Theory of the Heavy Electron

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Quantum electrodynamics with a generalized minimal conserved current of the form $j_\mu=\alpha_1\gamma_\mu+\alpha_2P_\mu$ is investigated. Fixing the maximum value of α_2 by the anomalous magnetic moment of the electron (muon), we obtain an excited state of mass $m'\geqslant(1+4\pi/\alpha)m$ [881 MeV for e'], and of magnetic moment $\leqslant 3$ in units of $1/2m'$, that can decay to the ground state. The excited state has negative norm, but can be made entirely consistent in the second-quantized theory. The equations are interpreted as those describing a partly "dressed" particle; thus a new form of the perturbation series is suggested.

I. INTRODUCTION

ON the subject of the breakdown of quantum electrodynamics, Low pointed out in 1965[1] that the only theoretically consistent way to describe this problem was in terms of the coupling of the electrons to other particles. In particular, he considered the possibility of a heavy electron e' and the process $e'\to e+\gamma$. The heavy electron has subsequently been the subject of a number of theoretical and experimental investigations.[2]

In this paper, we examine a generalization of the minimal electromagnetic coupling which has, *as a consequence*, a heavy electron of the above type. We have arrived at this theory from an entirely different consideration, namely, we have tried to understand the significance of "convective" currents proportional to the total momentum, which occur in infinite-component field equations.[3]

Because there is a unique parity-conserving vector operator γ_μ in the algebra of Dirac matrices [i.e., the four-dimensional representation of the Lie algebra of $O(4,2)$], any more general vector coupling must involve momenta. The next simple nontrivial coupling is given by the current of Eq. (2.1) below. In order that this coupling should still be applicable to quantum electrodynamics, we must interpret it as an equation describing a partly "dressed" particle. In our case, the fundamental vertex coupling already has the complete observed anomalous magnetic moment of the particle. In other words, the new "free" particle equation describes a fermion with the correct anomalous magnetic moment. As a consequence, we obtain an excited state of the particle. In a first-quantized theory, the excited state does not satisfy the normalization condition, a situation analogous to the negative energy states of the first quantized Klein-Gordon equation. However, a completely consistent second-quantized theory is possible and is developed in detail in Sec. III.

II. CONSERVED CURRENT

We postulate that the conserved electromagnetic current for a spin-$\frac{1}{2}$ particle-antiparticle system is of the form

$$j_\mu^{\mathrm{em}}=\alpha_1\gamma_\mu+\alpha_2P_\mu\,, \tag{2.1}$$

where α_1 and α_2 are constants (more generally, tensors depending on the possible internal quantum numbers of leptons) and $P_\mu=(p'+p)_\mu$ is the total momentum at the vertex. With this choice we deviate from the usual theory, but of course we shall require that the total charge, as well as observable consequences of the theory, agree with experiment. Eq. (2.1) represents the most general parity-conserving current operator linear in both the momenta and the generators of the rest-frame algebra [in the present case, the $O(4,2)$ algebra of Dirac matrices].[4] The current in the configuration space is

$$j_\mu^{\mathrm{em}}(x)=-e\bar{\psi}_1(x)\{\alpha_1\gamma_\mu-i\alpha_2\overleftrightarrow{\partial}_\mu\}\psi_2(x)\,, \tag{2.2}$$

where

$$\bar{\psi}_1\overleftrightarrow{\partial}_\mu\psi_2=\bar{\psi}_1(\partial_\mu\psi_2)-(\partial_\mu\bar{\psi}_1)\psi_2\,.$$

We shall give a momentum-space formulation of the theory and treat the Lagrangian formulation and the second quantization at the end.

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[4] We note that terms like q_μ and $\sigma_{\mu\nu}P^\nu$ in the current can never be conserved, whereas $\sigma_{\mu\nu}q^\nu$ is always conserved. For a discussion of the most general vertex operator, we refer to a subsequent paper, see Ref. 9.

We denote the spinorial wave functions between which j_μ of Eqs. (2.1) acts, by

$$|njm;p]=e^{i\xi\cdot\mathbf{M}}|njm], \tag{2.3}$$

where $n=\pm\frac{1}{2}$ is the "principal quantum number" (eigenvalue of $\frac{1}{2}\gamma_0$) ($+\frac{1}{2}$ for particle, $-\frac{1}{2}$ for antiparticle), (jm) are the usual spin quantum numbers, M_i are the generators of pure Lorentz transformations $\frac{1}{2}i\gamma^i\gamma^0$, and ξ is the relativistic velocity parameter such that the state (2.3) has the momentum $P_\mu=(m\cosh\xi, \hat{\xi}m\sinh\xi)$.[5] The complete configuration-space state vector is

$$\psi_{njm\mathrm{p}}(x)=|njm;p]e^{-2inp\cdot x}\rightarrow|njm;p\rangle$$
$$p\cdot x=+(\mathbf{p}^2+m^2)^{1/2}x^0-\mathbf{p}\cdot\mathbf{x}. \tag{2.4}$$

Now we require that the current (2.1) be conserved,[6] i.e.,

$$\langle n,p|j_\mu q^\mu|n',p'\rangle=0. \tag{2.5}$$

It immediately follows that n is a conserved quantum number.

In contrast to the usual simple current γ_μ, we now have more than one mass value in the theory; Eq. (2.5) gives

$$(2n)m_1[n|j_0e^{i\xi\cdot\mathbf{M}}|n']=m_2[n|e^{i\xi\cdot\mathbf{M}}j_0|n'](2n'), \tag{2.6}$$

or

$$(m_1-m_2)[\alpha_1-\alpha_2(m_1+m_2)]=0. \tag{2.7}$$

This equation is satisfied either for

$$m_1=m_2, \tag{2.8}$$

or for

$$m_1+m_2=\alpha_1/\alpha_2. \tag{2.9}$$

Thus the theory gives two mass values:

$$m_1 \quad \text{and} \quad m_2=\alpha_1/\alpha_2-m_1, \tag{2.10}$$

so that we have two types of transitions

(a) $ee\gamma$ or $e'e'\gamma$,

(b) $ee'\gamma$

and, as we shall see explicitly, the transitions of type (b) are of purely magnetic type—as it should be by gauge invariance. Because of this second solution, we have effectively doubled our Hilbert space, so we write the states, introducing a new quantum numbers τ, as $|n,p,\tau\rangle$ and evaluate the vertex amplitude

$$V_\mu{}^{n'\tau',n\tau}(p',p)=[n'p'\tau'|(\alpha_1\gamma_\mu+\alpha_2P_\mu)|np\tau]. \tag{2.11}$$

Using the identity

$$i\sigma_{\mu\nu}(2np_\tau-2n'p_{\tau'})^\nu=2n'p_{\tau'\mu}{}'+2np_{\tau\mu}$$
$$-2n'\gamma^\mu(p_{\tau'})_\mu\gamma_\mu-2n\gamma^\mu(p_\tau)_\mu\gamma_\mu, \tag{2.12}$$

[5] For the introduction of the quantum number n, see A. O. Barut, Phys. Rev. Letters 20, 893 (1968).

[6] Unless explicitly stated, we shall write n short for njm. The square bras and kets denote the nonunitary spinorial wave functions, the usual bras and kets denote the state vectors.

we can write (2.11) in the form

$$V_\mu{}^{n'\tau',n\tau}(p',p)=[n'p'\tau'|[\alpha_1-\alpha_2(m_\tau+m_{\tau'})]\gamma_\mu$$
$$-i\alpha_2\sigma_{\mu\nu}(2np_\tau-2n'p_{\tau'})^\nu|np\tau]. \tag{2.13}$$

The first term is zero for $\tau'\neq\tau$ because of the mass equation (2.9), so that the transition $e'\rightarrow e+\gamma$ is only magnetic and corresponds exactly to the interaction Hamiltonian[1]

$$H_I=(e\lambda/m_{e'})\bar{\psi}_{e'}\sigma_{\mu\nu}\psi_eF^{\mu\nu}+\text{H.c.} \tag{2.14}$$

For $\tau=\tau'$, we see that (2.13) ascribes an anomalous magnetic moment to the particle. Before determining the numerical values, we have to normalize our spinors by requiring that

$$V_\mu{}^{n'\tau',n\tau}(p'=p)=1. \tag{2.15}$$

Thus the spinors must be divided by a factor $(\alpha_1-2\alpha_2m_\tau)^{1/2}$. This is fine for the lowest mass state m_1; however, for the higher mass state $\alpha_1-2\alpha_2m_2=-(\alpha_1-2\alpha_2m_1)$, so that these states are normalized to -1! This difficulty is the same as the well-known normalization problem in the Klein-Gordon equation and can only be overcome by a second-quantized formalism. On the basis of a first-quantized effective theory, we should throw away this second solution.[7]

On the other hand, if we accept a second-quantized formalism (see Sec. III), we have the following numerical results:

(i) Let the lower mass state be the electron and let us determine the maximum value of α_2 by equating the anomalous magnetic moment resulting from (2.13) to the experimental value, i.e.,

$$\alpha_2/(\alpha_1-2\alpha_2m_e)=K_e/2m_e, \quad K_e=\alpha/2\pi+O(\alpha^2).$$

Now the ratio α_1/α_2 is determined: $\alpha_1/\alpha_2\geq 2m_e/K_e+2m_e$. The mass of the excited state is then

$$m_{e'}\geq m_e(1+2/K_e)=881.76 \text{ MeV}. \tag{2.16}$$

(ii) The anomalous magnetic moment of the excited state is determined from

$$\alpha_2/|\alpha_1-2\alpha_2m_{e'}|=K_{e'}/2m_{e'}$$

or

$$K_{e'}=(K_e+2)\cong 2 \text{ in units of } 1/2m_{e'}. \tag{2.17}$$

The total magnetic moment is $\mu_{e'}\cong 3$ in units of $1/2m_{e'}$.

(iii) The transition amplitude $e'\rightarrow e+\gamma$ is proportional to

$$\alpha_2(\alpha_1-2\alpha_2m_e)^{-1/2}|(\alpha_1-2\alpha_2m_{e'})|^{-1/2}$$
$$=\alpha_2(\alpha_1-2\alpha_2m_e)^{-1}=\lambda/m_{e'}.$$

[7] The infinite-component relativistic equations describing H atom or the model of hadrons (Ref. 3) have a set of solutions with an increasing mass spectrum and a positive normalization condition that are physical, but another set of solutions with a decreasing mass spectrum for which the normalization condition is negative.

Hence the parameter λ in (2.14) is

$$\lambda \leq 1. \qquad (2.18)$$

The experimental upper limit for λ for $m_{e'} \approx 881$ MeV is about 0.1 (see Boley *et al.*[2]). The values for $m_{e'}$ and λ given in (2.16) and (2.18) are the extreme values: If α_2 is smaller (i.e., if we account only for part of the anomalous magnetic moment), $m_{e'}$ becomes larger and λ becomes smaller.

III. LAGRANGIAN AND SECOND-QUANTIZATION FORMALISM

A. Equation of Motion

The Lagrangian density which gives rise to the current (2.1) or (2.2) is

$$\mathcal{L}(x) = -\tfrac{1}{2}\bar{\psi}(x)\{-i\alpha_1\gamma^\mu \overleftrightarrow{\partial}_\mu + \kappa\}\psi(x) - \alpha_2\bar{\psi}(x)\overleftarrow{\partial}^\mu \overrightarrow{\partial}_\mu \psi(x). \qquad (3.1)$$

The corresponding equation of motion becomes

$$(i\alpha_1\gamma_\mu\partial^\mu + \alpha_2\partial^\mu\partial_\mu - \kappa)\psi(x) = 0. \qquad (3.2)$$

The solution of this equation of the form (2.4) leads to the mass equation

$$\alpha_1 m - \alpha_2 m^2 - \kappa = 0, \qquad (3.3)$$

which is equivalent to Eq. (2.7). We determine κ from the mass of the lower state

$$\kappa = \alpha_1 m_e - \alpha_2 m_e^2. \qquad (3.4)$$

With (3.4), Eq. (3.2) factorizes in the form

$$\alpha_2(i\gamma^\mu\partial_\mu - m_e)[i\gamma^\mu\partial_\mu - (\alpha_1/\alpha_2) + m_e]\psi(x) = 0, \qquad (3.5)$$

whose general solution can be written as the sum of the two Dirac solutions, corresponding to masses m_e and $m_{e'}$, Eq. (2.10), respectively:

$$\psi(x) = \psi_e(x) + \psi_{e'}(x),$$

with the quantum number τ denoting the two states of different mass, we write the general solution as

$$\psi(x) = \sum_{njm\tau} \int d^3p \left[\frac{m_\tau}{(2\pi)^3\omega_\tau}\right]^{1/2} \times b_{njm}{}^\tau(\mathbf{p}) u_{njm}{}^\tau(p) e^{-2inp_\tau\cdot x}. \qquad (3.6)$$

B. Orthogonality

We first check that the states with $\tau \neq \tau'$ are orthogonal; we have

$$\int \bar{\psi}_e(x) j_0(x) \psi_{e'}(x) d^3x = \sum_{n'm'nm} \int \frac{d^3p_e d^3p_{e'}}{(2\pi)^3} \left(\frac{m_e m_{e'}}{\omega_e \omega_{e'}}\right)^{1/2} b_{n'e}{}^\dagger(\mathbf{p}_e) b_{n'e'}(\mathbf{p}_{e'}) \times \int d^3x\, e^{i2n_e p_e\cdot x} [np_e | (\alpha_1\gamma_\mu - i\alpha_2 \overleftrightarrow{\partial}_0) | n'p_{e'}] e^{-2in_{e'}p_{e'}\cdot x}.$$

After taking the derivatives and the x integration, the relevant factor is

$$[np | [\alpha_1\gamma_0 - \alpha_2(2n\omega_e + 2n'\omega_{e'})] | n'p'] \times \delta^{(3)}(2n'\mathbf{p}' - 2n\mathbf{p}), \qquad (3.7)$$

which by the identity (2.12) is equal to

$$[np | \{\alpha_1 - \alpha_2(m_e + m_{e'})\}\gamma_0 - i\alpha_2\sigma_{0i}(2np_e - 2n'p_{e'}) | n'p'] \delta^{(3)}(2n'\mathbf{p}' - 2n\mathbf{p}).$$

The first term is zero by virtue of the mass formula (2.10), and the second term vanishes because of the $\delta^{(3)}$ function. Thus,

$$\int \bar{\psi}_1 j_0(x) \psi_2 d^3x = 0. \qquad (3.8)$$

As a consequence of (3.7), we can determine the expansion coefficients in (3.6):

$$b_{njm}{}^\tau(\mathbf{p}) = \int d^3x \left(\frac{m_\tau}{(2\pi)^3\omega_\tau}\right)^{1/2} \times e^{2inp\cdot x} \bar{u}_{njm}{}^\tau(p_\tau) j_0(x) \psi(x),$$
$$b_{njm}{}^{\tau\dagger}(\mathbf{p}) = \int d^3x \left(\frac{m_\tau}{(2\pi)^3\omega_\tau}\right)^{1/2} \times \bar{\psi}(x) j_0(x) u_{njm}{}^\tau(p_\tau) e^{-2inp\cdot x}. \qquad (3.9)$$

The vertex function now becomes

$$\bar{\psi}_{n'j'm'}{}^{\tau'}(x) j_\mu(x) \psi_{njm}{}^\tau(x) \propto e^{i(2n'p' - 2np)\cdot x} \times b_{n'j'm'}{}^{\tau'\dagger}(\mathbf{p}') [n'p'\tau' | \{\alpha_1\gamma_\mu - \alpha_2(2np_\mu + 2n'p_\mu')\} \times | np\tau] b_{njm}{}^\tau(\mathbf{p}), \qquad (3.10)$$

and we get back the same normalization condition as in (2.15).

C. Second Quantization

Now, in order to second-quantize the theory, we derive from the Lagrangian density (3.1) the canonically conjugate fields

$$\pi(x) = \partial\mathcal{L}_0/\partial\psi_{,0}(x) = \tfrac{1}{2}i\alpha_1\bar{\psi}\gamma^0 - \alpha_2\bar{\psi}_{,0}$$

and

$$\bar{\pi}(x) = \partial\mathcal{L}_0/\partial\bar{\psi}_{,0}(x) = -\tfrac{1}{2}i\alpha_1\gamma^0\psi - \alpha_2\psi_{,0}, \qquad (3.11)$$

and assume the equal-time anticommutation relations

$$\{\psi(x), \pi(y)\}_{x^0=y^0} = \{\bar{\psi}(x), \bar{\pi}(y)\}_{x^0=y^0} = i\delta^3(\mathbf{x} - \mathbf{y}). \qquad (3.12)$$

In order to derive the anticommutation relations of the creation and annihilation operators b and $b^\dagger$, we evaluate

$$\{\psi(x), \pi(y)\}_{x^0=y^0} = \sum \int \frac{d^3p d^3p'}{(2\pi)^3} \{b_{njm}{}^\tau(\mathbf{p}), b_{n'jm'}{}^{\tau'}(\mathbf{p})\} \times e^{-2inp\cdot x + 2in'p'\cdot y} | np\tau] \times [n'p'\tau' | (\tfrac{1}{2}i\alpha_1\gamma^0 - 2in'\alpha_2\omega_{\tau'}). \qquad (3.13)$$

We shall show that the proper anticommutation relations for b's are

$$\{b_{nm}{}^{\tau}(\mathbf{p}),b_{n'm'}{}^{\tau'}(\mathbf{p}')\}=\frac{\alpha_1-2\alpha_2 m_e}{\alpha_1-2\alpha_2 m_\tau}\delta^{\tau\tau'}\delta_{nn'}\delta_{mm'}\delta^3(\mathbf{p}-\mathbf{p}')\,. \tag{3.14}$$

Indeed, using the completeness relation

$$\sum_m |njmp\tau][njmp\tau|=\frac{\gamma^\mu(p_\tau)_\mu+2nm_\tau}{2m_\tau(\alpha_1-2\alpha_2 m_e)} \tag{3.15}$$

and (3.14) in Eq. (3.13), we obtain

$$\{\psi(x),\pi(y)\}_{x^0=y^0}=\sum_{n\tau} i\int\frac{d^3p}{(2\pi)^3}e^{2in\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})}\frac{\gamma^\mu(p_\tau)_\mu+2nm_\tau}{\omega_\tau(\alpha_1-2\alpha_2 m_\tau)}\times(\tfrac{1}{2}\alpha_1\gamma_0-2n\alpha_2\omega_\tau)$$

or, explicitly writing the sum over n and changing the dummy index $\mathbf{p}$ into $-\mathbf{p}$ in the case of $n=-\frac{1}{2}$, we get

$$\{\psi(x),\pi(y)\}_{x^0=y^0}=\sum_\tau i\int\frac{d^3p}{(2\pi)^3}\times e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})}\left[\frac{1}{2}+\frac{\mathbf{p}\cdot\boldsymbol{\gamma}}{\alpha_1-2\alpha_2 m_\tau}\right]. \tag{3.16}$$

The second term does not contribute when summed over τ, and we get back the desired equal-time commutation relations (3.12).

The different signs in the commutation relations (3.14) for the two mass states (±1) is essential in this derivation.

D. Positive-Definite Energy

Now we show that with (3.12) and (3.14) the energy is positive definite. The Hamiltonian

$$H=\int d^3x\!:\!\{\pi(x)\psi_{,0}(x)+\bar\psi_{,0}(x)\bar\pi(x)-\mathcal{L}(x)\}\!: \tag{3.17}$$

becomes, in our case,

$$H=\int d^3x\!:\!\{\tfrac{1}{2}i\alpha_1\bar\psi\gamma^0\psi_{,0}-\alpha_2\bar\psi_{,0}\psi_{,0}-\tfrac{1}{2}i\alpha_1\bar\psi_{,0}\gamma^0\psi-\alpha_2\bar\psi_{,0}\psi_{,0}+\tfrac{1}{2}\bar\psi(-i\alpha_1\gamma\cdot\partial+\kappa)\psi+\tfrac{1}{2}\bar\psi(i\gamma\cdot\partial\alpha_1+\kappa)\psi-\alpha_2\partial^\mu\bar\psi\partial_\mu\psi\}\!: \tag{3.18}$$

or, using (3.6),

$$H=\sum_{nm\tau,\,n'm'\tau'}\int d^3p d^3p'\left(\frac{m_\tau m_{\tau'}}{\omega_\tau\omega_{\tau'}}\right)^{1/2}\tfrac{1}{2}\delta^{(3)}(2n\mathbf{p}-2n'\mathbf{p}')\times[np\tau|\{\alpha_1\gamma^0-\alpha_2(2n\omega_\tau+2n'\omega_{\tau'})\}|n'p'\tau']\times(2n\omega_\tau+2n'\omega_{\tau'})\!:\!b_{nm}{}^{\tau\dagger}(\mathbf{p})b_{n'm'}{}^{\tau'}(\mathbf{p}')\!:. \tag{3.19}$$

Because of the vanishing of the factor (3.7), we get a factor $\delta_{\tau\tau'}$ on the right-hand side. Further, we use the relations

$$\begin{aligned}[nmp|\gamma^0|n'm'p]&=\omega\delta_{n'n}\delta_{m'm}/m_\tau(\alpha_1-2\alpha_2 m_e)\,,\\ [nmp|nm'p]&=2n\delta_{mm'}/(\alpha_1-2\alpha_2 m_e)\end{aligned} \tag{3.20}$$

and finally write

$$H=H_e+H_{e'} \tag{3.21}$$

with

$$H_e=\sum_m\int d^3p\,\omega_p\{b_{\frac{1}{2}m}{}^\dagger(\mathbf{p})b_{\frac{1}{2}m}(\mathbf{p})+b_{-\frac{1}{2}m}(\mathbf{p})b_{-\frac{1}{2}m}{}^\dagger(\mathbf{p})\} \tag{3.22}$$

and

$$H_{e'}=-\sum_m\int d^3p\,\omega_p\times\{b_{\frac{1}{2}m}{}^\dagger(\mathbf{p})b_{\frac{1}{2}m}(\mathbf{p})+b_{-\frac{1}{2}m}(\mathbf{p})b_{-\frac{1}{2}m}{}^\dagger(\mathbf{p})\}\,. \tag{3.23}$$

The term $H_{e'}$ formally appears to be negative definite, but, because of the commutation relations (3.14), $b^\dagger$ has to be reinterpreted as the operator "minus the Hermitian conjugate of b," *for e' states.* Thus, H is positive definite. The procedure amounts to an indefinite metric in the τ space and is entirely consistent.[8] The procedure amounts to the prescription that, for the excited state, $b^\dagger$ always goes with a negative sign. This makes the signs in (3.14) and (3.23) always $+1$.

E. Charge

Finally, the charge

$$Q=\int d^3x\!:\!\psi(x)j_0(x)\psi(x)\!: \tag{3.24}$$

can be written as

$$Q=Q_e+Q_{e'}\,. \tag{3.25}$$

For the first term we get

$$\begin{aligned}Q_e&=\sum_{nmn'm'}\int\frac{d^3pd^3p'm_e}{(\omega\omega')^{1/2}}\times\!:\!b_{n'm'}{}^\dagger(\mathbf{p}')b_{nm}(\mathbf{p}')\!:\!\delta^{(3)}(2n\mathbf{p}-2n'\mathbf{p}')\times[n'm'p|(\alpha_1-2\alpha_2 m_e)|nmp]\\&=\sum_m\int d^3p\{b_{\frac{1}{2}m}{}^\dagger(\mathbf{p})b_{\frac{1}{2}m}(\mathbf{p})-b_{-\frac{1}{2}m}(\mathbf{p})b_{-\frac{1}{2}m}{}^\dagger(\mathbf{p})\}\,,\end{aligned} \tag{3.26}$$

which shows that ($n=-\frac{1}{2}$) states contribute with the opposite sign to the charge with respect to $n=+\frac{1}{2}$ states.

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[8] Similar situations have been discussed by A. O. Barut and G. H. Mullen, Ann. Phys. (N. Y.) **20**, 184 (1962); **20**, 203 (1962), by R. Norton, J. Math. Phys. **6**, 981 (1965).

For the operator $Q_{e'}$, we get a change of sign from the operation of Hermitian conjugation and another from the normalization condition $\alpha_1 - 2\alpha_2 m_{e'} = -(\alpha_1 - 2\alpha_2 m_e)$, so that $n=\frac{1}{2}$, e' state has the same charge as the $n=\frac{1}{2}$, e state, and similarly for $n=-\frac{1}{2}$. This completes the second-quantized formalism for our theory.

IV. FURTHER COMMENTS AND CONCLUSIONS

We have studied the consequences of a more general linear conserved current (2.1) describing two fermions with different masses and anomalous magnetic moments. This more general current can thus be viewed as an effective current describing a dressed particle when compared with the usual Dirac coupling. In the latter formalism, all anomalous magnetic moment effects are ascribed to higher-order terms in the perturbation theory, whereas in the present formalism part of the effect of the radiation is built in the lowest-order term itself. The current (2.1) does not completely contain all the radiation effects because we do not get the dependence of the form factors on the momentum transfer q^2. For this, one has to go to more general currents[9] and to higher dimensional representations of the group $O(4.2)$.[3] With the current (2.1) we can formulate a new perturbation theory. How such a theory will account for higher-order terms and how it will be renormalized remains to be seen.

The new current (2.1) is minimal[3] in the sense that it is the most general parity conserving, conserved operator, linear in the momenta and linear in the generators γ_μ. There is still the anomalous term

$$\alpha_3 \sigma_{\mu\nu} q^\nu$$

which is conserved by itself. The addition of such a term to the current (2.1) does *not* change the mass spectrum, but of course will modify the magnetic moments of the particles and bring a new parameter into the theory, and one can give to $m_{e'}$ any desired value instead of (2.16). Such a theory would be more phenomenological. However, it is important to notice that if we choose $\alpha_3 = -\alpha_2$, the anomalous magnetic moments of the particles vanish, there is no transition of the type $e' \to e+\gamma$; in other words, the two solutions of Eq. (3.5) become completely uncoupled. This theory is then equivalent to two uncoupled Dirac equations, hence of trivial character as far as a two-mass theory is concerned.

At this stage it is appropriate to comment on a paper by Rosen[10] who also considers a second-order Lagrangian and is able to obtain the μ-meson mass together with the electron. Rosen's equation is in fact of the type mentioned in the last paragraph, namely, two entirely uncoupled Dirac equations. His paper is based on the observation that $m_e/m_\mu \cong \frac{2}{3}\alpha$ is the coefficient of the d^3x/dt^3 term in the classical equation of motion (radiation reaction).

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