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Ion Motion and Vortex-Ring Formation in Pure Liquid He^4 and He^3-He^4 Solutions Between 0.05 and 0.5 K

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Measurements of the behavior of charge carriers produced by α particles in pure liquid He^4 and in He^3-He^4 solutions in the temperature range 0.05–0.5 K are reported. Attempts to study very small vortex rings with less than 0.1-eV energy were not successful due to stray electrostatic fields. The origin of these fields was investigated in detail. Above 0.36 K negative ions under pressure in pure He^4 can be accelerated to the Landau velocity for roton production, but it was found that they produce only vortex rings at lower temperatures. The mobilities of both positive and negative ions in He^3-He^4 solutions show temperature dependences in disagreement with all previously proposed theories; however, the negative-ion mobility increases sharply below $T = 0.1$ K in a fashion similar to the predicted phonon-limited mobility. The scattering per He^3 atom is not a simple function of He^3 concentration and does not show any appreciable change as the solution varies from a dilute to a mildly degenerate Fermi system. At 0.2 K the pressure dependence of the negative-ion mobility is consistent with the change of bubble radius derived from other experiments, but at 0.08 K it is different.

I. INTRODUCTION

It has been shown that ions in superfluid He^4 can form vortex rings of quantized circulation. For energies greater than 1 eV, the rings obey a dispersion relation derived from classical hydrodynamics.[1] In a previous report,[2] one of us described experiments on the magnetic deflection of vortex rings of energy $\geqslant 0.1$ eV. It was hoped that from these experiments one could obtain the form of the vortex-ring dispersion relation for very small rings, gain insight into the process by which an ion in liquid helium forms a ring, and determine whether or not there is a minimum size of ring. While the latter experiments did prove the existence of rings of energy ≈ 0.1 V, the data did not allow unambiguous selection of the best form for the dispersion relation. Therefore we decided to attempt to measure directly $V(E)$, the velocity as a function of energy, for very low energy rings. We planned these experiments for temperatures well below 0.1 K, where thermal scattering of the rings would be insignificant. Our apparatus is described in Sec. II. Unfortunately, experimental difficulties associated with the use of small electric fields prevented an exact determination of $V(E)$. These difficulties and a probable cause are discussed in Sec. III. Since we could measure the behavior of ions and rings in He^4 and He^3-He^4 solutions under various conditions of temperature and pressure without modification of the apparatus, we conducted survey experiments in this area. In Sec. IV we report details of ring formation in He^4 under pressure and in dilute isotopic solutions at zero pressure. In Sec. V we report the ion mobilities in isotopic solutions. For simplicity, by the term "ion" we mean a mobile charge carrier, the velocity of which increases with increasing applied electric field, and by "ring" we mean a carrier whose velocity decreases with increasing field.

II. APPARATUS

The cryostat and He^3-He^4 dilution refrigerator are shown schematically in Fig. 1. The main He^4 refrigerant bath, B_1, was kept at 4.2 K except when condensing liquid He into the refrigerator or into the copper experimental cell, C. Bath B_1 was surrounded by an annular liquid-nitrogen chamber, N, and the vacuum enclosure, V. Access to the refrigerator was provided by removing the tail of the vacuum enclosure and copper radiation shields (not shown) which bolted onto V, N, and B_1, respectively. The inner, helium temperature, shield sealed to the bottom of bath B_1 with a Pb O-ring, thus allowing the use of exchange gas for precooling the refrigerator. Sometimes a short indium wire connecting the refrigerator still, D, to B_1 served as a heat switch which could be opened by breaking the wire with a thread attached to a winch at the top of the cryostat. A small He^4 bath, B_2, contained

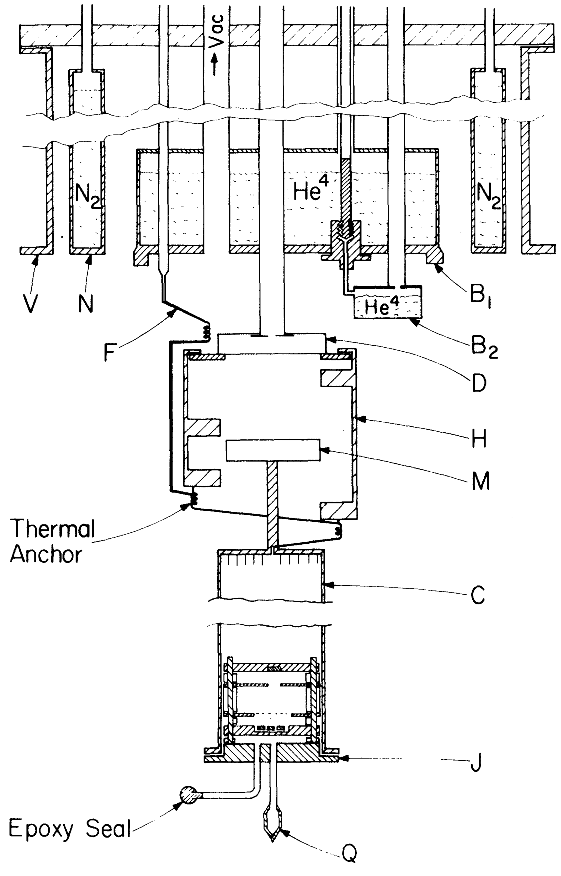

FIG. 1. Schematic diagram of the apparatus, not to scale. Actual grid geometries shown in Fig. 2.

the He^3 recirculation condenser and was kept at 1.1 K. It was continuously replenished from B_1 via a needle valve and choke capillary. The cell C was silver soldered to the stem of the refrigerator mixing chamber, M, made of sintered copper. A spiral corrugated copper foil was silver-soldered along its edge to the top inside surface of C; this provided 10^3 cm^2 of surface area for thermal contact to the experimental liquid He.

Measured amounts of helium gas were passed through a molecular sieve trap at 77 K, condensed in a tube passing through the main bath, and reached cell C via the 0.024 cm i.d. filling capillary, F. F had a 0.010-cm i.d. constriction near the top to restrict film flow and vapor reflux. Fifteen-cm lengths of F were thermally anchored to the still and to two of the refrigerator heat exchangers, H, by low melting-point solders. When the experimental liquid was pressurized, F was necessarily filled with liquid. The resulting heat leak to D increased the He^3 circulation rate in the refrigerator; however, the minimum temperature produced in C was unaffected. Cell C contained about 44 cm^3 of liquid when filled, and had an inside length of 7 cm.

The easily removable base, J, was sealed to the bottom of C with a Pb O-ring. The electrical wires for a carbon thermometer, the ion current collectors, and other electrodes entered C through epoxy seals[3] on the ends of four small stainless tubes in J. Other carbon thermometers were mounted on M and elsewhere. The internal thermometer and all electrodes were rigidly held within C by nylon rods threaded into base J, and were separated by nylon spacers on the rods. The pinch tube Q, at the bottom of the cell, had a replaceable tubular section made of soft solder. Each time the experimental cell was opened this section was replaced. The cell was then initially evacuated to a few microns pressure through a valve and vacuum system connected to the bottom of Q. The valve was then closed, and the cell filled with clean He gas through F. Tube Q was then pinched and its lower end cut off, and the pinch was coated with a layer of $Sn_{0.5}In_{0.5}$ solder to assure a seal. Thereafter C was evacuated and flushed through F.

During a given run, several different mole fractions of He^3 could be produced in C by repeated additions of He^3 gas from a pipette system. At the end of a run the entire contents of C were pumped out through an integrating flowmeter, thus providing a measure of the He^3 mole fraction accurate to ~ 5%. For experiments with mixtures under pressure, the cell had to be kept full of liquid for the entire run, which prohibited repeated additions of He^3. However, repeated application and release of pressure with He^4 gas produced no detectable change in the ion mobility at zero pressure, indicating that the heat flush in the filling capillary F allowed very little He^3 to escape during the release of pressure and the consequent flow of liquid up F.

Static pressures in the system above F were read by a mercury manometer or, for pressures > 1 atm, by a Heise bourdon gauge[4] accurate to $\frac{1}{2}$ psi.

The two electrode geometries used are illustrated in Fig. 2. The arrangement depicted in Fig. 2(a) served as a single-gate velocity spectrometer.[5] Ions in the He, created near the rectangular surface (9.5×3.2 mm) of the Po^{210} α source S(1 to 7 μc) were drawn out of this region toward the grid P by the field ϵ_{SP}. The dc field between each pair of electrodes was established independently by mercury batteries and potentiometers. The two outer collectors were connected together to one electrometer and the center collector was connected to a second Cary vibrating-reed electrometer. All three collectors were of the same rectangular dimensions as the source. This allowed a qualitative determination of ion- or ring-beam spreading. For actual measurements the center collector was used. The same collector geometry was used throughout all experiments. Beams of ions did not spread, so that the current registered by the "outer" electrometer was at most a few percent of the "inner" current. For rings, however, the "outer" and "inner" currents were about equal, under most circumstances. The Fig. 2(a) geometry was intended for creating rings of energy V_{SP}, reducing their energy (if desired) with a reversed field V_{PR}, and measuring their velocity by gating in the region RA. In practice it was found that, at low ring energies, any significant reversed field caused the ring beam to diverge[6] so that the remaining collector current became too small for velocity measurements. Best results were obtained by gating the region PR, with a dc field from R to the collectors.

Figure 2(b) depicts the double-gate spectrometer used in the later experiments. The grids PP′ and AA′ served as gates when driven with equal, in-phase square-wave voltages. The circuit supplying the grid voltages was similar to that used by Anderson *et al.*[7] The dc voltages between all grids, including the gates, were again provided by mercury batteries. The square wave was provided by a diode clamp, driven by a Hewlett-Packard model 211A generator. The diode clamp added a reverse voltage to the gates during the closed portion of the cycle, and was neutral during the open portion. The dc voltages were measured by a Hewlett Packard 740A differential voltmeter. The magnitude of the reverse voltage was not critical, so long as it was sufficient to stop the current and clear the gate regions of ions during the closed portion of the cycle. The purpose of grid S′ will be discussed later. In both geometries all of the electrodes, including the rectangular 80-mesh grids, were gold plated to avoid surface oxides.

Since the apparatus was originally designed for experiments with vortex rings in He^4, no magnetic thermometer was included because the ring properties were expected to be independent of temperature at sufficiently low temperatures. The carbon thermometers were calibrated against the vapor pressure of He^3, and the temperature values quoted in our data refer to the extrapolated calibrations of these thermometers. We believe the temperature values have a systematic error $< 10\%$ and a random error $< 2\%$ at $T \geqslant 0.1$ K. However, comparison with other similar thermometers show that at $T < 0.1$ K, our values would probably be systematically greater than the thermodynamic temperature. A value quoted as 0.06 K therefore represents a thermodynamic temperature $0.04 \leqslant T \leqslant 0.06$ K.

III. EXPERIMENTAL DIFFICULTIES WITH LOW-ENERGY RINGS

Successful conduct of experiments on low-energy rings, as originally planned, was rendered impossible by the presence of stray currents and electric fields within the experimental cell. The following remarks apply to both geometries of Fig. 2. At the outset of any run, once the temperature was brought well below 0.3 K, a positive current $\sim 3 \times 10^{-14}$ A appeared on the electrometers with no electric fields applied within the cell. In either geometry of Fig. 2 this current evidently originated at or near the source S, since it could always be stopped with a negative potential V_{SP} of the order of a few tenths of a volt, even when large positive fields were applied from P to the collectors. This spontaneous current behaved as an uncollimated beam of rings with a spread of energies, and a maximum energy of about 0.3 V (compare Ref. 13).

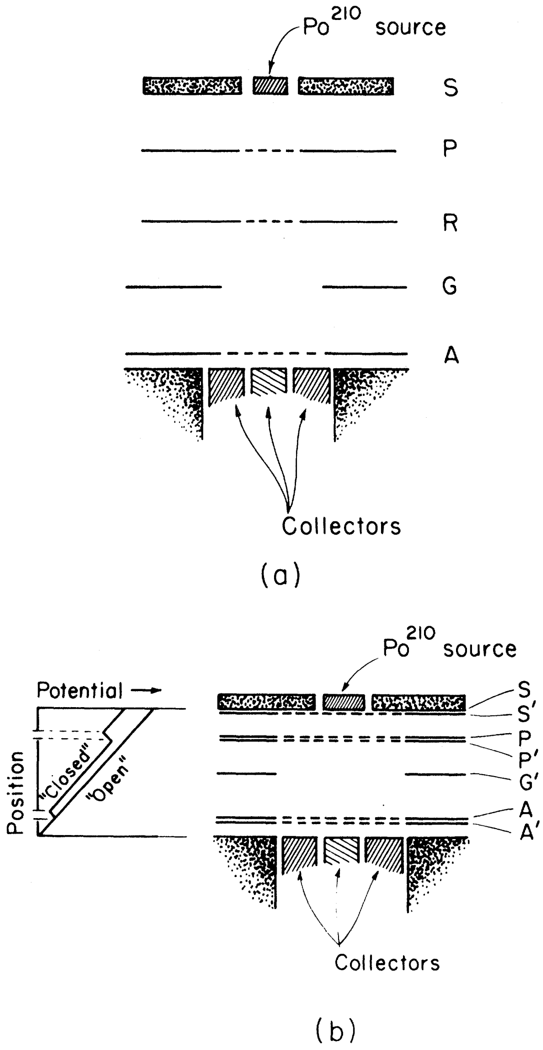

FIG. 2. (a) Grid geometry of single-gate spectrometer. (b) Grid geometry of double-gate spectrometer. The Po^{210} was a thin layer plated on the bottom surface of the "source" piece.

When a negative field ≥ 1 V/cm was applied to the cell for ≥ 1 h and then the applied field returned to zero, the stray positive current crossing the cell was increased by an order of magnitude and required as much as 0.5 V to be stopped. Positive fields of several V/cm, applied for several hours, would generally reduce the zero-field stray current to $\leq 2\times 10^{-15}$ A. By applying either a positive or a negative field between P and the collectors, with V_{SP} zero, we found that sometimes the source region was simultaneously ejecting both positive and negative currents through the grid P, and either sign of current could be stopped by application of a potential V_{SP} of opposite polarity. The current was therefore not due to the uncharged excitation described by Surko and Reif.[8] Application of a positive field from S to the collectors for some time would reduce the positive component of the current and increase the negative component. These stray currents across a field-free region were observed neither at $T > 0.3$ K, nor in He^3-He^4 mixtures, presumably due to the scattering and resultant energy loss of the rings. Repeated "sweeping" of the cell with fields of alternate polarities would occasionally remove all traces of the zero-field stray current crossing the cell; however, the stray current would always re-establish itself over a period of a few hours or less. There may be two processes responsible for this stray current: (1) Formation of positively and negatively charged rings in or near the track of the α particles, and/or (2) The presence of stray fields which could deflect, stop, or add energy to the rings. The changes brought about by "sweeping" the cell would indicate that the variable stray fields may be due to an accumulation of charged impurities on the electrode surfaces. Suppose, for example, that an impurity particle were left positively charged by the removal of an electron from a state of energy level W. Then if W were above the Fermi level of the metal electrode, the impurity could not be neutralized by proximity to the electrode,[9] and the impurity would be bound to the electrode by image forces. Application of a field could gradually remove the charged impurity layer from the surface of the electrode. Although similar undesired effects have been reported previously,[2,7,10] the energy-conserving nature of rings at $T < 0.1$ K enabled us to see the details of these effects more clearly.

By using the region between grids P and R [Fig. 2(a)] as a single-gate spectrometer, we were able to obtain velocity measurements on positively charged rings with $V_{SP} \geq 0.1$ V when the cell seemed least affected by stray fields. The velocity data scatter along and below the $V(E)$ values listed in Table I of Ref. 2, with none of the data coming near the $V(E^*)$ values. This does not, however, give positive evidence for preference of a dispersion relation other than the classical one, since the effects of stray fields, if present, would have been such as to give the rings an actual energy greater than V_{SP}. We did not find certain proof that freely traveling rings of $E < 0.1$ V do not exist; rather, our experiments were always limited by the rapid decrease in collector current as V_{SP} was lowered to 0.1 V. Reducing the SP distance from 6.3 to 3.1 mm, so as to increase the field for a given energy, did not help. In the future, it would be advantageous to use magnetic or electrostatic focusing to direct fast, low-energy rings to a collector.

TABLE I. Fermi temperatures for various C_3.

C_3	1.55×10^{-4}	2.9×10^{-3}	2.89×10^{-2}	4.4×10^{-2}
T_f	0.007 K	0.05 K	0.23 K	0.30 K

In the geometry of Fig. 2(b), the grid S′ was located 0.015 cm from S, which is less than the range of a Po^{210} α particle in liquid helium. It was hoped that, if all of the stray fields formed near S, the stray current might be suppressed by application of a suitable voltage $V_{SS'}$ while rings of unique energy could be drawn from the ionization region below S′. This effort met with only partial success. Much of the stray current could indeed be stopped in this manner, but varying currents were still ejected through grid P′ with the applied $V_{S'P}$ and $V_{PP'}$ both zero. After suitable "sweeping" of the cell with fields of both polarities, a positive field below P′ would draw a positive current to the collector, and this current could be cut off by a reverse potential $V_{PP'} \approx -0.3$ V. A similar situation was true for negative currents, indicating that both polarities of rings were being simultaneously ejected from the ionization in region S′P. If the cell were left field-free or negatively polarized for about one hour, no negative current could be detected, but the intensity and energy of the positive current would be increased, indicating formation of a positive stray field between the grids S′ and P. If the cell were left positively polarized, a similar negative stray field would build up. Qualitatively this indicated that stray fields would develop between grids, although their development was always more pronounced near the source. In later measurements on mobilities of ions in He^3-He^4 mixtures, we found evidence for similar stray fields in the drift space P′A.

IV. ION CRITICAL VELOCITIES AND RING FORMATION

Negative Ions in Pure He^4 Under Pressure

Meyer and Reif[11] and Rayfield[12,13] have shown that, at $T \geq 0.36$ K and pressures of 12–15 atm, negative ions in He II can be accelerated to the

Landau velocity for roton production without producing rings. This possibility of suppressing ring production led us to attempt an experimental verification of Takken's thesis[14] that, even in the presence of a very strong field, ions should be prohibited from measurably exceeding the Landau velocity. The results of Ref. 11 indicated such an effect, but were influenced by scattering due to the presence of thermally exicted rotons and phonons in the fluid. Such scattering should not be significant at temperatures below 0.1 K. Accordingly, we examined the behavior of negative ions under pressure in He^4 in the temperature range 0.5 K $\geq T \geq$ 0.05 K. To our surprise we found that negative ions under pressure at sufficiently low temperatures would create rings only.[15] Attempts at verification of the details of Takken's results were therefore postponed.

At pressures $\geq$ 16 atm and $T \approx$ 0.5 K, our results are consistent with Refs. 11, 12, and 13: Vortex rings comprise a minor fraction of the total current of negative charge carriers. However, as the temperature is reduced, rings comprise an increasing fraction of the current until at all $T \leq$ 0.3 K the current consists of rings only. These results were demonstrated in both geometries of Fig. 2.

In the single-gate spectrometer, for a single species of charge carrier, the current falls off linearly with increasing gating frequency. The carrier velocity is given by $V = 2f_cX$, in which f_c is the frequency at which the current becomes zero, and X is the spacing in the gate region, 0.658 cm. In the double-gate spectrometer, each species of carrier yields a current versus frequency curve which is a series of triangular peaks. The velocity is given by $V = f_mX$, with f_m being the peak separation in cycles per second and X = distance PA = 0.72 cm.

The collector current as a function of frequency, obtained with the single-gate system, is shown in Fig. 3. The gate-closed voltage was sufficient to stop rings as well as ions. The steep drop of current at low frequencies is due to vortex rings. (This was verified at $T <$ 0.1 K by measuring ring energy and velocity in a field-free region.) The current remaining at high frequencies is due to ions. Since rings can be formed in the source region, the ion velocities do not necessarily represent the velocity at which the ions create rings, but do show that rings can be formed by negative ions under 16-atm pressure, while the ions themselves can be accelerated to near the Landau velocity.

Direct recordings of current versus frequency, obtained with the double-gate system, are shown in Fig. 4. The gate-closed voltage was equal and opposite to the gate-open voltage so that ions were gated while rings were unaffected. The peak at 6.2 kc/sec, representing ions at a velocity of 45 m/sec disappears as T is reduced, while the background current due to rings rises. The existence of the rings was again verified by measurements of their velocities and by their ability to pass a potential barrier. The results at 16, 20, and 24 atm were nearly identical. In either geometry, only rings were present at $T \leq$ 0.3 K.

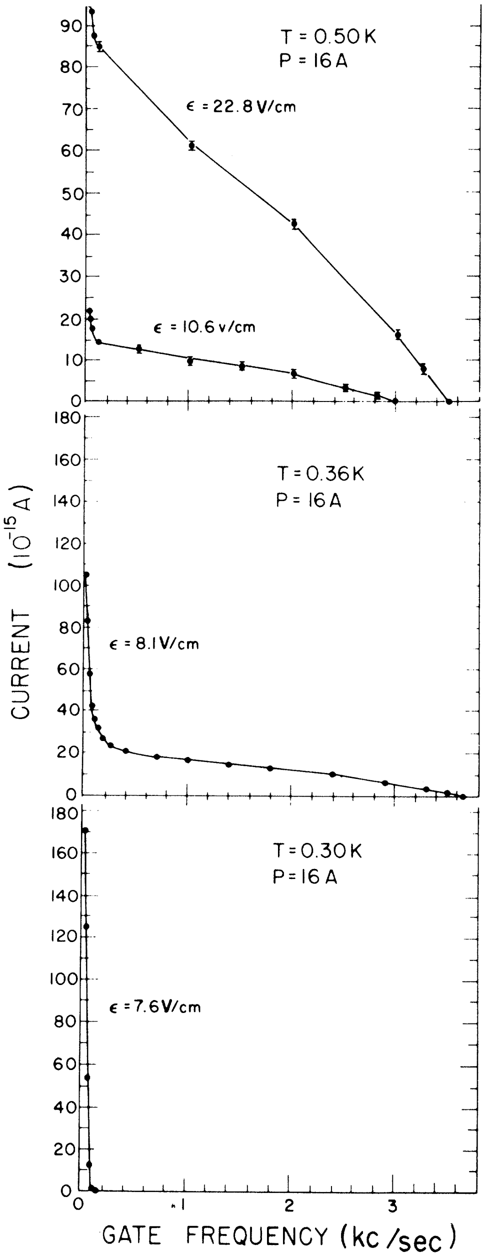

FIG. 3. Collector current as a function of frequency for the single-gate spectrometer.

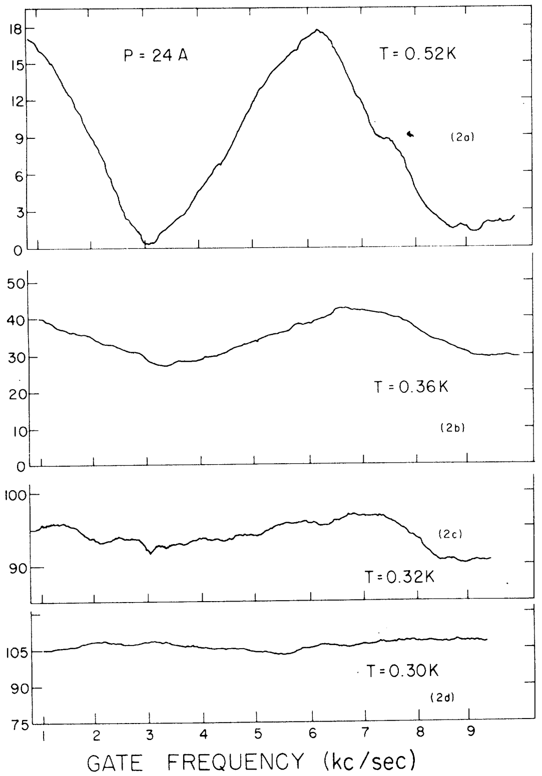

FIG. 4. Collector current as a function of frequency for the double-gate spectrometer. $\epsilon = 25$ V/cm.

Ion Motion in He^3-He^4 Solutions

Field Dependence of the Drift Velocity

In dilute isotopic solutions, $V(\epsilon)$, the ion velocity as a function of field, might be expected to be similar in form to $V(\epsilon)$ when scattering by thermal quasiparticles predominates.[16] That is, V would be proportional to ϵ at low fields, followed by a region where $V(\epsilon)$ curves *below* this linear dependence, followed by a sharp decrease caused by the formation of vortex rings at sufficiently high fields. We have studied ion velocities in dilute He^3-He^4 solutions as a function of electric field up to $\epsilon = 60$ V/cm, He^3 mole fraction (concentration) $1.5\times10^{-4} < C_3 < 4.5\times10^{-2}$, and temperature $0.05 \leqslant T \leqslant 0.5$ K. At low fields the ion velocities do increase linearly with field as expected. At the lower C_3 and at lower temperatures we increased the field until the velocities reached a peak and then decreased, as is characteristic of ring formation. However, the shape of the $V(\epsilon)$ curves at intermediate ϵ was unexpected.

Figure 5 shows $V_+(\epsilon)$, the velocity of positive carriers. As the temperature is increased, the peak velocity and corresponding electric field increases. Note, however, that prior to the peak the velocity rises *above* the extrapolation of the linear low field slope. This may indicate that the scattering is actually reduced by a growing flow pattern as the ion approaches the transition to a stable ring, contrary to what Rayfield finds for thermal quasiparticle scattering at higher temperatures.[13] Alternatively, we could state that the hydrodynamic nature of the carrier appears to change at a velocity significantly below the peak velocity. Rayfield[17] interprets his data as showing that the drag force due to He^3 scattering on a small ring is reduced by the presence of the trapped charge. Our data would indicate that the drag force on the ion is also reduced by the presence of the induced flow before formation of a stable ring.

Figure 6 shows $V_-(\epsilon)$ for the negative carriers. For $T \geqslant 0.1$ K, $V_-(\epsilon)$ also rises above the extrapolation of the low field slope, as does $V_+(\epsilon)$. However, at $T = 0.06$ K, $V_-(\epsilon)$ behaves as expected, curving below the extrapolation. The data of Figs. 5 and 6 were taken with the double-gate system, but the qualitative features of the curves were also produced by the single-gate spectrometer. Hence we feel that the shapes of the curves do not result from some systematic

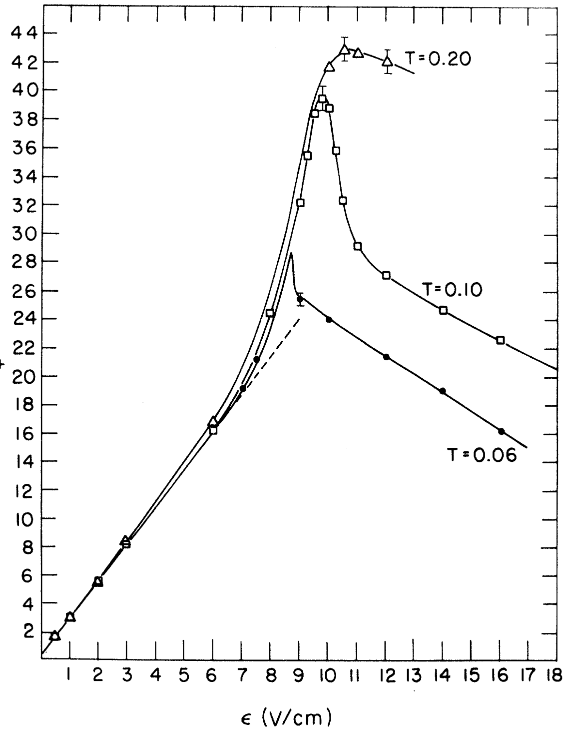

FIG. 5. Positive-ion velocity as a function of applied field. $C_3 = 1.55\times10^{-4}$. The sharp peak at $T = 0.06$ K was revealed by closely spaced data points at $C_3 = 1.82\times10^{-4}$. Half-length of error bars is $\leqslant 2\%$ of data at points where none are shown.

error. Figures 5 and 6 corroborate[17] that under predominantly He^3 scattering the positive carriers form a continuous transition between the ion and ring spectra, while $V_-(\epsilon)$ may be double valued.

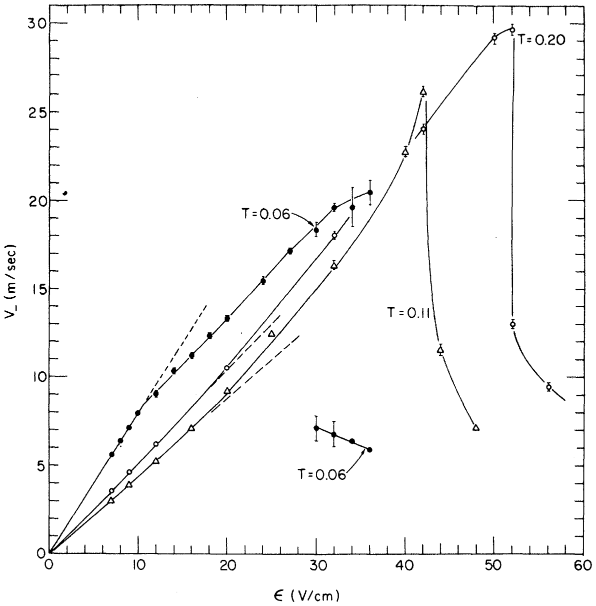

FIG. 6. Negative-ion velocity as a function of applied field. $C_3 = 1.82 \times 10^{-4}$. Half-length of error bars is $\leq 2\%$ of data at points where none are shown.

Discussion of Critical Velocities

In all these discussions it should be kept in mind that the measured drift velocity of the ions is the average not only over many ions with a velocity distribution which is not necessarily given by the Maxwell-Boltzmann distribution, but also an average over the velocity changes between collisions. The observed changes in drift velocity can be due to events taking place at actual ion velocities differing by an appreciable factor from the average shown by the drift velocity. Rayfield[12,13] hypothesized that ring production by an ion is governed by the relation $RV_c = \text{const}$, in which R is the ion radius, V_c is the critical ion velocity at which rings are made, and the constant is different for the positive and negative ions. He reasons that increasing the pressure on the negative ion causes R to decrease, with a consequent rise in V_c until V_c becomes greater than the Landau velocity at some pressure between 12 and 15 atm. His measurements of V_c as a function of pressure[13] agree with other data[18] for the pressure dependence of the negative-ion radius. Presumably, the constant RV_c is determined by the superfluid velocity field near the ion and should therefore be temperature independent. Yet, our experiments would indicate that at some temperature near 0.3 K the critical velocity in pure He^4 suddenly becomes less than the Landau velocity at all pressures ≤ 24 atm.

Assuming the peak value of $V(\epsilon)$ is the critical velocity for ring formation, we can make other comparisons with Rayfield's data. He finds a critical velocity for positive ions of 40 m/sec, independent of temperature and He^3 concentration.[13,19] It can be seen in Fig. 5 that our peak velocity for positive ions at $T = 0.2$ K roughly agrees with his result, but it decreases with decreasing temperature to about 29 m/sec at $T = 0.06$. In pure He^4 at zero pressure, Rayfield found for negative ions a critical velocity of 34 m/sec near $T = 0.6$ K,[13] but with C_3 in the range $(1.25 \text{ to } 4.4) \times 10^{-4}$ at $T = 0.3$ K this was reduced to 31 m/sec.[19] In Fig. 6 it is evident that above $T = 0.1$ K our peak value of $V_-(\epsilon)$ also roughly agrees with his, but it decreases with decreasing temperature in a manner similar to the peak value of $V_+(\epsilon)$. In general, then, the peak velocity associated with ring formation in isotopic solutions is roughly independent of temperature above 0.3 K, but decreases as the temperature is reduced far below this.

In view of our present knowledge of the temperature, pressure, and C_3 dependence of the critical velocities, we can conclude that any or all of the following may be true:

It is not possible to define a unique critical velocity for ring formation resulting from the quantization of circulation only. Instead, the observed peak ion velocity depends also on the scattering experienced by the ion or ring during the period of ring formation. The similarity of the temperature dependence of the peak velocities of the two ions in dilute solutions supports this argument. Furthermore, if the relation $RV_c = \text{const}$ were strictly imposed by the properties of the superfluid, then one might question why both we and Rayfield[12] find some rings as well as negative ions in pure He^4 at $T > 0.36$ K, in the region of pressures where V_c for ring formation is supposedly greater than the Landau velocity for roton formation and the formation of rings should be strictly forbidden. If one assumes the classical vortex-ring dispersion relation, then conservation of energy and momentum requires that the creation of a ring in an instantaneous process would require V_c greater than the Landau velocity.[20] The formation of a ring is therefore probably spread out over some interval of space and time, during which energy and impulse are supplied by the electric field. In this case, formation of a ring could be inhibited by scattering from thermally excited quasiparticles. If the apparent rule $RV_c = \text{const}$ were a result of the effects of scattering, the constant for pure He^4 would change below some temperature at which the number of thermally excited quasiparticles becomes too small. The fact that qualitatively the inhibition of ring formation in pure He^4 increases with temperature and pressure, as does the roton number density, supports such a point of view.

The critical velocity for roton production may increase with decreasing temperature and at $T = 0.3$ K become greater than the critical velocity for ring formation. Lorenz Kramer (private communication) finds theoretical evidence for this, but the predicted temperature dependence of the critical velocity for roton production is too small to account for the fact that negative ions produce rings in pure He^4 at low temperatures and high pressures.

As suggested by Rayfield[19] and Dahm,[21] the structure of the negative-ion bubble may be changed in the presence of He^3 impurities, and this might in some way account for the behavior of its critical velocity in dilute solutions. However, this leaves the similar temperature dependence of the critical velocity of the quite different positive ion unexplained.

V. LOW-FIELD MOBILITIES IN He^3 - He^4 SOLUTIONS

Mobilities at Zero Pressure

For $C_3 < 10^{-3}$ we could not measure drift velocities of the ions which were less than the thermal velocities because the corresponding low electric fields drew insufficient current from the source. Preferentially increasing the field in the source region did not add greatly to the collector current, presumably because much of the additional current drawn from the source was driven to the grid at which the field was discontinuous. It was fortunate for the experiments, therefore, that $V(\epsilon)$ of both positive and negative ions remained linear to velocities larger than thermal. Generally, $V(\epsilon)$ data were obtained at several values of ϵ in the linear region, and the mobility μ was determined as the slope of the best linear fit. In some cases we found that the V versus ϵ line did not go through the origin, thereby indicating the existence of stray fields in the spectrometer drift space similar to the stray fields discussed earlier. An example is shown in Fig. 7, where the effective field was the applied field plus 0.2 V/cm. The stray field varied with the recent electrical history of the cell, but was generally in the range of zero to a few tenths of a V/cm positive. (That is, for negative ions, the $V = 0$ intercept was at the origin or to the right of it.) Error limits for the mobility data were determined as the maximum and minimum possible slopes of a line which could be drawn within the velocity error bars.

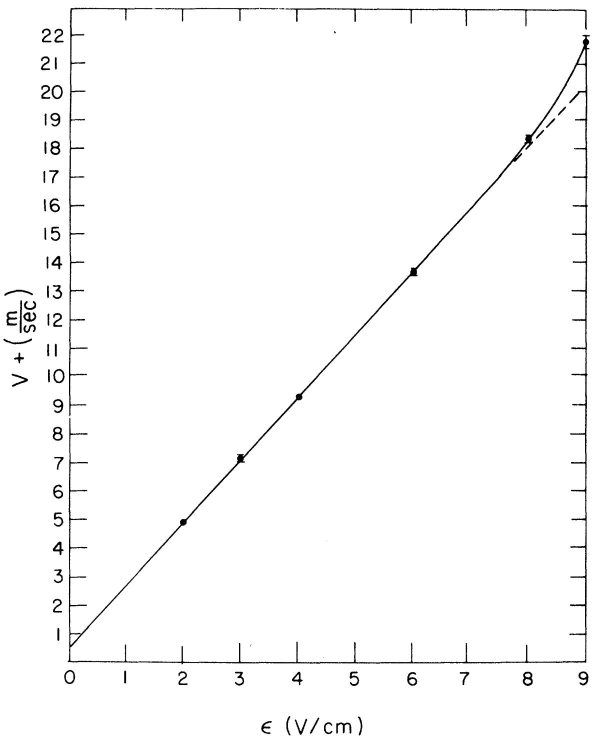

FIG. 7. Detail of positive-ion velocity as a function of applied field. $C_3 = 1.82 \times 10^{-4}$; $T = 0.06$ K. Half-length of error bars is 1% of data.

The product of positive ion mobility and C_3 (representing the scattering per He^3 atom) is shown as a function of temperature in Fig. 8. Data at $C_3 = 4.4 \times 10^{-2}$ were obtained with the single-gate spectrometer, and therefore may have a scaling error ≈ 5% with respect to the other data. For $C_3 < 2 \times 10^{-4}$ there is a slight minimum in $\mu_+ C_3$ near $T = 0.15$ K which is not truly resolved. Smooth monotonic curves could be drawn within the error bars. However, those two curves, which were obtained in separate experimental runs, and other data (not shown) obtained with the single-gate spectrometer, all display this slight minimum. All data for higher C_3 are monotonic in T. We conclude, therefore, that $\mu_+(T)$ probably has a slight minimum near 0.15 K for C_3 of the order of 10^{-4}.

The C_3 dependence of the product $\mu_+ C_3$ is shown in Fig. 9. If one considers the positive ion to be a solid ball, the ion radius varies roughly as $P_f^{-1/4}$, in which P_f is the freezing pressure. At $T > 1.4$ K $\partial P_f / \partial C_3$ is positive,[22] and $\mu_+ C_3$ should increase monotonically with C_3. Experimental data[10] in this temperature range qualitatively agree with this prediction. In the range of C_3 and T of our experiments, $\partial P_f / \partial C_3$ is again probably positive, and we would thereby expect the curves of Fig. 9 to have positive slopes. The more complicated form of the curves

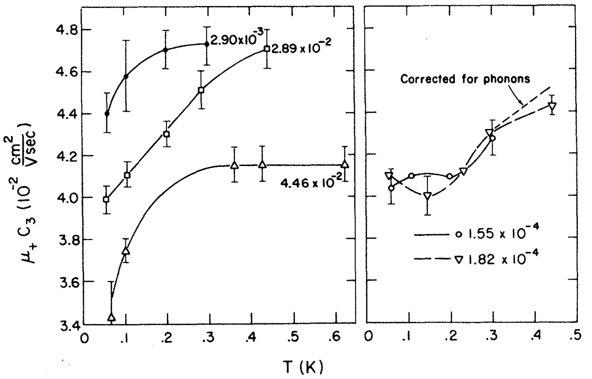

FIG. 8. Product of positive-ion mobility and C_3 as a function of temperature. The small correction for phonon scattering was calculated using the phonon-limited mobility given by L. Meyer and F. Reif, Phys. Rev. Letters 5, 1 (1960). The value of C_3 is indicated for each curve.

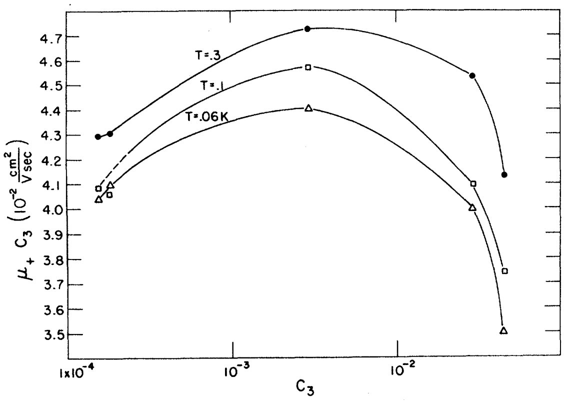

FIG. 9. Product of positive-ion mobility and C_3 as a function of C_3.

may indicate that $\mu_+ C_3$ depends on C_3 not only via the freezing pressure. We note, however, that $\partial P_f/\partial T$ must be negative at the lower end of our temperature range, and tends to zero at the upper end. Qualitatively this would cause $\mu_+ C_3$ to have the opposite of the temperature dependence shown by the curves on the left side of Fig. 8, but quantitatively $\partial P_f/\partial T$ is probably too small to account for the change in $\mu_+ C_3$.

The product $\mu_- C_3$ as a function of T, is shown in Fig. 10, and the C_3 dependence is illustrated in Fig. 11. In Fig. 10 the existence of a mobility minimum is clear, with the temperature of the minimum increasing as C_3 is increased. At $C_3 = 4.4 \times 10^{-2}$, μ_- was below the convenient range of our regular instrumentation, so these data were obtained by using the geometry of Fig. 2(a) with different electronics as a pulsed time-of-flight spectrometer similar to that of Schwarz.[23] Unfortunately, we did not extend these measurements to sufficiently high temperatures to show an eventual minimum. It is apparent that, at least for the lower C_3, the negative-ion mobility reaches a maximum and then again decreases as the temperature is increased. Unless the scattering of the negative ion by phonons is greater in He^3-He^4 solutions than in pure He^4, this latter decrease is not due solely to the onset of phonon scattering. The most striking characteristic of $\mu_-(T)$ is the steep rise as the temperature is lowered below 0.1 K.

Discussion of Mobilities

Simple models for the drag force on an ion predict that the drift velocity will be linear in ϵ at sufficiently small velocities:

(A) If the scattering can be discussed in the relaxation-time approximation, the time between collisions is determined by the free path of the ion between collisions and its thermal velocity. The superimposed drift velocity is due to the momentum acquired per unit ion mass from the field during a characteristic time. As long as the drift velocity is much less than the thermal velocity the time between collisions will remain constant and $V(\epsilon)$ will be proportional to ϵ. If

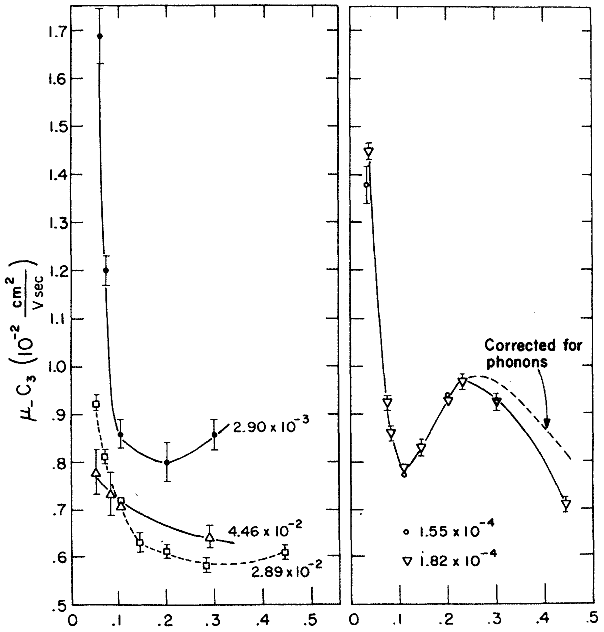

FIG. 10. Product of negative-ion mobility and C_3 as a function of temperature. The correction for phonon scattering was calculated using the phonon-limited mobility given in Ref. 23. The value of C_3 is indicated for each curve.

ϵ is increased so that the kinetic energy acquired by the ion between collisions is much greater than the thermal energy, then $V \sim \epsilon^{1/2}$. Using the measured values of the effective masses,[24] we find the thermal velocities at 0.1 K to be about 4 and 2 m/sec for the positive and negative ions, respectively. However, since the ion masses are much greater than the He^3 effective mass,[25] a collision between an ion and a He^3 atom leads only to incomplete momentum exchange, and we do not expect the relaxation time approximation to be valid.

(B) Since the ion mass is much greater than the He^3 mass, the ion velocity changes only slightly in each scattering event. If the mean-free path l_{33} for He^3-He^3 collisions is much greater than R_1, the ion radius, then individual scattering events account for the momentum loss of the ion. This type of analytical approach was most recently outlined by Arkhipov.[26] With scattering by phonons only, $V(\epsilon)$ should be proportional to ϵ so long as $V \ll c$, the phonon velocity. This has been demonstrated experimentally for the negative ion.[23] The reason V remains linear in ϵ to such high velocities can be seen qualitatively by considering the following one-dimensional example with elastic hard-sphere scattering. The momentum of a phonon is the same in a moving frame of reference as in the laboratory frame. If we therefore attach our coordinates to the moving ion, the momentum distribution of the phonons is unchanged. However, due to the motion of the ion, there are more phonon collisions per unit time on the leading side of the ion than on the trailing side, and this number difference is exactly proportional to the ion velocity. If we apply this one-dimensional example to scattering by He^3 impurities of velocity V_3 we find that the momentum exchange per collision is different at the front and back of the ion, but that the net momentum loss is again proportional to V, so long as $V < V_3$. (The corresponding restriction in three dimensions is $V \ll V_3$.) In the range of temperatures of our experiments, V_3 would be ~15 m/sec, and due to the broad thermal distribution of V_3, we should expect $V(\epsilon)$ to be linear only if $V \ll 15$ m/sec. In fact, Figs. 5, 6, and 7 show that the ion velocities remain linear in ϵ to unexpectedly large V.

(C) Finally, if $l_{33} \ll R_i$, the mobility should result from drag described by an effective viscosity of the scattering centers. For our purposes, we may estimate that l_{33} is determined by the geometrical radius of the He^3 in He^4, $R_3 \approx 2.4$ Å.[27] Then l_{33} is about 60 Å at $C_3 = 4.4\%$. Since this is much larger than the ion radii, we expect model B to be most applicable to our data.

He^3 dissolved in He^4 is known to behave as an ideal Fermi gas. Table I shows the Fermi temperatures for the He^3 concentrations of our experiments. In our experimental temperature range, the He^3 certainly behaves as a Boltzmann gas for $C_3 < 2 \times 10^{-4}$. If we consider the He^3 to be a Boltzmann gas and the He^3-ion scattering to be elastic with a geometrical cross section, the mobility is given classically by[26,28]

$$\mu_\pm = \frac{3e}{8N(R_{i\pm}+R_3)^2(2\pi \times 2.38\, m_3 kT)^{1/2}} \qquad (1)$$

in which $2.38\, m_3$ is the He^3 effective mass, k is Boltzmann's constant, and N is the number of He^3 atoms per unit volume. Using $R_{i+} = 6$ Å,[24] $R_{i-} = 18$ Å,[18,29] and $T = 0.1$ K, we obtain a comparison of the values predicted by classical theory with our data as given in Table II. The predicted magnitude could be brought into agreement by adjusting the cross sections, but the measured temperature dependences of the mobilities do not resemble a $T^{-1/2}$ law. Davis and Daggonier[28] predict that the mobilities should be proportional to

$$[(kT_f)^2 + \tfrac{1}{3}\pi^2(kT)^2]^{-1}$$

in the region where statistics are important in determining the He^3 properties, but where $(m_3 T_f / M_i T)^{1/2} \ll 1$. These conditions are most nearly satisfied by our negative-ion data for $C_3 > 2 \times 10^{-3}$ and $T \sim 0.1$ K. However, we obviously do not observe this temperature dependence either.

TABLE II. Comparison of the mobilities (cm^2/V sec) measured at 0.1 K with those derived from classical theory.

	Classical theory	Experiment
$\mu_- C_3$	2×10^{-2}	0.8×10^{-2}
$\mu_+ C_3$	12×10^{-2}	4.1×10^{-2}

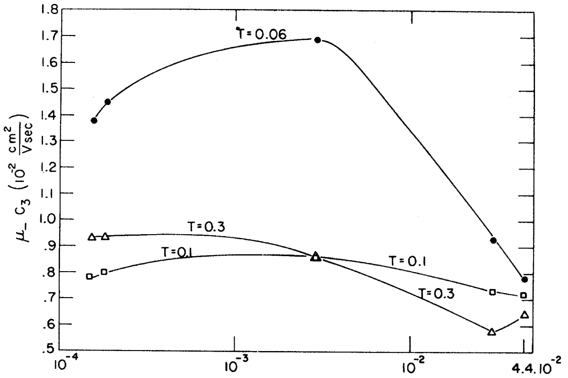

FIG. 11. Product of negative-ion mobility and C_3 as a function of C_3.

Furthermore, for all of our data, μ is roughly proportional to C_3^{-1}, which is not the dependence in Davis' expression where the concentration enters implicitly via its influence on the Fermi temperature. It may be possible that for $T \ll T_f$ μ_- approaches the T^{-2} dependence predicted for the $T \to 0$ limit,[28,30,31] but over our entire temperature range the slope of $\mu_+(T)$ is positive, which is the *opposite* of all previous theoretical predictions.

The unusual temperature dependence of μ_- above 0.1 K, and the fact that μ_- rises sharply below 0.1 K for all C_3, leads us to conclude that the observed behavior is not due to the statistics of the He^3, but rather due to the scattering properties of the negative ion itself. Baym, Barrera, and Pethick[32] have shown from a detailed consideration of the partial-wave scattering amplitudes that the phonon-limited mobility of the negative ion in He^4 is expected to rise rapidly below 0.2 K. A similar treatment of the negative-ion He^3 scattering may lead to similar results. The temperature of 0.1 K at which the change in mobility occurs seems somewhat too low to be related to the freezing out of the internal motions of the negative-ion bubble.[33]

Mobility of the Negative Ion Under Pressure

We have measured the negative-ion mobility as a function of pressure at $T = 0.23$ and 0.08 K with $C_3 = 1.82 \times 10^{-4}$. As the pressure was raised above zero, these measurements became more and more difficult for two reasons: First, as the pressure was raised, a higher field was required to draw suitable currents from the source. Second, as expected, the negative-ion mobility also increased. The combination of these two effects made it increasingly difficult to obtain data in the region of velocities where $V(\epsilon)$ is linear. At $T = 0.08$ K, for $P \geq 8$ atm, it was in fact impossible to verify that $V_-(\epsilon)$ was truly linear, and the data points in Fig. 12 at $T = 0.08$, $P \geq 8$ are simply V/ϵ at the lowest possible ϵ.

If we neglect the possible dependence of the scattering cross section on the radius of the He^3 impurity, then we might expect the mobility to vary as $1/R_{i-}^2$. In Fig. 12 we show $\mu(P)/\mu(0)$, the mobility normalized to the zero-pressure value, as a function of pressure. The solid line represents

$$[R_{i-}(P)/R_{i-}(0)]^{-2},$$

taken from Springett's data.[18] It can be seen that the pressure dependence of $\mu_-(P)$ at $T = 0.23$ K agrees with the pressure dependence of $R_{i-}^{-2}(P)$ to within the limits of error of the two experiments. Our data at $T = 0.08$ could be susceptible to larger errors than indicated, due to the difficulties mentioned above. However, at $T = 0.08$, the shape of the $\mu_-(P)$ curve is decidedly different from that of the $R_{i-}^{-2}(P)$ curve indicating again that the scattering properties of the negative ion change near 0.1 K.

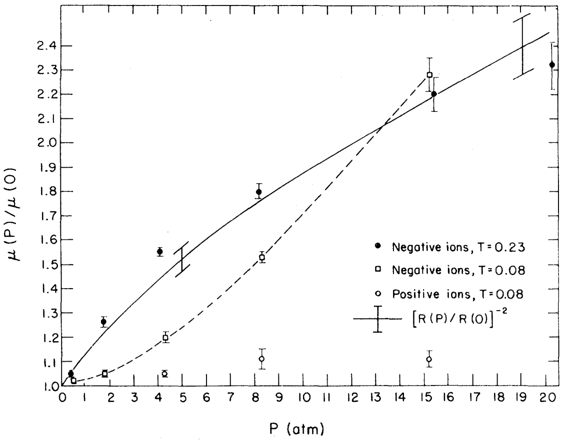

FIG. 12. Ion mobility as a function of pressure, normalized to the value at zero pressure. The solid curve and corresponding error bars represent the pressure dependence of the negative-ion radius. $C_3 = 1.82 \times 10^{-4}$.

VI. SUMMARY

The drift velocity of positive and negative charge carrier has been studied as a function of electric field in pure He^4 and He^3-He^4 solutions with a He^3 mole fraction in He^4 between 1.5×10^{-4}–4.5×10^{-2} in the temperature range 0.05–0.5 K.

The main results are as follows: It was known that in pure He^4 negative ions produce vortex rings when accelerated beyond thermal velocities at zero pressure but they can be accelerated to produce rotons in liquid He^4 under pressure. Surprisingly, we find at temperatures below 0.3 K negative ions in He^4 under pressure produce not rotons but vortex rings only.

Attempts to make vortex rings of an energy less than 0.1 V were not successful due to stray fields in the region of the Po^{210} source which emitted the α particles which in turn produced the ions.

The formation of vortex rings in He^3-He^4 solutions showed novel features below 0.3 K. At low electric fields the drift velocity of the ions is a linear function of the field, showing a well-defined mobility. In all cases except that of the negative ions below 0.1 K, when the field is increased the drift velocity increases *faster* than the field until the steep decline characteristic of the formation of vortex rings appears. The drift velocity at the peak is a function of the temperature.

The observed temperature dependence of the low-field mobility of the positive charge carriers in He^3-He^4 solutions is inconsistent with existing theories. The He^3-limited mobility of negative ions increases sharply below 0.1 K indicating a change in scattering behavior near this temperature. The product of mobility and He^3 concentration, which is a measure of the scattering per He^3 atom in the solution, is not a simple function of the He^3 concentration, and depends apparently very little on the statistical state of the He^3: i.e., whether it is a dilute Fermi gas or mildly degenerate.

VII. ACKNOWLEDGMENTS

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[*] Present address: Los Alamos Scientific Laboratories, Los Alamos, New Mexico, 87544.

[1] G. W. Rayfield and F. Reif, Phys. Rev. 136, A1194 (1964).

[2] Lothar Meyer, Phys. Rev. 148, 145 (1966).

[3] Stycast 2850 GT, available from Emerson and Cummings, Inc., Canton, Massachusetts.

[4] Heise Bourdon Tube Co., Newton, Massachusetts.

[5] See, for example, S. Cunsolo, Nuovo Cimento 21, 76 (1961) or D. J. Tanner, Phys. Rev. 152, 121 (1966).

[6] This divergence can be caused by a slight component of the electric field perpendicular to the velocity of the rings. See Appendix A of K. W. Schwarz, Phys. Rev. 165, 323 (1968).

[7] A. C. Anderson, M. Kuchnir, and J. C. Wheatley, Phys. Rev. 168, 261 (1968).

[8] C. M. Surko and F. Reif, Phys. Rev. Letters 20, 582 (1968); Phys. Rev. 175, 229 (1968). It seems unlikely that the large magnitudes of stray currents which we observed could be caused by the interaction of the neutral excitations with the grid S.

[9] See Manfred Kaminsky, Atomic and Ionic Impact Phenomena on Metal Surfaces, (Academic Press Inc., New York, 1965) p. 263.

[10] Yu. Z. Kovdyra and B. N. Yeselson, Zh. Eksperm. i Teor. Fiz. 55, 86 (1968).

[11] Lothar Meyer and F. Reif, Phys. Rev. 123, 727 (1961).

[12] G. W. Rayfield, Phys. Rev. Letters 16, 934 (1966).

[13] G. W. Rayfield, Phys. Rev. 168, 222 (1968). Ph.D. thesis, University of California, Berkeley, 1964 (unpublished), p. 48. This reference also includes a discussion of the source effect on ring creation.

[14] E. H. Takken, Ph.D. thesis, (treats roton production by ions as Cerenkov radiation) Ohio State University, 1967 (unpublished).

[15] D. A. Neeper, Phys. Rev. Letters 21, 274 (1968).

[16] F. Reif and Lothar Meyer, Phys. Rev. 119, 1164 (1960).

[17] G. W. Rayfield, Phys. Rev. Letters 19, 1371 (1967).

[18] B. E. Springett, Phys. Rev. 155, 139 (1967).

[19] G. W. Rayfield, Phys. Rev. Letters 20, 1467 (1968).

[20] See discussion in Ref. 15.

[21] A. J. Dahm, to be published, gives theoretical evidence for condensation of He^3 atoms on the surface of a bubble in He^4.

[22] K. W. Taconis and R. De Bruyn Ouboter in Progress in Low Temperature Physics, edited by C. J. Gorter, (North-Holland Publishing Co., Amsterdam, 1964), Vol. IV p. 38. See especially p. 84.

[23] K. W. Schwarz and R. W. Stark, Phys. Rev. Letters 21, 967 (1968).

[24] Arnold J. Dahm and T. M. Sanders, Jr., Phys. Rev. Letters 17, 126 (1966).

[25] J. Bardeen, G. Baym and D. Pines, Phys. Rev. 156, 207 (1967).

[26] R. G. Arkhipov, Usp. Fiz. Nauk. 88, 185 (1966) [English transl.: Soviet Phys. – Usp. 9, 174 (1966)].

[27] We estimate R_3 from the volume occupied by a He^3 atom. See Ref. 25.

[28] H. Ted Davis and R. Daggonier, J. Chem. Phys. 44, 4030 (1966).

[29] J. A. Northby and T. M. Sanders, Jr., Phys. Rev. Letters 18, 1184 (1967).

[30] Ryuzo Abe and Ko Aizu, Phys. Rev. 123, 10 (1961).

[31] R. C. Clark, Proc. Phys. Soc. (London) 82, 785 (1963).

[32] Gordon Baym, R. G. Barrera, C. J. Pethick, Phys. Rev. Letters 22, 20 (1969).

[33] V. Celli, Morrel H. Cohen, and M. J. Zuckerman, Phys. Rev. 173, 253 (1968).