. Rev. 126, 766 (1962).

where the removal of spinor factors from (5) is consistent with the eventual writing of the differential cross section in the simple form (9) below. The solution of (3) with (4) for $i=2$, according to an approximate method of Baker and Blankenbecler,[12] is

$$H_{22}(b^2,s)=\frac{H_c(b^2,s)+H_d(b^2,s)}{1-I(s)[H_c(b^2,s)+H_d(b^2,s)]}\,, \qquad (7)$$

where

$$I(s)=\frac{1}{\pi}\int_{s_0}^\infty \frac{ds'\rho_2(s')}{s'-s}=\frac{1}{(4\pi)^{1/2}}\left(i+\ln\frac{s}{s_0}\right) \qquad (8)$$

is an integral over the two-body density-of-states factor, and s_0 cuts off the integral so that values of s' which contradict the limit under which (3) holds are not considered. Here $s_0\gg(2M)^2$. During the operation of the computer program, s_0 is chosen to be 16 GeV^2.

The T-matrix element is obtained by the substitution of (5)–(8) into (2), and the differential cross section of $d\sigma/dt$ for proton-proton elastic scattering is given by

$$d\sigma(s,t)/dt=|T(s,t)|^2\,. \qquad (9)$$

The total cross section is obtained from (9) by integration. The contribution of the contact-interaction term in (5) is determined once a functional form for G is chosen. The conventional dipole expression

$$G(t)=(1-t/t_0)^{-2}\,, \qquad (10)$$

with $t_0=0.71$ GeV^2, is selected here because it leads to an analytic result for (5) in terms of a modified Bessel function of the second kind, and because it causes a rather slow approach of $T(s,t)$ to the Pomeranchuk limit. From the latter point of view, the parametrization of form factors given by Wataghin[13] may be better than (10), but no closed analytic result for (5) seems to be available in that case.

Because $B(s,t)$ contains all of the relevant strong-interaction physics, exact evaluation of (6) is not possible. However, all that is required here is an expression for $H_d(b^2,s)$ which permits the deduction of a differential cross section consistent with experimental observations. The choice

$$H_d(b^2,s)=\frac{(4\pi)^{1/2}}{i} \times\frac{(\alpha+i\beta)\exp\{-t_0b^2/R_0^2[1+\ln(s/s_0)]\}}{1+(\alpha+i\beta)\exp\{-t_0b^2/R_0^2[1+\ln(s/s_0)]\}} \qquad (11)$$

is therefore suitable, since it reproduces indirectly[10] the exponential fall in $d\sigma/dt$ with t for small values of momentum transfer. The logarithmic dependence on s in (11) is inserted to model the shrinkage of the diffraction peak in strong scattering. Abarbanel, Drell, and

[12] M. Baker and R. Blankenbecler, Phys. Rev. 128, 415 (1962).
[13] V. Wataghin, Nuovo Cimento 54A, 840 (1968).

Gilman[10] have regarded g^2, α, β, and a factor similar to R_0^2 as free parameters to be determined either by best fits to experimental data or by certain boundary conditions. The former method has led to their estimate of

$$g^2/4\pi \approx 5.1\,, \tag{12}$$

a medium-strong coupling which will be assumed below. The other boundary conditions used are

$$(d\sigma/dt)_{t=0} \approx 80 \text{ mb/GeV}^2\,, \tag{13}$$
$$d\sigma/dt = (d\sigma/dt)_{t=0} e^{10t}\,, \quad \text{for} \quad t \approx 0$$

and

$$d\sigma/dt \propto G^4(t)\,, \quad \text{for large } |t|\,. \tag{14}$$

Justification[10] for conditions (13) may be found in many compilations of experimental data. The same is true of (14), but that relation is historically important because it has led to the original suggestion of the contact-interaction theory.[9]

The previous equations are intended above all to imply (14). Abarbanel, Drell, and Gilman first proposed a differential cross section of the type

$$d\sigma(s,t)/dt = d\sigma(s,0)/dt[aG^4(t) + f(t)e^{(s/s_0)\ \ln 2[\alpha(t)-1]}] \tag{15}$$

[where $\alpha(t)$ is a Regge trajectory, probably the vacuum trajectory], in which the contact term involving G became exposed and stopped the shrinkage of the diffraction peak from the second term beyond some finite negative value of t. The importance of such a behavior for the explanation of pion cascades has already been mentioned in Sec. I. While the approximately unitary calculation based on (2) cannot be reduced to the evaluation of one simple expression like (15) and, in fact, requires numerical computation, the use of (14) nevertheless ensures that the physical interpretation from (15) is preserved.

III. CHARACTERISTICS OF SIMULATED SHOWERS

To avoid the complications that come with large numbers of independent amplitudes, let us ignore neutrons and assume that only one process of proton-proton inelastic scattering which is free from the contact interaction competes with elastic scattering. A simulated shower is made up of the following reactions:

$$p+p \rightarrow p+p\,, \tag{16}$$
$$p+p \rightarrow p+N_{11}^{*+}(1470)\,, \tag{17}$$
$$N_{11}^{*+}(1470) \rightarrow p+\pi^- +\pi^+\,, \tag{18}$$
$$N_{11}^{*+}(1470) \rightarrow p+\pi^0\,, \tag{19}$$
$$\pi^0 \rightarrow \text{electromagnetic cascade}\,, \tag{20}$$
$$\pi^+ \rightarrow \mu^+ + \nu_\mu\,, \quad \pi^- \rightarrow \mu^- + \bar{\nu}_\mu\,. \tag{21}$$

The 1470-MeV nucleon isobar in (17) is the best-known object of spin and isospin $\frac{1}{2}$ (chosen to remove spin and isospin complications from the discussion) which can be present in an isobar model of production of secondary cosmic-ray pions.

It is assumed that an isotropic flux of protons of energies between 10^{14} and 10^{20} eV is incident at the top of a spherically symmetric standard[14] atmosphere. The number density of the protons in this range of energy is given an $E^{-5/2}$ power dependence, in approximate agreement with a summary[15] of observational evidence. The average energy per proton is thus 3×10^{14} eV. A cascade detector in the form of a square array whose side length is 1.2 km is taken to lie on the surface of the earth. Directly below its central point, and separated from it by a vertical depth of 2000 cm of mercury, is a hypothetical high-energy muon detector with a square top face having sides 800 cm long. These specifications resemble fairly closely the descriptions of the apparatus which has been used for cascade[1,15] and muon[2] measurements. In the program, a search is made for pion events from (20) at the earth's surface and for muons from (21) with sufficiently high energy to penetrate to the subterranean detector through the outer crust, an imaginary shell of mercury with a thickness of 2000 cm.

Cross sections for (16) are calculated numerically by the methods of Sec. II. For the competing reaction (17), whose asymptotic behavior seems to set in at rather low energies, the following values[16] are used:

$$\begin{aligned} \sigma_{17} &= \text{const} = 0.7 \text{ mb}\,, \\ d\sigma_{17}/dt &= 6e^{18t} \text{ mb}, \quad |t| < 0.25 \end{aligned} \tag{22}$$

where t is measured in units of GeV^2. When a nucleon isobar is formed by (17), it is assumed to decay immediately through (18) or (19), the relative frequencies of these two reactions being[17] 0.45 and 0.55.

The program follows the path of a proton through the atmosphere by numerical integration up to one collision depth for the reaction (16). If this proton is a representative of N protons with energy variable s, and if σ_{16} is the cross section for (16), then $N\sigma_{16}(s)/[\sigma_{16}(s)+\sigma_{17}]$ protons are assumed to undergo (16), and $N\sigma_{17}/[\sigma_{16}(s)+\sigma_{17}]$ protons take part in (17) at the point of collision. The appropriate angular distributions of the secondaries are obtained from (9) and (22), respectively.

The protons emerging from this stage of the calculation are used as initial protons for another iteration in the same loop of the program. A proton is discarded when its energy has fallen to 3×10^{11} eV or less.

As an initial step, the program calculates the angles θ_π and θ_μ subtended at the point of entry of the primary

[14] C. W. Allen, *Astrophysical Quantities* (Athlone Press, London, 1964).
[15] C. B. A. McCusker, in Proceedings of the Tenth International Conference on Cosmic Radiation, Calgary, 1967 (unpublished).
[16] E. W. Anderson, E. J. Bleser, G. B. Collins, T. Fujii, J. Menes, F. Turkot, R. A. Carrigan, R. M. Edelstein, N. C. Hien, T. J. McMahon, and I. Nadelhaft, Phys. Rev. Letters **16**, 855 (1966).
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into the atmosphere by the pion cascade and muon detectors. If the path of any proton in a shower ever leaves a cone of apex angle $\frac{3}{2}\theta_\pi$, it is discarded. A similar criterion is applied to neutral pions when the apex angle is θ_π, and to charged pions and muons when it is θ_μ.

Reactions (18) and (19) are treated in the barycentric system of the decaying isobar. In that system, the momenta of all but one of the products of the decay are obtained by random-number generation. The value for the last of the products is assigned according to the conservation of energy and momentum. Momenta are then transformed back to the system, fixed with respect to the earth.

The quantity relevant to the π^0 which is measured in a ground-based cascade detector[15] is rp_L/h, where r is the separation of the cores of a pair of cascades at ground level, h is the height of production of the π^0, and p_L is its component of momentum along the original direction of the primary particle. This quantity is a measure of the momentum of the pion perpendicular to the axis of the shower. The values of p_L and h are calculated immediately upon the production of a neutral pion of energy greater than or equal to 10^{12} eV, and the position of the intersection of its path with the pion detector is recorded also. The last item of information is thus available for the determination of r for pairs of events when the simulation is completed, and a collection of values of rp_L/h is thus computed.

The program follows the tracks of charged pions of energy greater than 2×10^{11} eV from their points of production toward the muon detector, making allowance for pions absorbed by collisions with atmospheric protons and muons [from atmospheric decay (21) of pions] absorbed by nuclei of mercury in the "earth." Muons that are considered to have reached the detector are recorded by zenith angle of arrival and by whether they have energies greater than 10^{11} eV or energies greater than 7.5×10^{11} eV. The latter distinction is made for comparison with the measurements of Nash and Wolfendale,[18] who state that they do not have evidence to support the results of the Utah experiment[2] in the range of energy between 5×10^{11} and 10^{12} eV.

IV. SUMMARY OF NUMERICAL RESULTS

For pion cascades, plots[1,15] of rp_L/h against primary energy E (if one accepts the principle that 10^9 times the number of particles in a cascade is roughly equal to the primary energy in eV) show a decided gap between energies of 10^{14} and 10^{15} eV. Above 10^{15} eV, values of rp_L/h are scattered widely in the range 1–100 GeV/c, as distinct from the clustering about a mean of 0.5 GeV/c when the primary energy is less than 10^{14} eV.

If $y=\log_{10}(rp_L/h)$, and $x=\log_{10}E$, no straight-line plot of y against x from the results of the program is satisfactory. However, a good fit is obtained by the curve $y=(x-14)^2$ for $14\leqslant x\leqslant 16.5$. This tends to undervalue the distribution of y presented by McCusker,[15] but its trend is in the right direction to match the experimental results. Because of the low-energy criteria quoted in Sec. III to cut off the operation of certain sections of the program, there is a cluster of events around $E=10^{14}$ eV, for which $y\leqslant 0$. To examine the ability of the program to reproduce lower-energy values of y correctly, the lower limit of simulated primary energies should be reduced by one or two orders of magnitude, but this implies an increase in the amount of computing time which seems to be out of all proportion to the modest scope of the present project. Nevertheless, this crude simulation has proved to be able to generate values of rp_L/h up to 7 GeV/c, while previous Monte Carlo simulations[15] have not been able to do much better than 0.5 GeV/c. Thus a medium-strong contact interaction between protons may have a significant role to play in the explanation of cosmic-ray phenomena at high energies.

Muon detection is expected to show a "secθ" enhancement, where θ is the zenith angle, on the basis of an argument[2] which balances the possibility of nuclear interaction of potential parent particles in the atmosphere near $\theta=0$ against the possibility of muon production by decay on the more rarefied atmospheric paths as $\theta\to\frac{1}{2}\pi$. Hence, the power n of secθ in the distribution of muons with direction of arrival at the detector is expected, on nonrigorous grounds, to be 1. By contrast, the Utah group reports a value of $n=0.25\pm0.09$, and therefore concludes[4] that the muons are produced directly or by the decay of a short-lived parent with properties quite different from those of the particles normally considered to be constituents of extensive air showers. The only prospect that the theory presented in this paper can modify the argument is a geometrical one, because the geometrical properties of the development of a shower are affected somewhat, through (2) and (9), by the presence of the medium-strong interaction. Hence, the calculation of the muon distribution has been approached more as an entertaining addition to the pion calculation than as anything else.

The first conclusion to be drawn from the output of the program is that 2000 cm of mercury is a very efficient absorber of muons. Although the output is sorted into 5° bins, there are probably only enough "events" to compare the number arriving between angles of 0 and ϕ with the number between ϕ and 2ϕ. Since the maximum reliable angle in the Utah experiment is reported[2] to be 80°, the ratio

$$\int_0^\phi \sec^n\theta d\theta \Big/ \int_\phi^{2\phi} \sec^n\theta d\theta, \quad \phi=40°$$

has been compared with the output, and the value of n which gives the best agreement in each case has been noted. Where the muon energy E_μ is greater than 10^{11}

[18] W. F. Nash and A. W. Wolfendale, Phys. Rev. Letters **20**, 698 (1968).

eV, the best exponent is 0.74. Where $E_\mu > 7.5 \times 10^{11}$ eV, n falls to 0.61. Although some way from the Utah results, these estimates of n suggest a trend that may help to explain the discrepancy between that result and the measurements of Nash and Wolfendale.[18]

V. REMARKS ON DIRECT PRODUCTION OF MUONS

Of the various models which have been advanced for muon production, the only type more efficient than the present model is one which predicts copious (i.e., fairly strong) production of muons by a process directly linked to the primary particles and possibly mediated by an intermediate vector boson. A difficulty in such schemes is that some new form of strong coupling of the vector boson or mediator to muons (but presumably not electrons) must be postulated.[19] That coupling has been proposed as a source for the muon-electron mass difference,[5] and its implications for the Utah experimental results have been examined further by several authors.[20,21] Apart from the theory in Sec. II, there is the possibility that it is actually the medium-strong interaction that may be involved in direct production. Ne'eman[22] has already suggested the interaction as the $SU(3)$-breaking agent, and it is tempting to consider that the muon-electron mass difference is of the same order as the basic mass differences between particles in any one $SU(3)$ multiplet.

The intermediate vector boson does not fit very happily into $SU(3)$-symmetry-breaking schemes. Moreover, recent estimates[6] of its mass make it into a dynamically unusual particle. The widths of heavy resonances tend to increase as their mass increases,[17] which implies a situation at above, say, 10 GeV, in which their mutual interference produces only a continuum. One can draw on analogies in the behavior of complex electrical and mechanical systems for illustration. Therefore, an effect at a mass of about 30 GeV with a width small enough to allow its classification as a distinct elementary particle is unlikely to be found. Thus, one appears to be compelled to return to contact interactions or a local four-fermion process in the sense of the weak interaction.

[19] S. V. Pepper, C. Ryan, S. Okubo, and R. E. Marshak, Phys. Rev. **137**, B1259 (1965).
[20] S. Pakvasa, S. F. Tuan, and T. T. Wu, Phys. Rev. Letters **20**, 1546 (1968).
[21] J. D. Bjorken, S. Pakvasa, W. Simmons, and S. F. Tuan, Phys. Rev. (to be published).
[22] Y. Ne'eman, Phys. Rev. **172**, 1818 (1968).

The contact term in (1) is reminiscent of the weak current-current interaction, but in that case the current in (1) possesses only a (pp) bilinear part. If a $(\mu\mu)$ part is added, direct muon-pair production by proton-antiproton annihilation becomes possible, but then there is a prediction of a muon-proton interaction stronger than electromagnetism that is contradicted by measurements of low-energy scattering and the magnetic moment of the muon. Hence, if the effect exists at all, it must be suppressed until energies of the order of 10^{12} eV are reached. It is interesting to note that the conventional weak interaction, also a contact interaction, needs modification at just this characteristic energy to remove troubles with unitarity. The cures for the problems of these two interactions may therefore be the same. After trying various other unprofitable alternatives,[8] it is difficult to avoid the conclusion that the idea of a fixed coupling constant given from "outside" the theory should be abandoned in favor of some dynamical procedure of determining the (probably variable) coupling strength, which in the interests of economy may even link the two interactions.

The above discussion is intended to touch on some of the problems raised by theories of direct production of muons at very high energies, and thus to indicate why the less efficient theory of indirect production given in Secs. II and III, which does not face the same problems, has been chosen for the present work.

VI. CONCLUSIONS

The results reported in Sec. IV show that the introduction of a medium-strong contact interaction into a simulation of a cosmic-ray shower has an appreciable effect, which is of the correct type to assist in explanations of recently observed high-energy phenomena. In particular, it seems to lead fairly naturally to the production of neutral pions with unusually large transverse momenta. It is possible that the effect of the contact interaction will become obscured if more of the conventional scattering reactions than those considered in the present model are added to the simulation, but it appears that the contact interaction itself should always be taken into account in more complex models.

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