High-Energy Collision Processes in Quantum Electrodynamics. II

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We consider here all the high-energy scattering processes $a+b \to a+b$ in quantum electrodynamics, where a and b can be fermions or antifermions. Aside from the photon pole terms, these high-energy amplitudes are shown to be conveniently expressed by the impact factors. The impact factors for the electron, the positron, and the static nucleus are explicitly given.

1. INTRODUCTION

In this series of papers, we shall present a systematic study of all of the two-body elastic scattering amplitudes in high-energy quantum electrodynamics. Among these amplitudes, those of Delbrück scattering, electron Compton scattering, and photon-photon scattering are fairly complicated. Their treatments will be delayed to papers III–VIII, which form the "hard core" of this series. In this paper, we shall study the very simple processes $a+b \to a+b$, where a and b are fermions or antifermions. The calculations in this paper are straightforward. Nevertheless, the conclusions which we may draw from these calculations are already interesting. As we shall see, a simple picture of high-energy scattering will emerge from our considerations.

2. ELECTRON-ELECTRON SCATTERING

The lowest-order diagrams for electron-electron scattering are of the second order. The dominant one is illustrated in Fig. 1 of paper I. The scattering amplitude corresponding to this diagram is given by Eq. (2.1) in paper I. In the high-energy limit $s \to \infty$ (t fixed), we have $p_2 \sim p_1 \sim r_2$, $p_2' \sim p_1' \sim r_3$; hence,

$$\bar{u}(p_2)\gamma_\mu u(p_1) \sim r_{2\mu} m^{-1} \delta_{21} \tag{2.1}$$

and

$$\bar{u}(p_2')\gamma_\mu u(p_1') \sim r_{3\mu} m^{-1} \delta_{2'1'}. \tag{2.2}$$

From (2.1) and (2.2), we easily obtain (2.2) of paper I. The diagram which is obtained from Fig. 1 with p_2 and p_2' interchanged gives an amplitude too small by a factor of s at high energy and is therefore neglected.

Since the amplitude (2.2) of paper I is proportional to s in the limit $s \to \infty$ with t held fixed, it contributes a nonvanishing finite amount to $d\sigma/dt$ in this limit. This amplitude is real. To obtain an imaginary amplitude which is proportional to s in the high-energy limit we must go to the fourth order. The relevant first-order diagrams are illustrated in Fig. 2 of paper I. The first diagram there gives an amplitude

$$\mathfrak{M}_1 = -ie^4(2\pi)^{-4} \int d^4q [(r_2-q)^2-m^2]^{-1} [(r_1+q)^2-\lambda^2]^{-1} [(r_3+q)^2-m^2]^{-1} [(r_1-q)^2-\lambda^2]^{-1}$$
$$\times [\bar{u}(p_2)\gamma_\nu(-\mathbf{q}+\mathbf{r}_2+m)\gamma_\mu u(p_1)][\bar{u}(p_2')\gamma_\nu(\mathbf{q}+\mathbf{r}_3+m)\gamma_\mu u(p_1')]. \tag{2.3}$$

Introducing Feynman parameters in the usual way, we obtain

$$\mathfrak{M}_1 = -6ie^4(2\pi)^{-4} \int d^4q' \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 d\alpha_4 \delta(1-\sum_{i=1}^4 \alpha_i)[q'^2+\alpha_1\alpha_3 s+\alpha_2\alpha_4 t-(\alpha_1+\alpha_3)^2 m^2-(\alpha_2+\alpha_4)\lambda^2]^{-4} \bar{u}(p_2)\gamma_\nu$$
$$\times[-\mathbf{q}'+(1-\alpha_1)\mathbf{r}_2+\alpha_3\mathbf{r}_3+(\alpha_2-\alpha_4)\mathbf{r}_1+m]\gamma_\mu u(p_1)\bar{u}(p_2')\gamma_\nu[\mathbf{q}'+\alpha_1\mathbf{r}_2+(1-\alpha_3)\mathbf{r}_3-(\alpha_2-\alpha_4)\mathbf{r}_1+m]\gamma_\mu u(p_1'), \tag{2.4}$$

where

$$q' = q-\alpha_1 r_2+\alpha_3 r_3+(\alpha_2-\alpha_4)r_1. \tag{2.5}$$

Carrying out the integration over q', we obtain

$$\mathfrak{M}_1 = e^4(4\pi)^{-2} \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 d\alpha_4 \delta(1-\sum_{i=1}^4 \alpha_i)[\alpha_1\alpha_3 s+\alpha_2\alpha_4 t-(\alpha_1+\alpha_3)^2 m^2-(\alpha_2+\alpha_4)\lambda^2]^{-2} \bar{u}(p_2)\gamma_\nu[(1-\alpha_1)\mathbf{r}_2+\alpha_3\mathbf{r}_3$$
$$+(\alpha_2-\alpha_4)\mathbf{r}_1+m]\gamma_\mu u(p_1)\bar{u}(p_2')\gamma_\nu[\alpha_1\mathbf{r}_2+(1-\alpha_3)\mathbf{r}_3-(\alpha_2-\alpha_4)\mathbf{r}_1+m]\gamma_\mu u(p_1') - \tfrac{1}{2}e^4(4\pi)^{-2} \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 d\alpha_4$$
$$\times \delta(1-\sum_{i=1}^4 \alpha_i)[\alpha_1\alpha_3 s+\alpha_2\alpha_4 t-(\alpha_1+\alpha_3)^2 m^2-(\alpha_2+\alpha_4)\lambda^2]^{-1}[\bar{u}(p_2)\gamma_\nu\gamma_\rho\gamma_\mu u(p_1)][\bar{u}(p_2')\gamma_\nu\gamma_\rho\gamma_\mu u(p_1')]. \tag{2.6}$$

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Let us choose the z axis to be parallel to the spatial part of p_1. Then at high energies, we have

$$u(p_1)\sim(\tfrac{1}{2}E/m)^{1/2}\begin{pmatrix}\chi_1\\ \sigma_3\chi_1\end{pmatrix},\qquad u(p_2)\sim(\tfrac{1}{2}E/m)^{1/2}\begin{pmatrix}\chi_2\\ \sigma_3\chi_2\end{pmatrix},$$
$$u(p_1')\sim(\tfrac{1}{2}E/m)^{1/2}\begin{pmatrix}\chi_1'\\ -\sigma_3\chi_1'\end{pmatrix},\quad u(p_2')\sim(\tfrac{1}{2}E/m)^{1/2}\begin{pmatrix}\chi_2'\\ -\sigma_3\chi_2'\end{pmatrix}. \tag{2.7}$$

In (2.7), E is the energy of the electron, χ is a two-component normalized spinor, and σ_3 is the third Pauli spin matrix. Now we have

$$\bar{u}(p_2)\gamma_\nu r_2\gamma_\mu u(p_1)=2r_{2\mu}\bar{u}(p_2)\gamma_\nu u(p_1)-\bar{u}(p_2)\gamma_\nu\gamma_\mu r_2 u(p_1)\sim 2r_{2\mu}r_{2\nu}/m\delta_{12} \tag{2.8}$$

and, similarly,

$$\bar{u}(p_2')\gamma_\nu r_3\gamma_\mu u(p_1')\sim 2r_{3\mu}r_{3\nu}/m\delta_{1'2'}. \tag{2.9}$$

From (2.8), we have

$$[\bar{u}(p_2)\gamma_\nu r_2\gamma_\mu u(p_1)][\bar{u}(p_2')\gamma_\nu r_2\gamma_\mu u(p_1')]\sim 2r_2^2\delta_{12}\delta_{1'2'}(r_2r_3)m^{-2}.$$

Similarly, from (2.9), we have

$$[\bar{u}(p_2)\gamma_\nu r_3\gamma_\mu u(p_1)][\bar{u}(p_2')\gamma_\nu r_3\gamma_\mu u(p_1')]\sim 2r_3^2\delta_{12}\delta_{1'2'}(r_2r_3)m^{-2}.$$

These two terms, therefore, are proportional to s in the high-energy limit.

We are now ready to make high-energy approximations for (2.6). First of all, the second integral in the right side of (2.6) can be neglected. This is because the numerator of its integrand has no s dependence except through $u(p)$ and $\bar{u}(p)$, and from (2.7) we find that the numerator is at most of the order of $E^2(\sim s)$. Since the integral

$$\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 d\alpha_4\delta(1-\sum_{i=1}^4\alpha_i)\times[\alpha_1\alpha_3 s+\alpha_2\alpha_4 t-(\alpha_1+\alpha_3)^2m^2-(\alpha_2+\alpha_4)\lambda^2]^{-1}$$

vanishes as $s\to\infty$, the second integral in (2.6) is small compared to s. Similarly, we may neglect r_1 and m in the numerator of the first integral in (2.6) and terms like $[\bar{u}(p_2)\gamma_\nu r_i\gamma_\mu u(p_1)][\bar{u}(p_2')\gamma_\nu r_i\gamma_\mu u(p_1')]$, $i=2,3$. Finally, we remark that since $\alpha_1\alpha_3$ is the coefficient of s in the denominator of (2.6), $\alpha_1\alpha_3$ is roughly of the order of s^{-1} and the term

$$\alpha_1\alpha_3[\bar{u}(p_2)\gamma_\nu r_3\gamma_\mu u(p_1)][\bar{u}(p_2')\gamma_\nu r_2\gamma_\mu u(p_1')]$$

in the numerator can also be discarded. With all these approximations, (2.6) becomes

$$\mathfrak{M}_1\sim e^4(4\pi)^{-2}s^2m^{-2}\delta_{12}\delta_{1'2'}I_1(s,t), \tag{2.10}$$

where

$$I_1(s,t)=\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 d\alpha_4\delta(1-\sum_{i=1}^4\alpha_i)(1-\alpha_1)(1-\alpha_3)[\alpha_1\alpha_3 s+\alpha_2\alpha_4 t-(\alpha_1+\alpha_3)^2m^2-(\alpha_2+\alpha_4)\lambda^2]^{-2}. \tag{2.11}$$

To evaluate (2.11) asymptotically in the limit $s\to\infty$ with t held finite, it is convenient to make a Mellin transform of I_1. We have

$$\bar{I}_1(\zeta)\equiv\int_0^\infty s^{-\zeta}I_1(s,t)ds=\pi\zeta(\sin\pi\zeta)^{-1}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 d\alpha_4\delta(1-\sum_{i=1}^4\alpha_i)(1-\alpha_1)(1-\alpha_3)(\alpha_1\alpha_3)^{-1+\zeta}$$
$$\times[\alpha_2\alpha_4 t-(\alpha_1+\alpha_3)^2m^2-(\alpha_2+\alpha_4)\lambda^2]^{-1-\zeta}=-\pi\zeta e^{-i\pi\zeta}(\sin\pi\zeta)^{-1}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 d\alpha_4\delta(1-\sum_{i=1}^4\alpha_i)$$
$$\times(1-\alpha_1)(1-\alpha_3)(\alpha_1\alpha_3)^{-1+\zeta}[\alpha_2\alpha_4|t|+(\alpha_1+\alpha_3)^2m^2+(\alpha_2+\alpha_4)\lambda^2]^{-1-\zeta}. \tag{2.12}$$

From (2.12), we see that $e^{i\pi\zeta}\bar{I}_1(\zeta)$ is real.

The behavior of $\bar{I}_1(\zeta)$ at $\zeta\sim0$ is related to the behavior of $I_1(s,t)$ for $s\to\infty$. Specifically, a term ζ^{-n} in $\bar{I}_1(\zeta)$, $n>0$, corresponds to a term $[(n-1)!]^{-1}(\ln s)^{n-1}s^{-1}$ in $I_1(s,t)$. Therefore, obtaining the asymptotic form of $I_1(s,t)$ for high energies is equivalent to obtaining all the pole terms in (2.12) at $\zeta=0$. Since the integral diverges at $\alpha_1=\alpha_3=0$ when $\zeta=0$, the leading term of $\bar{I}_1(\zeta)$ must come from the region $\alpha_1\ll1$, $\alpha_3\ll1$. Thus, (2.12) gives

$$\bar{I}_1(\zeta)\sim-e^{-i\pi\zeta}\int_0^1 d\alpha_2 d\alpha_4\delta(1-\alpha_2-\alpha_4)(\alpha_2\alpha_4|t|+\lambda^2)^{-1}\int_0^1 d\alpha_1\alpha_1^{-1+\zeta}\int_0^1 d\alpha_3\alpha_3^{-1+\zeta}$$
$$=-e^{-i\pi\zeta}\zeta^{-2}\int_0^1 d\alpha_2 d\alpha_4(1-\alpha_2-\alpha_4)(\alpha_2\alpha_4|t|+\lambda^2)^{-1}. \tag{2.13}$$

From (2.13), we conclude that

$$\mathrm{Re}I_1(s,t)\sim-(s^{-1}\ln s)\int_0^1 d\alpha_2 d\alpha_4\delta(1-\alpha_2-\alpha_4)(\alpha_2\alpha_4|t|+\lambda^2)^{-1} \tag{2.14}$$

and

$$\mathrm{Im}I_1(s,t)\sim\pi s^{-1}\int_0^1 d\alpha_2 d\alpha_4\delta(1-\alpha_2-\alpha_4)(\alpha_2\alpha_4|t|+\lambda^2)^{-1}. \tag{2.15}$$

Substituting (2.14) and (2.15) into (2.10), we obtain the leading asymptotic terms for $\mathrm{Re}\mathfrak{M}_1$ and $\mathrm{Im}\mathfrak{M}_1$ in the limit $s\rightarrow\infty$, and t held finite.

The real part of $\mathfrak{M}_1$, however, is canceled by that of $\mathfrak{M}_2$, the amplitude from the second diagram of Fig. 2. of paper I. We have

$$\mathfrak{M}_2=-ie^4(2\pi)^{-4}\int d^4q[(r_2-q)^2-m^2]^{-1}[(r_1+q)^2-\lambda^2]^{-1}[(r_3-q)^2-m^2]^{-1}[(r_1-q)^2-\lambda^2]^{-1}$$
$$\times[\bar{u}(p_2)\gamma_\nu(r_2-q+m)\gamma_\mu u(p_1)][\bar{u}(p_2')\gamma_\mu(r_3-q+m)\gamma_\nu u(p_1')]. \tag{2.16}$$

Introducing Feynman parameters and carrying out the integration over q, we get

$$\mathfrak{M}_2=e^4(4\pi)^{-2}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 d\alpha_4\delta(1-\sum_{i=1}^4\alpha_i)[\alpha_1\alpha_3 u+\alpha_2\alpha_4 t-(\alpha_1+\alpha_3)^2m^2-(\alpha_2+\alpha_4)\lambda^2]^{-2}$$
$$\times\bar{u}(p_2)\gamma_\nu[(1-\alpha_1)r_2-\alpha_3 r_3+(\alpha_2-\alpha_4)r_1+m]\gamma_\mu u(p_1)\bar{u}(p_2')\gamma_\mu[(1-\alpha_3)r_3-\alpha_1 r_2+(\alpha_2-\alpha_4)r_1+m]\gamma_\nu u(p_1')$$
$$+\tfrac{1}{2}e^4(4\pi)^{-2}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 d\alpha_4\delta(1-\sum_{i=1}^4\alpha_i)[\alpha_1\alpha_3 u+\alpha_2\alpha_4 t-(\alpha_1+\alpha_3)^2m^2-(\alpha_2+\alpha_4)\lambda^2]^{-1}$$
$$\times[\bar{u}(p_2)\gamma_\nu\gamma_\rho\gamma_\mu u(p_1)][\bar{u}(p_2')\gamma_\mu\gamma_\rho\gamma_\nu u(p_1')]. \tag{2.17}$$

As before, at high energies $\mathfrak{M}_2$ can be approximated by

$$\mathfrak{M}_2\sim e^4(4\pi)^{-2}s^2m^{-2}\delta_{12}\delta_{1'2'}I_2, \tag{2.18}$$

where

$$I_2=\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 d\alpha_4\delta(1-\sum_{i=1}^4\alpha_1)[\alpha_1\alpha_3|u|+\alpha_2\alpha_4|t|+(\alpha_1+\alpha_3)^2m^2+(\alpha_2+\alpha_4)\lambda^2]^2$$
$$\sim\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 d\alpha_4\delta(1-\sum_{i=1}^4\alpha_1)[\alpha_1\alpha_3 s+\alpha_2\alpha_4|t|+(\alpha_1+\alpha_3)^2m^2+(\alpha_2+\alpha_4)\lambda^2]^2. \tag{2.19}$$

Notice that I_2 and hence $\mathfrak{M}_2$ are real.

From (2.10) and (2.18), we obtain

$$\mathfrak{M}_0^{(--)}=\mathfrak{M}_1+\mathfrak{M}_2\sim e^4(4\pi)^{-2}s^2m^{-2}\delta_{12}\delta_{1'2'}(I_1+I_2). \tag{2.20}$$

To obtain the asymptotic form of $\mathfrak{M}_0^{(--)}$ in the limit $s\rightarrow\infty$ and t held finite, we make a Mellin transform of I_1+I_2.

$$\bar{I}_1(\zeta)+\bar{I}_2(\zeta)\sim(1-e^{-i\pi\zeta})\pi\zeta(\sin\pi\zeta)^{-1}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 d\alpha_4\delta(1-\sum_{i=1}^4\alpha_i)(1-\alpha_1)(1-\alpha_3)$$
$$\times(\alpha_1\alpha_3)^{-1+\zeta}[\alpha_2\alpha_4|t|+(\alpha_1+\alpha_3)^2m^2+(\alpha_2+\alpha_4)\lambda^2]^{-1-\zeta}. \tag{2.21}$$

Near $\zeta\sim0$, we have

$$\bar{I}_1(\zeta)+\bar{I}_2(\zeta)\sim i\pi\zeta^{-1}\int_0^1 d\alpha_2 d\alpha_4\delta(1-\alpha_2-\alpha_4)(\alpha_2\alpha_4|t|+\lambda^2)^{-1} \tag{2.22}$$

or

$$I_1+I_2\sim i\pi s^{-1}\int_0^1 d\alpha_2 d\alpha_4\delta(1-\alpha_2-\alpha_4)(\alpha_2\alpha_4|t|+\lambda^2)^{-1}. \tag{2.23}$$

Substituting (2.23) into (2.20), we obtain (2.4a) of paper I. Notice that $\mathfrak{M}_0^{(--)}$ is purely imaginary. This means that the real parts $\mathfrak{M}_1$ and $\mathfrak{M}_2$ cancel each other.

The coefficient of is for $\mathfrak{M}_0^{(--)}$ can be cast in a form which is physically more suggestive. By introducing Feynman parameters, it is easy to prove that

$$\int d\mathbf{q}_\perp[(\mathbf{q}_\perp+\mathbf{r}_1)^2+\lambda^2]^{-1}[(\mathbf{q}_\perp-\mathbf{r}_1)^2+\lambda^2]^{-1}=\pi\int_0^1 d\alpha_2 d\alpha_4\delta(1-\alpha_2-\alpha_4)(\alpha_2\alpha_4|t|+\lambda^2)^{-1}. \quad (2.24)$$

From (2.4a) of paper I and (2.24), we obtain (2.4b) of paper I.

3. ELECTRON-POSITRON SCATTERING

The treatment of electron-positron scattering is very similar to that of electron-electron scattering. The dominant second-order diagram is illustrated in Fig. 3 of paper I. The corresponding amplitude is given by (2.6) of paper I. Since in the high energy limit we have $\bar{v}(p_1')\gamma_\mu v(p_2')\sim r_{3\mu}m^{-1}\delta_{1'2'}$, we easily obtain (2.7) of paper I. The second-order Bhabha diagram illustrated in Fig. 1 gives an amplitude too small by a factor of s at high energy and can be neglected.

The lowest-order diagrams relevant to yielding an imaginary amplitude proportional to s at high energies are illustrated in Fig. 4 of paper I. The corresponding amplitude is given by (2.8) of paper I. At high energies, we have

$$v(p_1')\sim(\tfrac{1}{2}E/m)^{1/2}\begin{pmatrix}-\sigma_3\chi_1'\\ \chi_1'\end{pmatrix},\qquad v(p_2')\sim(\tfrac{1}{2}E/m)^{1/2}\begin{pmatrix}-\sigma_3\chi_2'\\ \chi_2'\end{pmatrix}, \quad (3.1)$$

and

$$\bar{v}(p_1')\gamma_\mu r_3\gamma_\nu v(p_2')\sim 2r_{3\mu}r_{3\nu}\delta_{1'2'}m^{-1}. \quad (3.2)$$

The rest of the calculation exactly parallels that in Sec. 2 and will not be repeated here. We found that (2.9) of paper I holds. In other words, if we express the electron-positron scattering amplitude $\mathfrak{M}_0^{(-+)}$ in the form of (2.16), then the impact factor of the positron introduced is equal to that of the electron.

The amplitude from the fourth-order Bhabha diagram illustrated in Fig. 2 can be shown to be negligible at high energies. It is equal to

$$\begin{aligned}\mathfrak{M}^{(B)}=ie^4(2\pi)^{-4}\int d^4q[(r_2-q)^2-m^2]^{-1}\\ \times[(r_1+q)^2-\lambda^2]^{-1}[(r_3+q)^2-m^2]^{-1}\\ \times[(-r_1+q)^2-\lambda^2]^{-1}[\bar{v}(p_1')\gamma_\nu(q-r_1+m)\gamma_\mu u(p_1)]\\ \times[\bar{u}(p_2)\gamma_\mu(q+r_1+m)\gamma_\nu v(p_2')]. \quad (3.3)\end{aligned}$$

Introducing Feynman parameters and carrying out the integration over q, we obtain

$$\begin{aligned}\mathfrak{M}^{(B)}\sim -e^4(4\pi)^{-2}\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 d\alpha_4\delta(1-\sum_{i=1}^4\alpha_i)\\ [\alpha_1\alpha_3 s+\alpha_2\alpha_4 t-(\alpha_1+\alpha_3)\lambda^2-(\alpha_2+\alpha_4)^2m^2]^{-1}\\ \times\bar{v}(p_1')\gamma_\nu(\alpha_1 r_2-\alpha_3 r_3)\gamma_\mu u(p_1)\\ \times\bar{u}(p_2)\gamma_\mu(\alpha_1 r_2-\alpha_3 r_3)\gamma_\nu v(p_2'). \quad (3.4)\end{aligned}$$

In obtaining (3.4), we have discarded r_1, q, and m. Now we may discard further terms proportional to $\alpha_1\alpha_3$. Then (3.4) becomes

$$\begin{aligned}\mathfrak{M}^{(B)}\sim -e^4(4\pi)^{-2}\int d\alpha_1 d\alpha_2 d\alpha_3 d\alpha_4\delta(1-\sum_{i=1}^4\alpha_i)\\ \times[\alpha_1\alpha_3 s+\alpha_2\alpha_4 t-(\alpha_2+\alpha_4)^2m^2-(\alpha_1+\alpha_3)\lambda^2]^{-2}\\ \times\{\alpha_1^2[\bar{v}(p_1')\gamma_\nu r_2\gamma_\mu u(p_1)][\bar{u}(p_2)\gamma_\mu r_2\gamma_\nu v(p_2')]\\ +\alpha_3^2[\bar{v}(p_1')\gamma_\nu r_3\gamma_\mu u(p_1)][\bar{u}(p_2)\gamma_\mu r_3\gamma_\nu v(p_2')]\}. \quad (3.5)\end{aligned}$$

By making approximations like

$$\begin{aligned}\bar{v}(p_1')\gamma_\nu r_2\gamma_\mu u(p_1)\sim 2r_{2\mu}\bar{v}(p_1')\gamma_\nu u(p_1),\\ \bar{u}(p_2)\gamma_\mu r_2\gamma_\nu v(p_2')\sim 2r_{2\mu}\bar{u}(p_2)\gamma_\nu v(p_2'),\\ \bar{v}(p_1')\gamma_\nu r_3\gamma_\mu u(p_1)\sim 2r_{3\nu}\bar{v}(p_1')\gamma_\mu u(p_1),\\ \bar{u}(p_2)\gamma_\mu r_3\gamma_\nu v(p_2')\sim 2r_{3\nu}\bar{u}(p_2)\gamma_\mu v(p_2'),\end{aligned} \quad (3.6)$$

we easily see that $\mathfrak{M}^{(B)}$ is negligible.

4. ELECTRON-NUCLEUS SCATTERING

The second-order diagram for electron-nucleus scattering is illustrated in Fig. 5 of paper I. We shall assume that the momentum transfer is so small that the nucleus can be considered as static. The corresponding amplitude is real and proportional to s at high energies. It is given by (2.10) of paper I and its high-energy approximation is given by (2.11) of paper I.

The fourth-order diagram for electron-nucleus scattering which gives an imaginary amplitude is illustrated in Fig. 6 of paper I. The corresponding amplitude is given by (2.12) of paper I. Performing the Feynman parametrization and neglecting r_1 and m, we obtain

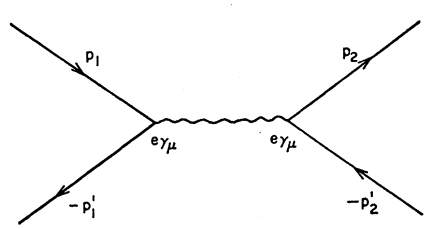

FIG. 1. The second-order Bhabha diagram for electron-positron scattering.

from (2.12) of paper I

$$\mathfrak{M}_0^{(-)} \sim -2Z^2e^4(2\pi)^{-3}\int d^3q \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3$$

$$\times\delta(1-\sum_{i=1}^{3}\alpha_i)\bar{u}(p_2)\gamma_0\mathbf{r}_2\gamma_0 u(p_1)(1-\alpha_1)$$

$$\times[-\mathbf{q}^2+\alpha_1^2E^2+\alpha_2\alpha_3 t-\alpha_1 m^2-(\alpha_2+\alpha_3)\lambda^2]^{-3}. \quad (4.1)$$

Now we have

$$\int d^3q(q^2+A)^{-3}=\tfrac{1}{4}\pi^2A^{-3/2}. \quad (4.2)$$

Thus, (4.1) becomes

$$\mathfrak{M}_0^{(-)} \sim Z^2e^4(8\pi)^{-1}E^2m^{-1}\delta_{12}$$

$$\times\int_0^1 d\alpha_1 d\alpha_2 d\alpha_3\delta(1-\alpha_1-\alpha_2-\alpha_3)(1-\alpha_1)$$

$$\times[-\alpha_1^2E^2-\alpha_2\alpha_3 t+\alpha_1^2m^2+(\alpha_2+\alpha_3)\lambda^2-i\epsilon]^{-3/2}. \quad (4.3)$$

Putting $x=\alpha_1 E$ and taking the limit $E\to\infty$, we obtain

$$\mathfrak{M}_0^{(-)} \sim Z^2e^4(8\pi)^{-1}Em^{-1}\delta_{12}\int_0^1 d\alpha_2 d\alpha_3\delta(1-\alpha_2-\alpha_3)$$

$$\times\int_0^\infty dx(-x^2-\alpha_2\alpha_3 t+\lambda^2-i\epsilon)^{-3/2}. \quad (4.4)$$

Deforming the contour of integration in x to the positive imaginary axis, we get

$$\mathfrak{M}_0^{(-)} \sim iZ^2e^4(8\pi)^{-1}Em^{-1}\delta_{12}\int_0^1 d\alpha_2 d\alpha_3\delta(1-\alpha_2-\alpha_3)$$

$$\times\int_0^\infty dy(y^2-\alpha_2\alpha_3 t+\lambda^2)^{-3/2}=iZ^2e^4(16\pi)^{-1}sm^{-1}M^{-1}\delta_{12}$$

$$\times\int_0^1 d\alpha_2 d\alpha_3\delta(1-\alpha_2-\alpha_3)(-t\alpha_2\alpha_3+\lambda^2)^{-1}, \quad (4.5)$$

where

$$s=2ME.$$

From (4.5) and (2.24), we obtain (2.13) of paper I.

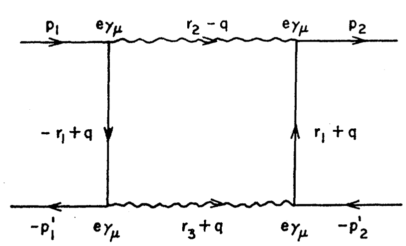

FIG. 2. The fourth-order Bhabha diagram for electron-positron scattering.

5. GENERALIZATIONS AND CONCLUSIONS

In the three different processes treated in the preceding sections, a general expression is found for the imaginary part of the scattering amplitudes at high energy. In each of the cases, it can be expressed in the form (2.16) of paper I, with the impact factor for electrons, and static nuclei given by (2.17) of paper I. The integration in (2.16) is over the two-dimensional space of the transverse momentum. This means that the virtual photons carry vanishing longitudinal momenta in the high-energy limit.

We may similarly treat other scattering processes $a+b\to a+b$, where a and b can be fermions or antifermions. For example, the amplitudes for the processes of positron-positron scattering, positron-proton scattering, electron-antiproton scattering, and positron-antiproton scattering can all be obtained in pretty much the same way. We shall not elaborate on the calculations of these amplitudes. It is significant that, as it turns out, (2.16) of paper I is obtained in all of the cases, with the impact factors for the particles given by exactly the same expressions as before.

In the lowest order of perturbation considered here, the impact factor of a fermion has the following properties: (i) It vanishes for spin-flip transitions; (ii) it is equal to the impact factor of the antiparticle; (iii) it is independent of $\boldsymbol{q}_\perp$ and $\boldsymbol{r}_1$.

A physical picture presents itself here. In high-energy scattering processes, each particle carries an impact factor. The particles interact through interchanging vector mesons of transverse momenta, with the impact factors serving as coupling constants. The coefficient of $i(2r_2r_3)$ of the scattering amplitude is equal to the product of the impact factors with the vector-meson propagators integrated over the two-dimensional space of transverse momentum.