Mass of the κ Meson and the Relative Sign of F_K and F_π from Broken Chiral Symmetry

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The Glashow-Weinberg analysis of broken chiral symmetry is reexamined in the light of the recently established equality $F_K^2=F_\pi^2$. It is shown that if the pole-dominance hypothesis for $J=0$ and $J=1$ mesons is valid, then (a) F_K and F_π must have *opposite* signs and (b) the κ meson must lie *above* the $K\pi$ threshold.

RECENTLY, under the assumption of pole dominance of spin-zero and spin-1 spectral functions, several important results have been derived within the framework of broken chiral symmetry.[1,2] The same assumption has also led to the establishment of the equality

$$F_K^2=F_\pi^2 \tag{1}$$

to all orders of the $SU(3)$-breaking interaction.[3]

The purpose of the present paper is to point out further consequences emerging from the framework of the hypotheses mentioned above. Specifically, we shall consider two cases:

$$\text{(a)}\ F_\kappa=0,\quad m_\kappa^2F_\kappa\neq 0 \tag{2}$$

$$\text{(b)}\ F_\kappa\neq 0,\quad m_\kappa^2<\infty\,. \tag{3}$$

Case (a) corresponds to removing the κ meson from the theory by taking the limit $m_\kappa\to\infty$. In this instance, the partial conservation law

$$\partial_\mu V_\kappa^{\ \mu}=F_\kappa m_\kappa^2\kappa(x)\,, \tag{4}$$

where $\kappa(x)$ is an interpolating field for the κ meson, represents the most general *nonresonant* characterizaion of $SU(3)$ breakdown within the framework of chiral symmetry. Case (b) corresponds to the possible existence of a κ meson with finite mass.

To discuss the two cases (a) and (b), we shall make use of the following relations established in Ref. 1[4]:

$$F_\pi Z_\pi^{1/2}=F_KZ_K^{1/2}+F_\kappa Z_\kappa^{1/2}\,, \tag{5}$$

$$m_\pi^2F_\pi Z_\pi^{-1/2}=m_K^2F_KZ_K^{-1/2}+m_\kappa^2F_\kappa Z_\kappa^{-1/2}\,, \tag{6}$$

$$f^+(0)=(F_K^2+F_\pi^2-F_\kappa^2)(2F_KF_\pi)^{-1}\,, \tag{7}$$

and in Ref. 2:

$$F_K^2+F_\kappa^2+2F_KF_\kappa Z_K^{1/2}Z_\kappa^{-1/2}=F_\pi^2\,. \tag{8}$$

We remark that the validity of Eq. (7) has recently been shown to be independent of how the chiral symmetry is broken.[5]

On combining Eqs. (1) and (8), we obtain either

$$F_\kappa=0 \tag{9}$$

or

$$(Z_K/Z_\kappa)^{1/2}=-\tfrac{1}{2}(F_\kappa/F_K)\,. \tag{10}$$

If Eq. (9) holds (with $m_\kappa^2F_\kappa\neq 0$), we are led to case (a). Then it is easy to see that the only consistent solution to the above system of equations is

$$F_\pi=+F_K\,,\quad F_\kappa=0\,. \tag{11}$$

Experimentally,[6] however,

$$\xi\equiv|F_K/F_\pi f^+(0)|=1.28\pm0.06\,. \tag{12}$$

We conclude that Eq. (11) is incompatible with Eq. (7). We wish to emphasize here that the only approximations involved in the demonstration of the incompatibility of Eqs. (7), (11), and (12) are pole dominance and the smoothness of vertex functions. These very assumptions have been successfully employed elsewhere, leading to very satisfactory agreement between theory and experiment.[7] We are therefore inclined to argue that the solution displayed in Eq. (11) should be rejected on *experimental* grounds.

We next consider the other possibility, Eq. (10). It is fairly obvious that the consistency between Eqs. (5) and (10), together with the positivity of the $Z^{1/2}$'s implies that F_K and F_π must have opposite signs, and further that $Z_K=Z_\pi$. Consequently, one finally obtains the result[8]

$$m_\kappa^2=m_K^2(1-1/\xi)^{-1}(1+m_\pi^2/m_K^2)\,. \tag{13}$$

We notice from Eq. (13) that the κ meson cannot possibly lie *below* the $K\pi$ threshold: For $m_\kappa\approx m_K+m_\pi$, Eq. (13) yields $\xi=2.9$, in disagreement with the experi-

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[4] F_π, etc., are defined by the one-particle matrix element in the usual fashion: $\langle 0|J^\mu(0)|\pi,p\rangle=F_\pi p^\mu(2\pi)^{-3/2}(2E_\pi)^{-1/2}$. Z_π, etc., are the renormalization constants, and $Z^{1/2}$ is positive (see Ref. 1); $f^+(0)$ denotes the renormalized K_{l3} form factor at $q^2=0$.

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[7] We have in mind the application to the decay of $A_1\to\rho+\pi$; see H. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1967).

[8] Equation (13) was first derived in Ref. 2, in the form

$$m_\kappa^2=m_K^2(1-\xi^{-1})^{-1}\left\{1-\frac{m_\pi^2}{m_K^2}\frac{F_\pi}{F_K}\left[\frac{Z_K}{Z_\pi}\right]^{1/2}\right\}.$$

mental value in Eq. (12). From Eqs. (12) and (13), we see that m_κ lies between 1015 MeV ($\xi=1.34$) and 1200 MeV ($\xi=1.22$). It is worth emphasizing that this range of values is consistent (as it should be) with the Glashow-Weinberg lower bound on m_κ[1,9]:

$$m_\kappa \geqslant |m_\pi + m_K|\,|F_\pi/F_\kappa| \approx 945 \text{ MeV}.$$

The solution $F_\pi=-F_K$, therefore, indicates a very large symmetry-breaking effect, and hence the very existence of a κ meson in this particular mass range implies a large symmetry-breaking effect. This raises an important question about the concept of "approximate symmetry." We believe that there exists no *a priori* reason why $SU(3)\otimes SU(3)$ symmetry should *not* be strongly broken. A recent experimental analysis of the process $K+N \to K+\pi+\Delta^{++}$ has indicated the existence of an $I=\frac{1}{2}$, $J^P=0^+$ resonance with mass at about 1100 MeV.[10] This value for m_κ is also suggested by a recent investigation based on spectral-function sum rules.[11] We are therefore of the opinion that although the solution

$$F_K=+F_\pi, \quad F_\kappa=0, \quad m_\kappa^2 F_\kappa \neq 0$$

cannot be excluded purely on the basis of broken chiral symmetry, there are nevertheless strong experimental indications in favor of the strong symmetry-breaking solution $F_K=-F_\pi$, $m_\kappa \approx 1100$ MeV.

Finally, we wish to point out a curious coincidence: Let us abandon the notion of κ dominance as expressed by Eq. (8), but choose instead to supplement the Glashow-Weinberg relations Eqs. (5) and (6) with the well-known Khuri result[12]

$$Z_K/Z_\pi = (F_K^2/F_\pi^2) m_K^4/m_\pi^4. \tag{14}$$

It is appropriate to remark here that Khuri's result actually does hold in both the gradient-coupling model and the σ model. Perhaps the simplest way of arriving at Eq. (14) is to observe that in both these models[13] one has $\partial_\mu A_\mu^{\pi,K} = m_0^2 \phi_0^{\pi,K}$, where m_0 is the common bare mass. From partial conservation of axial-vector current, we have $\partial_\mu A_\mu^{\pi,K} = F_{\pi,K} m_{\pi,K}^2 \phi^{\pi,K}$, where $\phi_{\pi,K} = \phi_{\pi,K}^0 / Z^{\frac{1}{2}}{}_{\pi K}$. The equality of the bare masses immediately yields Eq. (14).

If we now combine Khuri's result with the Glashow-Weinberg relations, one obtains (with $F_K=-F_\pi$)

$$m_\kappa^2 = 2(m_\pi^2 + m_K^2) F_\pi^2/F_\kappa^2. \tag{15}$$

This expression is *identical* to the one derived previously using κ dominance. We find this result very puzzling. As a final remark, we observe that if Khuri's result is accepted, then one can rule out the possibility $F_\pi = +F_K$, $F_\kappa=0$, $m_\kappa^2 F_\kappa \neq 0$ on *theoretical* grounds.

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[9] We are using $F_\kappa \approx 0.67 F_\pi$. This value, obtained with $\xi=1.28$, differs considerably from that obtained using *both* the Weinberg sum rules for the chiral partners K^* and K_A, which yield $F_\kappa^2 \approx 0.02 F_\pi^2$. This rather severe discrepancy may be due to the doubtful validity of the second sum rule, $g_{K^*}{}^2 = g_{KA}{}^2$. The mass relation $m_{K_A} = \sqrt{2} m_{K^*}$ can be maintained with $F_\kappa^2 \approx 0.44 F_\pi^2$ (and the first sum rule), provided $g_{K^*}{}^2 = 0.78 g_{KA}{}^2$.

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