Continuous-Moment Sum Rules and Absorptive Regge Cuts in the Process $\pi^- p \rightarrow \eta n$†

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The Regge-pole model with absorptive corrections is applied together with the continuous-moment sum rules (CMSR) to the process $\pi^- p \rightarrow \eta n$. It is found that a good fit to both the low- and high-energy scattering cross sections and the polarization at several values of the momentum transfer t can be obtained by including Regge absorptive cuts and three Regge poles corresponding to the $A_{2H}(1315)$, the $\pi_N(1016)$, and possibly a pole with the same trajectory parameters as the pion conspirator (π_c) found in previous studies of pion photoproduction.

I. INTRODUCTION

RECENTLY, a Regge-pole model containing absorptive corrections has been proposed by Arnold[1] and by Frautschi and Margolis.[2] The model is based on the idea that the absorptive corrections, which produce Regge cuts, are due to multiple Regge-pole exchange. The beauty of this model is enhanced by the fact that only the parameters of the exchanged Regge poles enter into the correction terms, and thus it is especially suited to phenomenological studies of scattering processes. Its success in fitting differential cross sections and polarizations for several reactions[2–5] makes it reasonable to attempt its use in conjunction with the finite-energy or continuous-moment sum rules (CMSR). This has been done recently for πN scattering with good results.[6]

We present here the results of the application of this model together with the CMSR[7] to the process $\pi^- p \rightarrow \eta n$. This process is particularly well suited for the study of the effects of absorptive corrections because only even signature trajectories with even parity, odd G parity, and isotopic spin 1 can be exchanged in the inelastic process. The only known particles with these quantum numbers are the $A_{2H}(1315)$, the $\pi_N(1016)$, and possibly the pion conspirator (π_c) which may correspond to the $A_{2L}(1270)$.[8] Assuming that the leading trajectory for the elastic process is the Pomeranchuk trajectory (P), then the leading absorptive cuts are produced by A_2-P interference.

Using the phase-shift analysis recently made for this process by Botke,[9] we computed the CMSR using an upper cutoff of $T_\pi = 2.4$ GeV (incident pion laboratory kinetic energy) for values of the continuous moment ranging from 0 to 2. We saturate the right-hand side by using (a) one Regge pole, (b) two poles, (c) one pole and absorptive cut, (d) two poles and absorptive cut, and finally, (e) three poles and absorptive cut. The results, although not as definite as one would like, because of uncertainty in the data, indicate that absorptive cuts are needed to fit both the low- and high-energy data. In addition, and very unexpectedly, we found that besides the A_2 pole, a second pole corresponding very well with the $\pi_N(1016)$ is certainly necessary. The evidence for the pion conspirator is not as compelling as for the A_2 and the π_N but the trajectory parameters we obtain agree with those found in recent CMSR studies of pion photoproduction.[7] However, the existence of Regge cuts in π^+ photoproduction, n-p charge exchange, and similar processes eliminates the need for conspiracies of Regge trajectories. The method we use here and the existing data do not allow us to decide between the existence or nonexistence of the π_c in this process, although it is certainly not needed within the statistical errors.

The model as it applies to the CMSR is presented in Sec. II followed in Sec. III by a discussion of the fitting procedure. The results are presented in Sec. IV

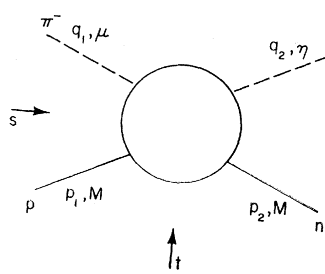

FIG. 1. Kinematics.

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[2] S. Frautschi and B. Margolis, Nuovo Cimento **56A**, 1155 (1968).
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[4] M. L. Blackmon, Phys. Rev. **178**, 2385 (1969).
[5] M. L. Blackmon and G. Goldstein, Phys. Rev. **179**, 1480 (1969).
[6] C. Ferro Fontan, R. Odorico, and L. Masperi, Nuovo Cimento **58A**, 534 (1968).
[7] The CMSR has been applied to the process $\gamma p \rightarrow \pi^+ n$ by the following authors: K. V. Vasavada and K. Raman, Phys. Rev. Letters **21**, 577 (1968); K. Raman and K. V. Vasavada, Phys. Rev. **175**, 2191 (1968); P. Di Vecchia *et al.*, Phys. Letters **27B**, 296 (1968); P. Di Vecchia *et al.*, Phys. Letters **27B**, 521 (1968).
[8] The quantum numbers of this meson are not yet well established. However, several authors have proposed models in which they assume that the $A_{2L}(1270)$ has the same spin and parity as the $A_{2H}(1315)$; D. M. Austin, J. V. Beaupre, and K. E. Lassila, Phys. Rev. **173**, 1573 (1968); J. V. Beaupre *et al.*, Phys. Rev. Letters **21**, 1849 (1968); T. J. Gajdicar and J. W. Moffat, Phys. Rev. **181**, 1875 (1969).
[9] J. C. Botke, Phys. Rev. **180**, 1417 (1969).

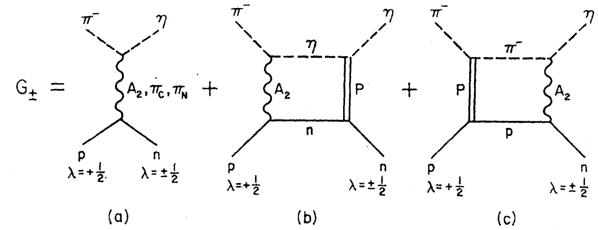

FIG. 2. Diagrams corresponding to first-order absorptive correction.

and a discussion of the parameters and the errors is given in Sec. V.

II. THE MODEL

The four-momenta of the external particles are defined in the usual manner[10] (Fig. 1). The Mandelstam variables are given by

$$s=(p_1+q_1)^2=(p_2+q_2)^2=W^2\,, \tag{1}$$

$$t=(p_1-p_2)^2=2(M^2-E_1E_2)+2|\mathbf{p}_1||\mathbf{p}_2|\cos\theta_s\,, \tag{2}$$

$$u=(p_1-q_2)^2=\Sigma-s-t\,, \tag{3}$$

$$\nu=(s-u)/4M\,, \tag{4}$$

where

$$4sp_1^2=[s-(M+\mu)^2][s-(M-\mu)^2]\,,$$
$$4sp_2^2=[s-(M+\eta)^2][s-(M-\eta)^2]\,,$$
$$E_{1,2}^2=p_{1,2}^2+M^2\,,$$
$$\Sigma=2M^2+\eta^2+\mu^2\,.$$

The two independent t-channel helicity amplitudes are given in terms of the usual A and B invariant amplitudes by

$$f_{++}^t(\nu,t)=-[4(t-4M^2)^{1/2}]^{-1}\times[(t-4M^2)A(\nu,t)-4M^2\nu B(\nu,t)]\,, \tag{5a}$$

$$f_{+-}^t(\nu,t)=\tfrac{1}{8}\sin\theta_t\{[t-(\eta+\mu)^2][t-(\eta-\mu)^2]\}^{1/2}\times B(\nu,t)\,. \tag{5b}$$

We define also the t-channel amplitudes which are free of kinematic singularities:

$$\tilde{f}_{++}^t(\nu,t)=(t-4M^2)^{1/2}f_{++}^t(\nu,t)\,, \tag{6a}$$

$$\tilde{f}_{+-}^t(\nu,t)=\frac{f_{+-}^t(\nu,t)}{\sin\theta_t\{[t-(\eta+\mu)^2][t-(\eta-\mu)^2]\}^{1/2}}\,. \tag{6b}$$

These amplitudes satisfy the following crossing relations:

$$\tilde{f}_{++}^t(\nu,t)=\tilde{f}_{++}^t(-\nu,t)\,, \tag{7a}$$

$$\tilde{f}_{+-}^t(\nu,t)=-\tilde{f}_{+-}^t(-\nu,t)\,. \tag{7b}$$

The s-channel helicity amplitudes are defined as[1]

$$G_+=W(f_1+f_2)\cos\tfrac{1}{2}\theta_s\,, \tag{8a}$$

$$G_-=W(f_1-f_2)\sin\tfrac{1}{2}\theta_s\,. \tag{8b}$$

Using Eqs. (6) and (8) and the definition of f_1 and f_2 in terms of the A and B amplitudes, the following s-t crossing relations are easily derived:

$$\tilde{f}_{++}^t=\gamma_{11}G_++\gamma_{12}G_-\,, \tag{9a}$$

$$\tilde{f}_{+-}^t=\gamma_{21}G_++\gamma_{22}G_-\,, \tag{9b}$$

where

$$\gamma_{11}=(\pi/2W\cos\tfrac{1}{2}\theta_s)(a_+b_++a_-b_-)\,, \tag{10a}$$

$$\gamma_{12}=(\pi/2W\sin\tfrac{1}{2}\theta_s)(a_+b_+-a_-b_-)\,, \tag{10b}$$

$$\gamma_{21}=(\pi/4W\cos\tfrac{1}{2}\theta_s)(b_++b_-)\,, \tag{10c}$$

$$\gamma_{22}=(\pi/4W\sin\tfrac{1}{2}\theta_s)(b_+-b_-)\,, \tag{10d}$$

and

$$a_\pm=(4M^2-t)(M\pm W)+4M^2\nu\,, \tag{11a}$$

$$b_\pm=[(E_1\pm M)(E_2\pm M)]^{1/2}. \tag{11b}$$

We observe here that as $\nu\to\infty$ the crossing coefficients have the following limits:

$$\lim_{\nu\to\infty}\gamma_{11}=4\pi M\,, \tag{10a$'$}$$

$$\lim_{\nu\to\infty}\gamma_{12}=\frac{2\pi}{\sqrt{s}}\frac{(-t)}{\sin\frac{1}{2}\theta_s}\,, \tag{10b$'$}$$

$$\lim_{\nu\to\infty}\gamma_{21}=\pi/2M\nu\,, \tag{10c$'$}$$

$$\lim_{\nu\to\infty}\gamma_{22}=-\frac{\pi}{\nu(\sqrt{s})\sin\frac{1}{2}\theta_s}\,. \tag{10d$'$}$$

Since the A_{2H} is the dominant Regge pole exchanged in this process, we have kept only the absorptive corrections arising from A_{2H}-P interference and these only to first order. The eikonal phases are then defined by[4]

$$G_+=(p_1p_2)^{1/2}W\int_0^\infty db\,bJ_0(b\Delta)\times[\chi_0+\tfrac{1}{2}i(\chi_0\chi_P{}^{\eta N}+\chi_P{}^{\pi N}\chi_0)]\,, \tag{12a}$$

$$G_-=(p_1p_2)^{1/2}W\int_0^\infty db\,bJ_1(b\Delta)\times[\chi_f+\tfrac{1}{2}i(\chi_f\chi_P{}^{\eta N}+\chi_P{}^{\pi N}\chi_f)]\,, \tag{12b}$$

where

$$\Delta=2(p_1p_2)^{1/2}\sin\tfrac{1}{2}\theta_s\,, \tag{13}$$

$$\chi_0=(p_1p_2)^{-1/2}W^{-1}\int_0^\infty d\Delta\,\Delta J_0(b\Delta)G_+{}^{\text{Regge pole}}\,, \tag{14a}$$

$$\chi_f=(p_1p_2)^{-1/2}W^{-1}\int_0^\infty d\Delta\,\Delta J_1(b\Delta)G_-{}^{\text{Regge pole}}\,, \tag{14b}$$

[10] We use the system of units in which $\hbar=c=1$ and the metric in which $p\cdot q=p_0q_0-\mathbf{p}\cdot\mathbf{q}$. The normalization of our helicity amplitudes and the definitions of the scattering angles are those of L. L. Wang, Phys. Rev. **142**, 1187 (1966).

and

$$\chi_P^{(\pi N;\eta N)}=\frac{1}{(p_1;p_2)W}\int_0^\infty d\Delta\,\Delta J_0(b\Delta)G_{+,\mathrm{el}}^{(\pi N;\eta N)}. \quad (14c)$$

Equations (12a) and (12b) correspond to Figs. 2(a)–2(c).

We assume the following forms for the ith Regge-pole amplitudes.

$$(\tilde{f}_{++}{}^t)_{\mathrm{pole}}=\beta_{++}{}^i(t)e^{K\alpha_i' t}\epsilon_i(\alpha_i(t))\frac{(e^{-i\pi/2}\nu)^{\alpha_i(t)}}{\sin\frac{1}{2}\pi\alpha_i(t)}, \quad (15a)$$

$$(\tilde{f}_{+-}{}^t)_{\mathrm{pole}}=\beta_{+-}{}^i(t)e^{K\alpha_i' t}\epsilon_i(\alpha_i(t))\frac{(e^{-i\pi/2}\nu)^{\alpha_i(t)}}{\sin\frac{1}{2}\pi\alpha_i(t)}\frac{1}{\nu}, \quad (15b)$$

where

$$\alpha_i(t)=\alpha_i(0)+\alpha_i' t, \quad (16)$$

and K is an arbitrary parameter corresponding to a universal scale factor.

The $\epsilon_i(\alpha_i(t))$ are ghost-eliminating factors. For the A_2, we choose the Gell-Mann or nonsense-choosing mechanism, $\epsilon_{A_2}=\alpha_{A_2}(t)$. This is motivated by the lack of any dip in the differential cross sections in the region where the A_2 trajectory should cross $\alpha=0$. For the π_c, we also chose the nonsense mechanism in order to avoid the embarrassing prediction of a low-mass scalar meson. As will be discussed in Sec. IV, we found that a lower-lying trajectory which corresponds well with the $\pi_N(1016)$ was surprisingly necessary in our fits. For this meson, we chose $\epsilon_{\pi_N}=\alpha_{\pi_N}(t)+2$ in order to eliminate a possible ghost at $\alpha=-2$.[11]

Group-theoretical considerations[12] combined with analyticity and factorization determine that near $t=0$, the residues of the s-channel helicity amplitudes corresponding to the exchange of Regge poles of given M behave in the following manner:

$$G_+ \xrightarrow[t\to 0]{} \mathrm{const},\quad M=0; \qquad t,\quad M=1 \quad (17a)$$

$$G_- \xrightarrow[t\to 0]{} \sin\tfrac{1}{2}\theta_s,\quad M=0; \qquad \sin\tfrac{1}{2}\theta_s,\quad M=1. \quad (17b)$$

Using the crossing relations Eqs. (9) and (10) we find that, for the leading terms, the residues of the t-channel helicity amplitudes behave like

$$\beta_{+\pm}{}^i(t) \xrightarrow[t\to 0]{} \mathrm{const},\quad M=0; \qquad t,\quad M=1. \quad (18)$$

Therefore we assume that the residue functions $\beta_{+\pm}{}^i$ of Eqs. (15) are constants multiplied by the above-mentioned factors. Also for simplicity, we did not consider the problem of trajectory mixing where two straight-line trajectories cross. The A_2 was assumed to be a pure $M=0$ trajectory, the pion conspirator to be pure $M=1$ which it must be at $t=0$, and the $\pi_N(1016)$ to be pure $M=0$.

The s-channel Pomeranchuk amplitude was taken to be of the following form:

$$G_{+,\mathrm{el}}{}^{(\pi N;\eta N)}=\frac{\beta_{++}{}^{P(\pi N;\eta N)}}{4\pi M}e^{K\alpha_P' t}\frac{\alpha_P(t)}{\sin\frac{1}{2}\pi\alpha_P(t)}\times(e^{-i\pi/2}\nu)^{\alpha_P(t)}, \quad (19)$$

where $\alpha_P=1+\alpha_P' t$. We evaluate the residue $\beta_{++}{}^{P(\pi N)}$ by using the optical theorem in conjunction with the high-energy total cross section data for πN scattering, taking the $I=\frac{1}{2}$ asymptotic cross section $\sigma_T\sim 27$ mb.[13]

The total cross section for ηN scattering is not known, however, in view of the fact that the ηN branching ratios of the known $I=\frac{1}{2}$, πN resonances are much smaller than the elastic branching ratios,[9] we assume that $\beta_{++}{}^{P(\eta N)}\sim 0$.

By combining Eqs. (15), (19), (9), and (14), we can calculate the eikonal phases. In order to perform the resulting integrals, we assumed that factors like $\alpha(t)/\sin\frac{1}{2}\pi\alpha(t)$ could be replaced by their average values and removed from the integrands. The resulting integrands decay exponentially roughly as e^t so that we need average only over the range $-1\lesssim t\leqslant 0$. Then using Eqs. (12) and (9) the t-channel amplitudes result

$$\begin{aligned}\tilde{f}_{++}{}^t(\nu,t)&=\beta_{++}{}^{A_2}\alpha_{A_2}R(\alpha_{A_2},\nu)+\frac{1}{8\pi M^2}\beta_{++}{}^{P(\pi N)}\beta_{++}{}^{A_2}\\&\times\langle(\alpha_P/\sin\tfrac{1}{2}\pi\alpha_P)\rangle\langle(\alpha_{A_2}/\sin\tfrac{1}{2}\pi\alpha_{A_2})\rangle\\&\times\frac{e^{K\alpha_e' t}}{2(\mu+K)}\frac{1}{(\alpha_{A_2}'+\alpha_P')}(e^{-i\pi/2}\nu)^{\alpha_e(t)}\\&+\beta_{++}{}^{\pi_c}t\alpha_{\pi_c}R(\alpha_{\pi_c},\nu)\\&+\beta_{++}{}^{\pi_N}R(\alpha_{\pi_N},\nu)(\alpha_{\pi_N}+2),\end{aligned} \quad (20a)$$

$$\begin{aligned}\tilde{f}_{+-}{}^t(\nu,t)&=\beta_{+-}{}^{A_2}\frac{\alpha_{A_2}}{\nu}R(\alpha_{A_2},\nu)\\&+\frac{1}{8\pi M^2}\beta_{++}{}^{P(\pi N)}\left(\beta_{+-}{}^{A_2}+\frac{\alpha_{A_2}'}{\alpha_P'}\frac{\beta_{++}{}^{A_2}}{8M^2}\right)\langle\alpha_P/\sin\tfrac{1}{2}\pi\alpha_P\rangle\\&\times\langle\alpha_{A_2}/\sin\tfrac{1}{2}\pi\alpha_{A_2}\rangle\frac{e^{K\alpha_e' t}}{2(\mu+K)}\frac{\alpha_P'}{(\alpha_P'+\alpha_{A_2}')^2}(e^{-i\pi/2}\nu)^{\alpha_e(t)}\frac{1}{\nu}\\&+\beta_{+-}{}^{\pi_c}t\frac{\alpha_{\pi_c}}{\nu}R(\alpha_{\pi_c},\nu)\\&+\beta_{+-}{}^{\pi_N}\frac{1}{\nu}R(\alpha_{\pi_N},\nu)(\alpha_{\pi_N}+2),\end{aligned} \quad (20b)$$

[11] Our choice of the ϵ_i's is a simplification of the actual behavior of the residues near nonsense points. Since our fits are made for values of t in a small interval the effect of these approximations is negligible.

[12] D. Freedman and J. Wang, Phys. Rev. 160, 1560 (1967).

[13] High-Energy Physics Group, Department of Physics, University of Michigan Report (unpublished).

where

$$\mu = \ln\nu - \tfrac{1}{2}i\pi\,, \qquad (21a)$$

$$\alpha_e(t) = \alpha_{A_2}(0) + \left(\frac{\alpha_{A_2}'\alpha_P'}{\alpha_{A_2}'+\alpha_P'}\right)t\,, \qquad (21b)$$

and

$$R(\alpha,\nu) = \frac{e^{K\alpha' t}(e^{-i\pi/2}\nu)^{\alpha(t)}}{\sin\frac{1}{2}\pi\alpha(t)}\,. \qquad (21c)$$

We approximate

$$\langle\alpha_P(t)/\sin\tfrac{1}{2}\pi\alpha_P(t)\rangle \simeq 1 \quad\text{and}\quad \langle\alpha_{A_2}(t)/\sin\tfrac{1}{2}\pi\alpha_{A_2}(t)\rangle \simeq 0.7\,.$$

The CMSR for the amplitudes $\nu\tilde{f}_{++}{}^t(\nu,t)$ and $\tilde{f}_{+-}{}^t(\nu,t)$ are derived in the manner described by Della Selva *et al.*[14]

$$S_{++(\text{l.h.s.})}(t,\gamma) = \frac{M}{\nu_m^2}\nu_N^2 G_1 + \frac{1}{\nu_m^2}\left[\left(\frac{M^*-M}{2M}\right)(t-4M^2)\nu_p + 2M\nu_p^2\right]G_2 - \frac{1}{\nu_m^{\gamma+2}}\int_{\nu_\eta}^{\nu_m} d\nu[\nu^2-\nu_\eta^2]^{\gamma/2} \times \text{Im}[e^{-i\pi\gamma/2}\nu\tilde{f}_{++}{}^t(\nu,t)]\,, \qquad (22a)$$

$$S_{++(\text{r.h.s.})}(t,\gamma) = \beta_{++}{}^{A_2}G_4(\alpha_{A_2},\nu_m)\left(\frac{\alpha_{A_2}}{\alpha_{A_2}+\gamma+2}\right) - \frac{G_3}{\nu_m^2}\beta_{++}{}^{A_2}e^{-2K} \times I(\gamma+2+\alpha_e(t)) + t\beta_{++}{}^{\pi_c}G_4(\alpha_{\pi_c},\nu_m)\left(\frac{\alpha_{\pi_c}}{\alpha_{\pi_c}+\gamma+2}\right) + \beta_{++}{}^{\pi N}G_4(\alpha_{\pi N},\nu_m)\left(\frac{\alpha_{\pi N}+2}{\alpha_{\pi N}+\gamma+2}\right)\,, \qquad (22b)$$

$$S_{+-(\text{l.h.s.})}(t,\gamma) = \frac{1}{8M}G_1 + \frac{1}{4M}G_2 - \frac{1}{\nu_m^{\gamma}}\int_{\nu_\eta}^{\nu_m} d\nu[\nu^2-\nu_\eta^2]^{\gamma/2} \times \text{Im}[e^{-i\pi\gamma/2}\tilde{f}_{+-}{}^t(\nu,t)]\,, \qquad (22c)$$

$$S_{+-(\text{r.h.s.})}(t,\gamma) = \beta_{+-}{}^{A_2}G_4(\alpha_{A_2},\nu_m)\left(\frac{\alpha_{A_2}}{\alpha_{A_2}+\gamma}\right) + G_3\left[\beta_{+-}{}^{A_2} + \frac{\beta_{++}{}^{A_2}}{8M^2}\frac{\alpha_{A_2}'}{\alpha_P'}\right]\frac{\alpha_P'}{(\alpha_{A_2}'+\alpha_P')}I(\gamma+\alpha_e(t)) + t\beta_{+-}{}^{\pi_c}G_4(\alpha_{\pi_c},\nu_m)\left(\frac{\alpha_{\pi_c}}{\alpha_{\pi_c}+\gamma}\right) + \beta_{+-}{}^{\pi N}G_4(\alpha_{\pi N},\nu_m)\left(\frac{\alpha_{\pi N}+2}{\alpha_{\pi N}+\gamma}\right)\,, \qquad (22d)$$

and

$$S_{+\pm(\text{l.h.s.})}(t,\gamma) = S_{+\pm(\text{r.h.s.})}(t,\gamma)\,, \qquad (23)$$

where

M^* = mass of $P_{11}(1470)$,

$$\nu_N = (2M^2+t-\Sigma)/4M\,,$$

$$\nu_P = (2M^{*2}+t-\Sigma)/4M\,,$$

$$\nu_\eta = [2(M+\eta)^2+t-\Sigma]/4M\,,$$

$$G_1 = -\frac{\pi}{\sqrt{2}}\frac{g_{\pi NN}g_{\eta NN}}{\nu_m^{\gamma}}(\nu_\eta^2-\nu_N^2)^{\gamma/2}\,,$$

$$G_2 = -\frac{\pi}{2\sqrt{2}}\frac{g_{\pi NN^*}g_{\eta NN^*}}{\nu_m^{\gamma}}(\nu_\eta^2-\nu_P^2)^{\gamma/2}\,,$$

$$G_3 = \frac{1}{32\pi M^2}\frac{\beta_{++}{}^{P(\pi N)}}{\nu_m^{\gamma}}\langle\alpha_P/\sin\tfrac{1}{2}\pi\alpha_P\rangle\langle\alpha_{A_2}/\sin\tfrac{1}{2}\pi\alpha_{A_2}\rangle \times\frac{\exp[-K(\gamma+\alpha_{A_2}(0))]}{(\alpha_P'+\alpha_{A_2}')}\,,$$

$$G_4(\alpha,\nu_m) = e^{K\alpha' t}\frac{[\sin\frac{1}{2}\pi(\alpha+\gamma)]\nu_m^{\alpha}}{\sin\frac{1}{2}\pi\alpha}\,,$$

$$I(a) = \pi - 2\theta + \sum_{n=1}^{\infty}\frac{i^{n-1}}{n!}\frac{a^n}{n}[g_+{}^n - g_-{}^n]\,,$$

$$\theta = \tan^{-1}[(\ln\nu_m + K/\tfrac{1}{2}\pi)]\,,$$

$$g_\pm = \pm\tfrac{1}{2}\pi - i[\ln\nu_m + K]\,.$$

The nucleon pole for $\nu>0$ occurs at $\nu=-\nu_N$ which, for $-t>1.01$, lies above the ηN threshold. In that case, the sum rules given above need to be modified slightly. The unphysical cut, between the πN and the ηN thresholds, has been replaced by a pole obtained by using the narrow-resonance approximation to the P_{11} resonance.

III. THE METHOD

Our original intent was to determine the Regge-pole parameters by fitting the CMSR calculated from the low-energy data and then to compute the high-energy differential cross sections. This scheme was only moderately successful, owing to the experimental uncertainties in the data. We therefore decided to use both the low- and high-energy data at the same time, giving exact validity to the CMSR in connecting both regions. Fits to the differential cross sections[15] and the CMSR were made for several values of t simultaneously. The success of this approach was surprisingly good, allowing us to obtain the set of parameters given in Table I.

The upper cutoff in the CMSR was chosen as a balance between keeping it high enough that the Regge representation is a good approximation to the scattering

[14] A. Della Selva, L. Masperi, and R. Odorico, Nuovo Cimento **55A**, 602 (1968).

[15] O. Guisan *et al.*, Phys. Letters **18**, 200 (1965); Oregon Conference on Regge Theory, 1968 (unpublished).

TABLE I. Values of the Regge-pole parameters with their estimated errors.

$\alpha_{A_2}(0)=0.46\pm0.05$	$\alpha_{\pi_c}(0)=0\pm0.2$	$\alpha_{\pi_N}(0)=-0.6\pm0.1$
$\alpha_{A_2}'(0)=0.7\pm0.15$	$\alpha_{\pi_c}'(0)=1.4\pm0.3$	$\alpha_{\pi_N}'(0)=0.6\pm0.2$
$\beta_{++}{}^{A_2}=+7.5\pm2$	$\beta_{++}{}^{\pi_c}=-5\pm10$	$\beta_{++}{}^{\pi_N}=-0.4\pm0.3$
$\beta_{+-}{}^{A_2}=+6\pm1$	$\beta_{+-}{}^{\pi_c}=-1\pm10$	$\beta_{+-}{}^{\pi_N}=+0.5\pm0.4$
$K=2\pm1$	$\alpha_P'(0)=0\pm0.1$	

amplitude and low enough that one could evaluate the left-hand side using the partial-wave analysis. By choosing this to be $T_\pi=2.4$ GeV, we have integrated over a region $1.3<T_\pi<2.4$ GeV in which no experiments have been done and this is most certainly a major source of error in this work.

The fits were made over a total of 97 data points of which 70 correspond to the low-energy region and 27 to the high-energy differential cross sections. Each trajectory required four parameters; i.e., the intercept $\alpha_i(0)$, the slope α_i', and the two residues $\beta_{++}{}^i$ and $\beta_{+-}{}^i$. Also the slope of the Pomeranchuk trajectory (α_P') and the scale parameters K were freed. Therefore, the maximum number of parameters, corresponding to case (e), was fourteen.

The choice of the best set of parameters was performed by minimizing the function φ defined by

$$\varphi=\sum_{t,\gamma}\left[S_{++(\text{l.h.s.})}(t,\gamma)-S_{++(\text{r.h.s.})}(t,\gamma)\right]^2+\sum_{t,\gamma}\left[S_{+-(\text{l.h.s.})}(t,\gamma)-S_{+-(\text{r.h.s.})}(t,\gamma)\right]^2+\sum_{t,\gamma}\left[\left(\frac{d\sigma}{dt}\right)_{\text{expt}}-\left(\frac{d\sigma}{dt}\right)_{\text{Regge}}\right]^2. \quad (24)$$

The numerical work was performed on an IBM Model 360-65 computer using a nonlinear least-squares program (IBM Share program No. 1428). The values of φ for the different fits are shown in Table II.

IV. DISCUSSION OF RESULTS

The first model we tried was single Regge-pole exchange. Although this model has been reasonably successful in fitting the differential cross-section data,[16] we found that it totally failed to fit the CMSR, particularly for $-t>0.2$ (Fig. 3). This model also predicts no polarization. As we will discuss later, this is most likely wrong even though the scant amount of data available[17] is consistent with zero at $T_\pi=4.86$ GeV and $T_\pi=11.06$ GeV for $t\simeq-0.2$. We next tried a two-pole model (Austin *et al.*[8]), which is able to produce nonzero polarization, but found no significant improvement in the CMSR.

TABLE II. Values of the minimized function φ for the different fits.

Model	φ
A_2	75
A_2+cut	45
$A_2+\pi_N$+cut	30
$A_2+\pi_N+\pi_c$+cut	28

[16] R. Phillips and W. Rarita, Phys. Rev. Letters **15**, 807 (1965); M. Barmawi, Phys. Rev. **166**, 1857 (1968); F. Arbab, N. Bali, and J. Dash, *ibid.* **158**, 1515 (1967).

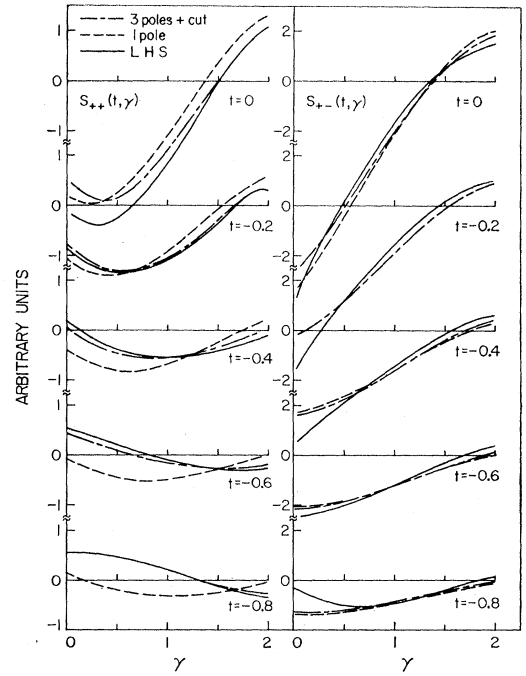

FIG. 3. Continuous-moment sum rules. 3 poles + cut is plotted only where it differs from the left-hand side. One pole is plotted only where it differs from 3 poles + cut.

Recently, Blackmon[4] has shown that the differential cross-section data for this process could be fitted rather well with one Regge pole and its first-order absorptive correction. We tried this model and found significant improvement in the CMSR, particularly in the large-t region where the one- and two-pole models were in complete disagreement with the CMSR. This model predicts[4] zero polarization near $t=-0.2$, in agreement with the higher-energy data,[17] and a large polarization in the region near $t=-0.8$. In general, a Regge model gives a weak energy dependence of the polarization, hence one cannot expect this type of model to produce the strongly energy-dependent polarization observed near $T_\pi=3.0$ GeV.[17] In fact, it has been shown in Ref. 9 that this polarization is most likely due to an interference between the $N(2650)$ and several other resonances. We found, however, that adding another pole produced additional improvement in the CMSR particularly in the small-t region. The need for this pole provided a pleasant surprise. Its parameters, as obtained from the fit, $\alpha(0)\simeq-0.6$ and $\alpha'(0)\simeq0.6$,

[17] D. D. Drobnis *et al.*, Phys. Rev. Letters **20**, 274 (1968); P. Bonamy, in *Proceedings of the Heidelberg International Conference on Elementary Particles*, edited by H. Filthuth (Wiley-Interscience, Inc., New York, 1968).

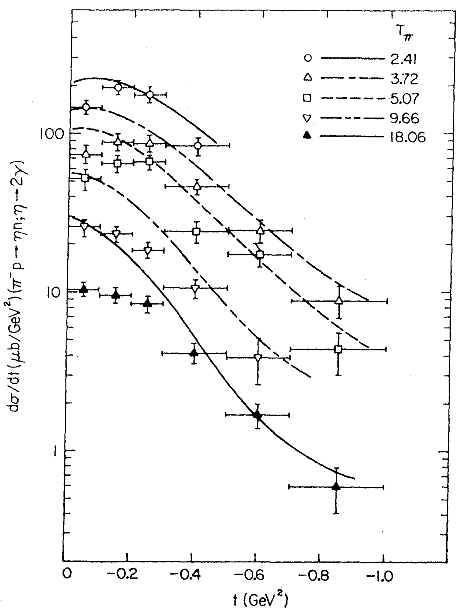

FIG. 4. Differential cross sections.

predict a scalar meson with a mass of about 1 GeV which could well correspond to the $\pi_N(1016)$.[18]

In order to study the possible effect of the existence of the π_c, we added another pole fixing its intercept at $\alpha(0)=-0.02$. We found a very slight improvement to a visual fit to the curves, but it was statistically insignificant compared to the cut and the $\pi_N(1016)$. The slope of this pole seems to coincide with the one given to the pion conspirator trajectory found in CMSR studies of pion photoproduction.[7]

In Table I, we present the parameters determined by our fits along with rough estimates of the errors in these parameters. The CMSR and the differential cross sections are presented in Figs. 3 and 4 and the polarization is given in Figs. 5 and 6.

The fit to $S_{++}(t,\gamma)$ is good except at $t=0$ for small γ. We believe that this discrepancy is due to errors in the low-energy phase-shift analysis and to the uncertainties on integrating over the unphysical region rather than to a failure in the Regge-pole model. Also, since for these values of t and γ, the left-hand side of the CMSR is just the integral of the total cross section and since this quantity has not been measured for T_π between 1.3 and 2.4 GeV, the errors committed might be quite large.

The fit to $S_{+-}(t,\gamma)$ is reasonable, although not as good as for $S_{++}(t,\gamma)$. It is possible that this could be corrected by using more complicated residue functions and/or by including higher-order absorptive corrections for the A_2 and the other trajectories, but the existing data are certainly not accurate enough to allow for the evaluation of such details. Until more low- and high-energy data (especially polarization measurements) are available it will be rather difficult to settle such points with any certainty.

The fits to the differential cross sections are reasonable except at large energies in the forward direction where the predicted cross sections are too large. We found that the differential cross sections could be fit using the same trajectories but with the residues reduced slightly. This again could be easily accounted for by assuming that the cross section in the region of T_π between 1.3 and 2.4 GeV has been overestimated.

The polarization predicted by this model agrees with the high-energy data (Fig. 5) near $t=-0.2$ and has the same general shape as a function of t (Fig. 6) as do those of Blackmon[4] and of Austin *et al.*[8]

V. CONCLUSION

We conclude with a discussion of the parameters. The slope and intercept of the A_2 are in reasonable agreement with previous work,[16] although the intercept lies somewhat lower than one would like in order to maintain exchange degeneracy with the ρ.[4,16] The residues of the A_2 are reasonably well determined. The signs of our residues are determined by the relative sign of the $pS_{11}(1550)\pi^-$ and $nS_{11}(1550)\eta$ coupling constants. According to Deans, Holladay, and Rush,[19] on the basis of SU_3 symmetry, this relative sign should be negative. The CMSR then give positive signs for the $\beta_{+\pm}{}^{A_2}$ residues. Computing the residues from the results of Blackmon,[4] we find $\beta_{++}{}^{A_2}=7.9$ and $\beta_{+-}{}^{A_2}=6.7$ which agrees with our work. This provides a check of the D/F ratio assignment of Ref. 19 and the exchange-degeneracy assumption of Ref. 4. The trajectory parameters of the π_N are reasonably well determined. The residues are small and only roughly determined. This pole, however, is more important to the fits than the small residues might seem to indicate, as can be seen from Table II.

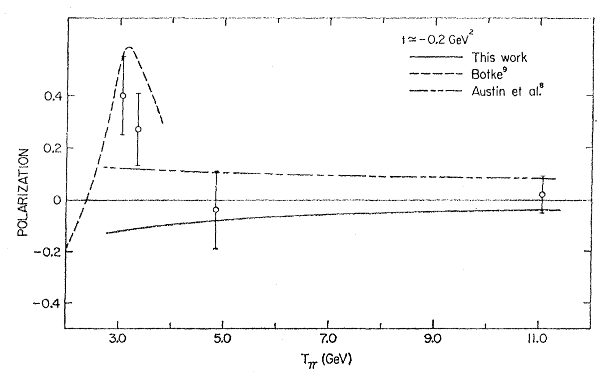

FIG. 5. Polarization for $t\simeq-0.2$ GeV as a function of energy. For comparison, the results of Austin *et al.* (Ref. 8) and Botke (Ref. 9) are also plotted.

[18] N. Barash-Schmidt, A. Barbaro-Galtieri, K. R. Price, A. H. Rosenfeld, P. Soding, C. G. Wahl, and M. Roos, Rev. Mod. Phys. **41**, 109 (1969); R. Ammar *et al.*, Phys. Rev. Letters **21**, 1832 (1968).

[19] S. R. Deans, W. G. Holladay, and J. E. Rush, University of South Florida Report (unpublished).

The existence of the π_c is statistically irrelevant to the fits. The parameters obtained here should be considered only as qualitatively determined once the trjaectory is inserted on the right-hand sides of Eqs. (22). Therefore, within the present experimental errors and the range of validity of the Regge absorptive model we cannot confirm the existence of the π_c trajectory. Of course, conspiracy relations for the π are not needed if there exist Regge cuts in π^+ photoproduction, n-p charge exchange, and similar processes.

We found the slope of the Pomeranchuk trajectory to be zero, in agreement with the model of Arnold and Blackmon.[3,4] For α_P' significantly different from zero, we were not able to obtain a reasonable fit to either the CMSR or the differential cross sections. In particular, as α_P' increased from zero, a dip appeared in the differential cross sections near $t=-0.7$, contrary to experiment. One must keep in mind, however, that this is a rather crude model and that if one were to include higher-order corrections the resulting α_P' might be different.

Because of the large experimental uncertainties in this problem, it is rather difficult to assign meaningful confidence limits to our parameters or to take very seriously the values of φ obtained in the different fits. The error estimates given in Table I represent only an educated guess of the limits based on the results of the many different fits.

Finally, although we have obtained what we think are encouraging results for the Regge absorptive model in conjunction with the CMSR, more experimental data, especially differential cross sections at intermediate energies and polarization at different values of t, are needed to assess the real value of this model. In particular, one would like to investigate the cutoff dependence of the CMSR as well as the effects of including higher-order absorptive corrections[20] and/or more complicated residue functions.

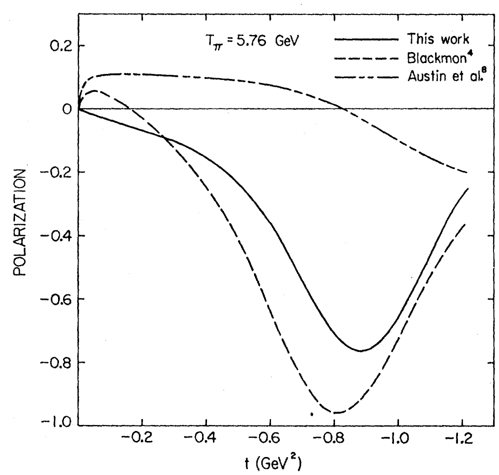

FIG. 6. Polarization for $T_\pi=5.76$ GeV as a function of t. For comparison, the results of Blackmon (Ref. 4) and Austin *et al.* (Ref. 8) are also plotted.

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[20] There is also the possibility that the next leading absorptive cut, corresponding to ρ-A_2 interference plays as important a role as the nonleading pole we have introduced in this work. We have not investigated this point in detail on the assumption that our parametrization of the Pomeranchuk trajectory corresponds to a phenomenological representation of the high-energy elastic amplitude. Inclusion of this cut will constitute indeed a first refinement of the model.