ions. At higher chromium concentrations oxidation and reduction experiments[10] show that the dominant optical absorption in the vicinity of 5500 Å is due to several percent of the total Cr concentration which is in the 4+ valence state.

In conclusion, the transverse Zeeman patterns of the purely electronic transition of the 8000-Å fluorescence can be consistently interpreted as the transition from the ^2E_g state to the $^4A_{2g}$ state of the chromium ion. Below the cubic to tetragonal phase transition at 107°K the v component of the ^2E_g state lies highest in energy, indicating that the tetragonal distortion at the chromium site is tensive in nature. This result is consistent with a c/a ratio >1 in the tetragonal phase.

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[10] B. W. Faughnan (private communication); see B. W. Faughnan and Z. J. Kiss [Phys. Rev. Letters **21**, 1331 (1968)] for a similar effect in $SrTiO_3$:Fe.

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Energy-Loss Straggling of a Helical Electron Beam in a Magnetized Foil

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The energy-loss process of a helical negatron or positron beam of a few MeV in a thin magnetized foil was studied. The straggling distributions were derived, taking into account spin-dependent terms of the Möller and Bhabha cross sections. These terms affect the shape of the straggling distributions by an amount which increases with increasing energy losses. Numerical calculations were carried out in some typical cases, which are of interest with a view to using the spin-polarization effect as a means of measuring the helicity of β rays.

1. INTRODUCTION

THE energy-loss straggling of a helical electron beam passing through a thin magnetized foil depends on the relative polarization of the beam and of the absorber. Recently, Braicovich[1] suggested that this effect could provide a new powerful method for β-ray helicity measurements. Since the available theoretical evaluations of the straggling[2] are valid in the absence of any spin-polarization effect, it has been considered worthwhile to carry out an analysis of the energy-loss process of negatrons and positrons in magnetized matter, taking into account the spin-dependent terms of the single-scattering cross section. This investigation provides the means of obtaining an evaluation of the magnitude of the polarization effect under conditions which are realistic with a view to using the effect for β-ray helicity measurements.

2. ENERGY STRAGGLING CURVE

Landau,[3] in his theory of the straggling, has shown that the straggling distribution $f(\Delta,x)$, giving the probability density of an energy loss Δ suffered by a particle passing through a layer of thickness x, may be obtained from the following expression:

$$f(\Delta,x)=\frac{1}{2\pi i}\int_{-i\infty+\sigma}^{+i\infty+\sigma} e^{p\Delta-xg(p)}dp\,, \tag{1}$$

where

$$g(p)=\int_0^{\epsilon_{\max}} W(\epsilon)[1-e^{-\epsilon p}]d\epsilon. \tag{2}$$

In Eq. (2), $W(\epsilon)$ is the probability density (per unit path length) of an energy loss ϵ in a single collision, and $\epsilon_{\max}$ is the maximum energy transfer in a single collision. It is assumed that the total energy loss in the path x is small compared with the initial energy, so that the scattering process can be considered to be adequately described by the same function $W(\epsilon)$ throughout the whole succession of scatterings which cause the slowing down.

In order to apply Eq. (1) while allowing for the spin effect in the present treatment, the following approximations have been made.

(i) The "soft collisions," in which the energy transfer is of the order of the atomic binding energies, are treated in the same way as in Landau's work, and no spin effect is considered. The integration interval in Eq. (2) is accordingly split into two parts: 0, ϵ_1 and ϵ_1, $\epsilon_{\max}$, where ϵ_1 is the separation energy defined by Landau. The integral in the first interval is calculated by replacing the exponential $e^{-p\epsilon}$ with $1-p\epsilon$. As a con-

[1] L. Braicovich, Letters Nuovo Cimento **1**, 340 (1969).
[2] See R. D. Birkhoff, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1958), Vol. 34, p. 53.
[3] L. D. Landau, J. Phys. USSR **8**, 201 (1944).

sequence, only the first moment of the cross section for bound electrons contributes to $g(p)$. In this connection it must be remembered that Blunck and Leisegang[4] also took into account the second moment; such an improvement is not always necessary, unless the scattering foil is very thin, as has been pointed out by Knop *et al.*[5]

(ii) In the collisions where the effect of atomic binding can be neglected ("intermediate" and "hard collisions," with $\epsilon > \epsilon_1$) the free negatron-negatron and positron-negatron scattering cross sections dependent on spin polarization are considered. The following approximate expression of the single scattering law $W(\epsilon)$ has been assumed:

$$W(\epsilon)=\sum_{n=-2}^{+1} a_n \epsilon^n \quad (\epsilon_1 \leq \epsilon \leq \epsilon_{\max}). \tag{3}$$

The term with $n=-2$ corresponds to the single-scattering law used by Landau. The coefficients with $n \geq -1$ depend on spin polarization and are specified below. For the purpose of the present work, the assumed representation of $W(\epsilon)$ is sufficiently accurate, and it is discussed later.

(iii) The longitudinal depolarization suffered by the beam while slowing down is neglected, on the basis of the known phenomenology.[6] This approximation is reasonable, since the total energy loss is assumed to be small with respect to the energy of the incident beam, and since the depolarization is small even when the relative energy loss is large.[7]

(iv) The energy loss due to bremsstrahlung emission is neglected, as in Landau's work. Thus, the present treatment is valid for primary energy not greater than a few MeV.

Remembering the above assumptions, we obtain the following expression for $g(p)$:

$$g(p)=\int_0^{\epsilon_1} \epsilon W(\epsilon) d\epsilon + \sum_{n=-2}^{+1} a_n \int_{\epsilon_1}^{\epsilon_{\max}} \epsilon^n (1-e^{-\epsilon p}) d\epsilon. \tag{4}$$

Accordingly as in point (i), the first integral is taken from Landau's work. The other integrals can be calculated in an elementary way; the result is put in a simpler form by remembering the condition $\epsilon_1 \ll \epsilon_{\max}$. We point out that the approximation $\epsilon_{\max} \to \infty$, which is made in Landau's work, has not been retained here. By choosing the imaginary axis as integration path in Eq. (1), one obtains for the straggling distribution the integral representation

$$\phi(\lambda,\rho)=f(\Delta,x)\frac{d\Delta}{d\lambda}=\frac{1}{\pi}\int_0^{\infty} \mathcal{A}(t,\rho)\mathcal{B}(t,\rho,\lambda)dt. \tag{5}$$

The functions $\mathcal{A}(t,\rho)$ and $\mathcal{B}(t,\rho,\lambda)$ are given by

$$\begin{aligned}\mathcal{A}(t,\rho)=\exp\{&-t[\tfrac{1}{2}\pi+\mathrm{Si}(t\rho)-(1-\cos(t\rho))/t\rho] \\ &-\mu[C+\ln(t\rho)-\mathrm{Ci}(t\rho)]-\nu[1-\sin(t\rho)/t\rho] \\ &-\xi[\tfrac{1}{2}+(1-\cos(t\rho))/(t\rho)^2-\sin(t\rho)/t\rho]\}\end{aligned} \tag{6}$$

$$\begin{aligned}\mathcal{B}(t,\rho,\lambda)=\cos\{&t[\lambda-\ln t+\sin(t\rho)/t\rho-\mathrm{Ci}(t\rho)] \\ &-\mu[\tfrac{1}{2}\pi+\mathrm{Si}(t\rho)]-\nu[1-\cos(t\rho)]/t\rho \\ &-\xi[\sin(t\rho)/(t\rho)^2-\cos(t\rho)/t\rho]\},\end{aligned} \tag{7}$$

where Si and Ci denote sine and cosine integrals, and C is Euler's constant. The variables ρ and λ are defined by

$$\rho=\epsilon_{\max}/a_{-2}x \tag{8}$$

and

$$\lambda=\Delta/a_{-2}x-\ln(a_{-2}x/\epsilon')-1+C, \tag{9}$$

where

$$\ln\epsilon'=\ln[(1-\beta^2)I^2/2m_0v^2]+\beta^2, \tag{10}$$

with I representing an average ionization energy of the atoms in the absorbing foil.

The coefficients μ, ν, ξ are defined by

$$\mu=a_{-1}x, \tag{11}$$

$$\nu=a_0 x \epsilon_{\max}, \tag{12}$$

$$\xi=a_1 x \epsilon^2_{\max}. \tag{13}$$

Note that Eq. (5) is an extension of Landau's results and that it reduces to the universal function $\phi(\lambda)$ obtained by this author when $\mu=\nu=\xi=0$ and the limit $\rho \to \infty$ is taken. In the present formulation the result depends not only on the dimensionless variable λ but also on the dimensionless variable ρ and on the energy of the incident beam through the explicit expressions for μ, ν, and ξ.

3. SINGLE-SCATTERING LAW

When the magnetization of the absorbing foil is along the direction of the helical beam, the function $W(\epsilon)$ may be obtained from the single-scattering differential cross sections for the parallel and for the antiparallel spin cases, $(d\sigma/d\epsilon)_p$ and $(d\sigma/d\epsilon)_a$. It is given by

$$\begin{aligned}W(\epsilon)=\tfrac{1}{2}NZ\{&[(d\sigma/d\epsilon)_p+(d\sigma/d\epsilon)_a] \\ &\pm f[(d\sigma/d\epsilon)_p-(d\sigma/d\epsilon)_a]\},\end{aligned} \tag{14}$$

where NZ is the number of atomic electrons per unit volume and f is the oriented fraction of target electrons. The upper (lower) sign holds when the helicity of the beam is positive (negative) in respect of the magnetization of the foil.

In both cases, negatron-negatron and positron-negatron scattering, the expressions of the spin-dependent cross sections have been taken from the work of Ford and Mullin,[8] whose calculations were based on the lowest-order Feynmann diagrams. The following

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[5] G. Knop, A. Minten, and B. Nellen, Z. Physik **165**, 533 (1961).
[6] L. Braicovich, B. De Michelis, and A. Fasana, Nucl. Phys. **63**, 548 (1965).
[7] L. Braicovich, B. De Michelis, and A. Fasana, Phys. Rev. **164**, 1360 (1967).
[8] G. W. Ford and C. J. Mullin, Phys. Rev. **108**, 477 (1957); **110**, 1485 (1958).

explicit expressions for the coefficients a_{-2}, μ, ν, and ξ are obtained:

$$a_{-2}=2\pi e^4NZ/m_0v^2 \tag{15}$$

(negatron-negatron scattering),

$$\mu=(1/\rho)(\epsilon_{\max}/T)[(1-2\gamma)/\gamma^2](1\pm f\gamma), \tag{16a}$$

$$\nu=(1/\rho)(\epsilon_{\max}/T)^2\gamma^{-2}[2\gamma^2-4\gamma+2 \pm f(3\gamma-\gamma^2-3)], \tag{16b}$$

$$\xi=(1/\rho)(\epsilon_{\max}/T)^3\gamma^{-2}[(1-2\gamma)(1\pm f\gamma)+2\gamma^2] \tag{16c}$$

(positron-negatron scattering),

$$\mu=(1/\rho)(\epsilon_{\max}/T)(\gamma^4+2\gamma^3+\gamma^2)^{-1}[-2\gamma^4-4\gamma^3+4\gamma+1 \pm f(2\gamma^2+\gamma^3-2\gamma^2-\gamma)], \tag{17a}$$

$$\nu=(1/\rho)(\epsilon_{\max}/T)^2(\gamma^4+2\gamma^3+\gamma^2)^{-1}[3\gamma^4-5\gamma^2-2\gamma+4 \pm f(-3\gamma^4-\gamma^3+6\gamma^2-\gamma-1)], \tag{17b}$$

$$\xi=(1/\rho)(\epsilon_{\max}/T)^3(\gamma^4+2\gamma^3+\gamma^2)^{-1}[-2\gamma^4+6\gamma^3-6\gamma^2 +2\gamma\pm f(2\gamma^4+2\gamma^3-10\gamma^2+6\gamma)]. \tag{17c}$$

In the above equations, m_0 and e are the rest mass and the charge of the electron, v is the beam velocity, T is the kinetic energy, and γ is the total energy in m_0c^2 units (laboratory system).

4. RESULTS AND DISCUSSION

Numerical calculations of the straggling distribution given by Eq. (5) have been carried out in some typical cases by means of an IBM 7040 computer. The set of values chosen for the initial energy of the beam, for the scatterer thickness, and for the oriented fraction of atomic electrons were, respectively, $\gamma=2.5$ and 4.5; $x=22.3$, 44.6, and 59 mg/cm^2; and $f=0.06$. The above values may be considered realistic in view of an experiment with β rays. In one set of calculations, the maximum energy transfer $\epsilon_{\max}$ was set equal to $0.5T$, which is the maximum value in negatron-negatron scattering and in forward positron-negatron scattering. In another set of calculations, the value $0.25T$ was assumed in order to exclude the effects of harder collisions. The results are nearly equal in both cases, indicating that a very accurate representation of the single-scattering law is unimportant in the region of very great energy transfers; thus the assumed approximate representation for $W(\epsilon)$ may be considered adequate.

As a measure of the spin polarization effect on the straggling, the relative difference $\delta=2(\phi_p-\phi_a)/(\phi_p+\phi_a)$ was calculated from the obtained distributions; the symbols ϕ_p and ϕ_a represent the straggling distributions for parallel and antiparallel polarization, respectively. The relative difference δ is plotted as a function of λ in Fig. 1 for negatrons and in Fig. 2 for positrons. One sees that the polarization effect increases with increasing values of λ, i.e., with increasing energy loss; this increase should be clearly revealed by means of a suitable experimental arrangement. As an example, in case a in Fig. 1, if one counts the particles which have suffered energy losses corresponding to λ values between 25 and 40, one should obtain a parallel-antiparallel counting rate asymmetry of the order of 2.5%. Note that the number of these particles is an appreciable fraction of the incident beam: The integral of the straggling curve between $\lambda=25$ and $\lambda=40$ resulting from the present calculations is equal to 1.6% of the total area. The set of the results obtained provides a quantitative basis for the design of a β-ray polarimeter.

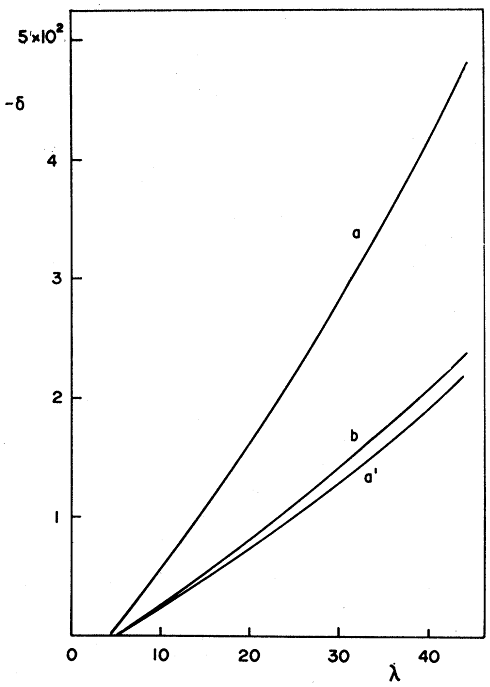

FIG. 1. Parallel-antiparallel relative difference δ of the straggling distributions for negatrons plotted as a function of λ. Curve a: $\gamma=2.5$; $\rho=100$ ($T=0.76$ MeV; $x=44.6$ mg/cm^2). Curve a′: $\gamma=2.5$, $\rho=200$ ($T=0.76$ MeV; $x=22.3$ mg/cm^2). Curve b: $\gamma=4.5$; $\rho=200$ ($T=1.79$ MeV; $x=59$ mg/cm^2).

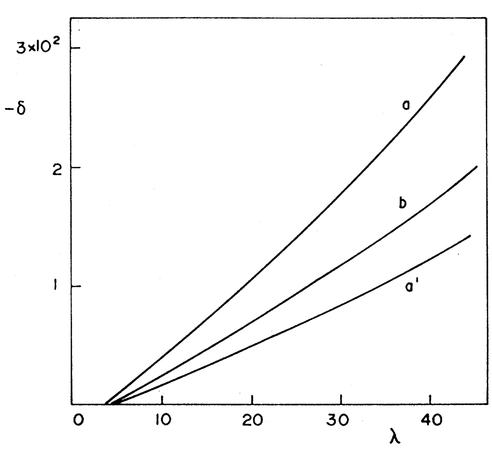

FIG. 2. Parallel-antiparallel relative difference δ of the straggling distribution for positrons, plotted as a function of λ. Curves a, a′, and b: same energies and thicknesses as in Fig. 1.

We remember that the full validity of the present treatment fails if the radiative energy losses are appreciable. Nevertheless, even at an initial energy of a few MeV, a substantial decrease in asymmetry can be avoided by the use of sufficiently thin target foils, as can be deduced from the calculations by Blunck and Westphal,[9] and by Schultz.[10]

APPENDIX

The presented calculations of energy straggling with allowance for spin can be easily extended to particles other than electrons. The case of helical muons has been considered. As can be derived from the cross section given by Backenstoss *et al.*,[11] in this case the coefficient a_{-2} is still expressed by Eq. (15), while

$$\mu=(1/\rho)[-(\gamma^2-1)/\gamma^2\pm f(\epsilon_m/m_\mu c^2)(1/\gamma)],$$

$$\nu=\frac{1}{\rho}\left[\frac{1}{2\gamma^2}\left(\frac{\epsilon_m}{m_\mu c^2}\right)^2\pm f\left(-\frac{1}{\gamma}\frac{\epsilon_m}{m_\mu c^2}+\frac{1}{2\gamma^2}\frac{\epsilon_m{}^2}{m_\mu{}^2c^4}\right)\right],$$

$$\xi=0,$$

where the maximum energy transfer $\epsilon_{\max}$ is given by

$$\epsilon_{\max}=m_\mu c^2\frac{m_0}{m_\mu}\frac{2(\gamma^2-1)}{1+(m_0/m_\mu)(2\gamma+m_0/m_\mu)}.$$

In the above formulas m_0 and m_μ are the electron and muon rest masses. In order to take into account the density effect, the definition (9) of the reduced variable γ must be modified by the addition of a corrective term.[12] Numerical calculations for $f=0$ gave very good agreement with the results obtained by Vavilov.[13] In the case of a magnetized target ($f=0.06$), a very low parallel-antiparallel difference was found in every realistic condition. This fact is due to the very great number of single scatterings which occur in a foil which is thick enough to give a mean energy loss comparable with the resolving power of an actual experimental apparatus.

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Nuclear Spin Relaxation by Translational Diffusion in Liquid Ethane*

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Torrey's theory for nuclear spin relaxation by translational diffusion has been extended to take into account the effect of the radial distribution function. By suitable expansions, the frequency dependence of the intermolecular relaxation rate has been made explicit and shown to be more significant than previously suspected. Measurements of self-diffusion and of the intermolecular relaxation of protons in liquid ethane have been made over a wide range of temperatures and at three frequencies in order to test the theory. Good agreement is obtained with the assumption of an rms flight distance which varies monotonically from about 0.8 to 1.3 times the molecular diameter over the liquid range.

I. INTRODUCTION

In his classic paper on nuclear spin relaxation by translational diffusion in 1953, Torrey pointed out that the essentially microscopic character of nuclear spin relaxation would reflect details of the process of random flights of which diffusion is only the limiting macroscopic approximation.[1] He predicted that studies of relaxation would allow the independent measurement of $\langle r^2\rangle$, the mean-squared flight distance, and τ, the mean time between flights. He well understood that measurement of the self-diffusion constant D would provide the familiar combination

$$D=\langle r^2\rangle/6\tau \qquad (1)$$

and he foresaw that the spin-lattice relaxation time T_1 would be a function of these parameters also.

To our knowledge, no one has yet experimentally investigated in detail these suggestions as they relate to motion in liquids. The 16-yr delay between Torrey's

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[1] H. C. Torrey, Phys. Rev. **92**, 962 (1953).